4-3 Coulomb's Law

Reading Assignment: *pp. 90-93*

Now, let' **determine** the electric field $\mathbf{E}(\bar{r})$ "produced" by charge density $\rho_v(\bar{r})$!

Q:

A: HO: Coulomb's Law

Q:

A: HO: Coulomb's Law for Charge Distributions

Coulomb's Law

Recall from **Coulomb's Law of Force** that a charge Q_2 located at point $\bar{r}_2$ applies a force $\mathbf{F}_1$ on charge Q_1 (located at point $\bar{r}_1$):

$$\mathbf{F}_1 = \frac{1}{4\pi\varepsilon_0} \frac{Q_1 Q_2}{R^2} \hat{a}_{21} = \frac{Q_1 Q_2}{4\pi\varepsilon_0} \frac{\bar{r}_1 - \bar{r}_2}{\left|\bar{r}_1 - \bar{r}_2\right|^3}$$

Likewise, from the **Lorentz Force Law**, we know that the force $\mathbf{F}_1$ on a charge Q_1 located at point $\bar{r}_1$ is attributed to an **electric field** located at $\bar{r}_1$:

$$\mathbf{F}_1 = Q_1 \mathbf{E}(\bar{r}_1) \quad \Rightarrow \quad \mathbf{E}(\bar{r}_1) = \frac{\mathbf{F}_1}{Q_1}$$

Inserting Coulomb's Law of Force into this equation, we get the electric field at location $\bar{r}_1$, generated by charge Q_2 located at $\bar{r}_2$!

$$\mathbf{E}(\bar{r}) = \frac{\mathbf{F}_1}{Q_1} = \frac{Q_2}{4\pi\varepsilon_0} \frac{\hat{a}_{21}}{R^2}$$

In general, we can say the electric field $\mathbf{E}(\bar{r})$ at location $\bar{r}$, generated by a charge Q at point $\bar{r}'$, is:

$$\mathbf{E}(\bar{r}) = \frac{Q}{4\pi\varepsilon_0}\frac{\hat{a}_R}{R^2} = \frac{Q}{4\pi\varepsilon_0}\frac{\bar{r}-\bar{r}'}{\left|\bar{r}-\bar{r}'\right|^3}$$

This is **Coulomb's Law** !! It describes the electric field $\mathbf{E}(\bar{r})$ at location $\bar{r}$ that is created by a charge Q at location $\bar{r}'$.

Note that:

$$\hat{a}_R \doteq \frac{\bar{r} - \bar{r}'}{\left|\bar{r} - \bar{r}'\right|}$$

Therefore, **if** the charge Q is at the **origin** (i.e., $\bar{r}' = 0$), then:

$$\hat{a}_R = \frac{\bar{r}}{\left|\bar{r}\right|} = \hat{a}_r$$

Recall that the base vector $\hat{a}_r$ always **points away** from the origin. In other words, a charge located at the origin creates an electric field vector that points in the direction of base vector $\hat{a}_r$ (i.e., **away from the origin**) at all points $\bar{r}$!

Likewise, **if** the charge is at the origin, then:

