The Electrostatic Equations

If we consider the **static** case (i.e., constant with time) of Maxwell's Equations, we find that the **time derivatives** of the electric field and magnetic flux density are **zero**:

$$\frac{\partial \mathbf{B}(\bar{r},t)}{\partial t}=0 \qquad \text{and} \qquad \frac{\partial \mathbf{E}(\bar{r},t)}{\partial t}=0$$

Thus, Maxwell's equations for **static fields** become:

$$\nabla \times \mathbf{E}(\bar{r})=0$$

$$\nabla \cdot \mathbf{E}(\bar{r})=\frac{\rho_v(\bar{r})}{\varepsilon_0}$$

$$\nabla \times \mathbf{B}(\bar{r})=\mu_0 \mathbf{J}(\bar{r})$$

$$\nabla \cdot \mathbf{B}(\bar{r})=0$$

Look at what has happened! For the static case (but **just** for the static case!), Maxwell's equations "**decouple**" into two **independent** sets of two equations.

The first set involves electric field $\mathbf{E}(\bar{r})$ and charge density $\rho_v(\bar{r})$ only. These are called the **electrostatic equations** in **free-space**:

$$\nabla x \mathbf{E}(\bar{r}) = 0$$

$$\nabla \cdot \mathbf{E}(\bar{r}) = \frac{\rho_v(\bar{r})}{\varepsilon_0}$$

These are the **electrostatic equations** for free space (i.e., a vacuum).

Note that the **static** electric field is a **conservative** vector field (do you see why ?)!

This of course means that **everything** we know about a conservative field is true **also** for the **static** field $\mathbf{E}(\bar{r})$.

Essentially, **this** is what the electrostatic equations **tell** us:

1) The **static** electric field is **conservative**.

2) The **source** of the static field is **charge**:

$$\nabla \cdot \mathbf{E}(\bar{r}) = \frac{\rho_v(\bar{r})}{\varepsilon_0}$$

In other words, the static electric field $\mathbf{E}(\bar{r})$ **diverges** from (or **converges** to) charge!

Chapters 4, 5, and 6 deal only with **electrostatics** (i.e., static electric fields produced by static charge densities).

In chapters 7, 8, and 9, we will study **magnetostatics**, which considers the **other** set of static differential equations:

$$\nabla \times \mathbf{B}(\bar{r}) = \mu_0 \mathbf{J}(\bar{r})$$

$$\nabla \cdot \mathbf{B}(\bar{r}) = 0$$

These equations are called the **magnetostatic equations** in free-space, and relate the static **magnetic flux density** $\mathbf{B}(\bar{r})$ to the static **current density** $\mathbf{J}(\bar{r})$.