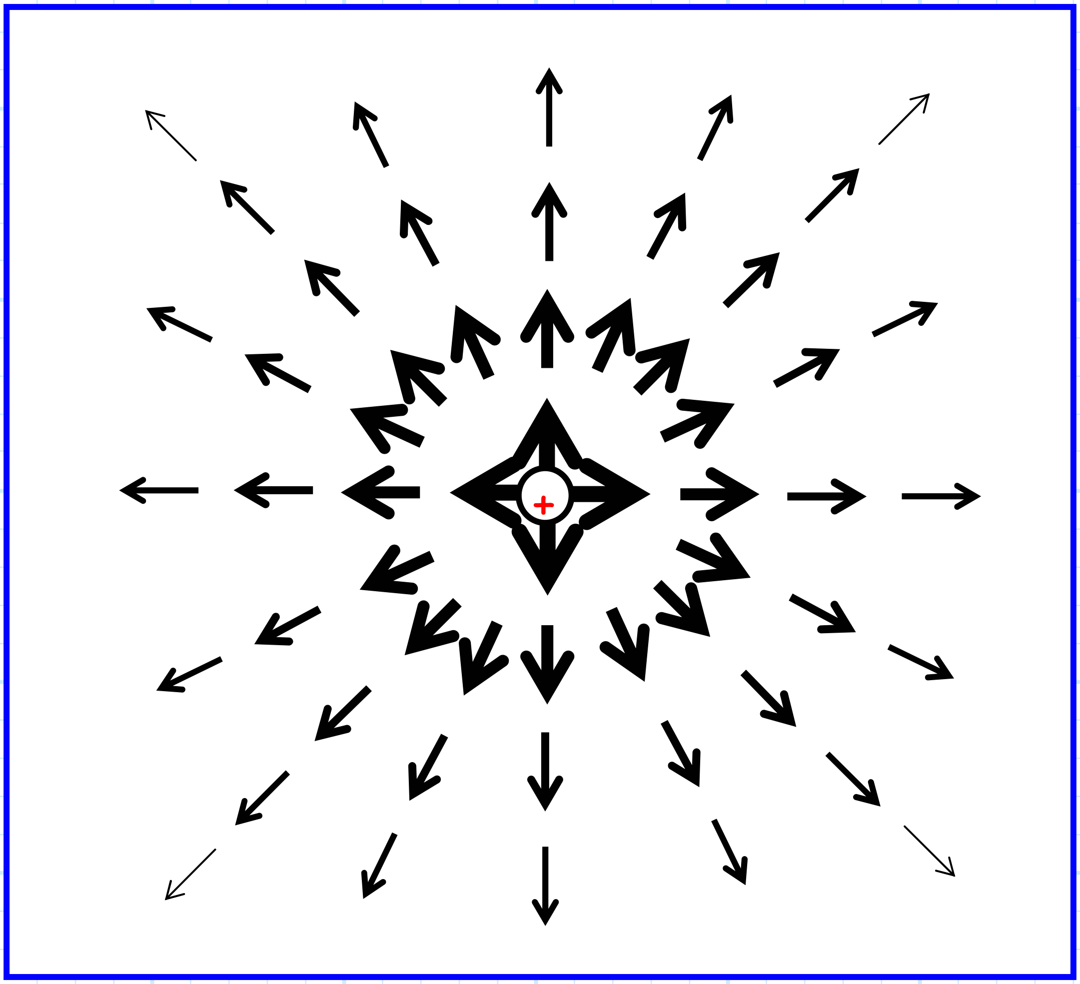
$$R = \left|\bar{r}\right| = r$$

In other words, the **magnitude** of the electric field vector is **proportional** to $1/r^2$. As a result, the magnitude of the electric field is dependent on its distance from the origin (i.e., distance from the charge). Therefore, **if** $\bar{r}' = 0$:

$$\mathbf{E}(\bar{r}) = \frac{Q}{4\pi\varepsilon_0} \frac{\hat{a}_r}{r^2} = \frac{Q}{4\pi\varepsilon_0} \frac{\bar{r}}{r^3}$$

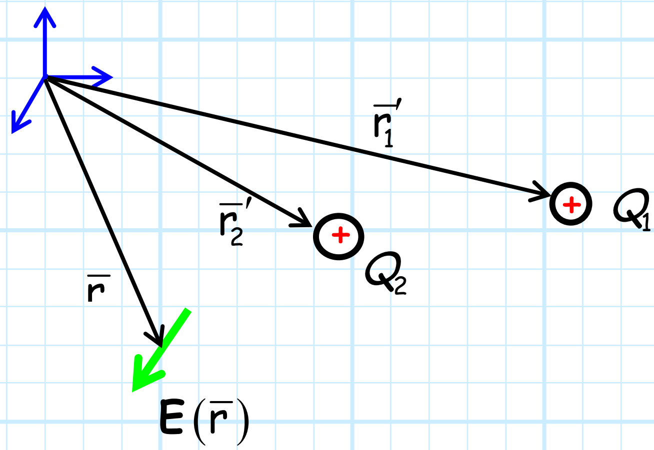
Q: *What is the **curl** of* $\mathbf{E}(\bar{r})$ *??*

A: $\nabla \times \mathbf{E}(\bar{r}) =$



Coulomb's Law for Charge Density

Consider the case where there are **multiple** point charges present. What is the resulting **electrostatic field** ?



The electric field produced by the charges is simply the **vector** sum of the electric field produced by each (i.e., **superposition**!):

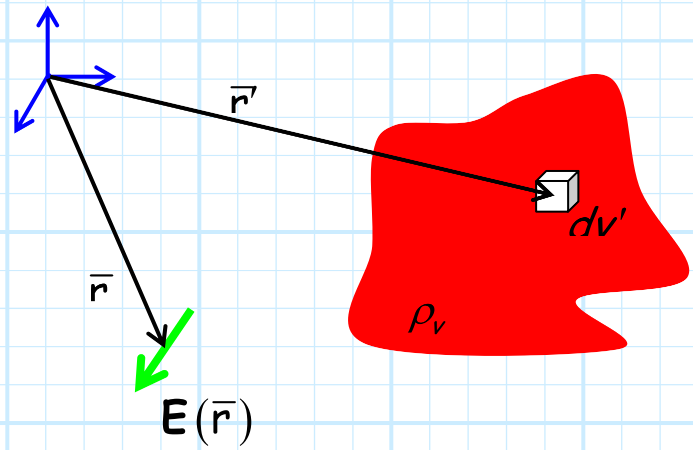
$$\mathbf{E}(\bar{r}) = \frac{Q_1}{4\pi\varepsilon_0}\frac{\bar{r}-\bar{r}_1'}{\left|\bar{r}-\bar{r}_1'\right|^3} + \frac{Q_2}{4\pi\varepsilon_0}\frac{\bar{r}-\bar{r}_2'}{\left|\bar{r}-\bar{r}_2'\right|^3}$$

Or, more generally, for N point charges:

$$\mathbf{E}(\bar{r}) = \sum_{n=1}^{N}\frac{Q_n}{4\pi\varepsilon_0}\frac{\bar{r}-\bar{r}_n'}{\left|\bar{r}-\bar{r}_n'\right|^3}$$

Consider now a volume V that is filled with a "cloud" of charge, described by **volume charge density** $\rho_v(\bar{r})$.

A very small differential volume *dv*, located at point $\bar{r}'$, will thus contain charge $dQ = \rho_v(\bar{r}')\,dv'$.



This differential charge produces an electric field at point $\bar{r}$ equal to :

$$\mathbf{dE}(\bar{r}) = \frac{\rho_v(\bar{r}')\,dv'}{4\pi\varepsilon_0}\,\frac{\bar{r}-\bar{r}'}{|\bar{r}-\bar{r}'|^3}$$

The **total** electric field at $\bar{r}$ (i.e., $\mathbf{E}(\bar{r})$) is the summation (i.e., **integration**) of all the electric field vectors produced by all the little differential charges *dQ* that make up the charge cloud:

$$\mathbf{E}(\bar{r}) = \iiint_V \frac{\rho_v(\bar{r}')}{4\pi\varepsilon_0}\,\frac{\bar{r}-\bar{r}'}{|\bar{r}-\bar{r}'|^3}\,dv'$$

Note: The variables of integration are the **primed** coordinates, representing the locations of the charges (i.e., **sources**).

Similarly, we can show that for **surface** charge:

$$\mathbf{E}(\bar{\mathbf{r}}) = \iint_S \frac{\rho_s(\bar{\mathbf{r}}')}{4\pi\varepsilon_0} \frac{\bar{\mathbf{r}}-\bar{\mathbf{r}}'}{\left|\bar{\mathbf{r}}-\bar{\mathbf{r}}'\right|^3} \, ds'$$

And for **line** charge:

$$\mathbf{E}(\bar{\mathbf{r}}) = \int_C \frac{\rho_\ell(\bar{\mathbf{r}}')}{4\pi\varepsilon_0} \frac{\bar{\mathbf{r}}-\bar{\mathbf{r}}'}{\left|\bar{\mathbf{r}}-\bar{\mathbf{r}}'\right|^3} \, d\ell'$$