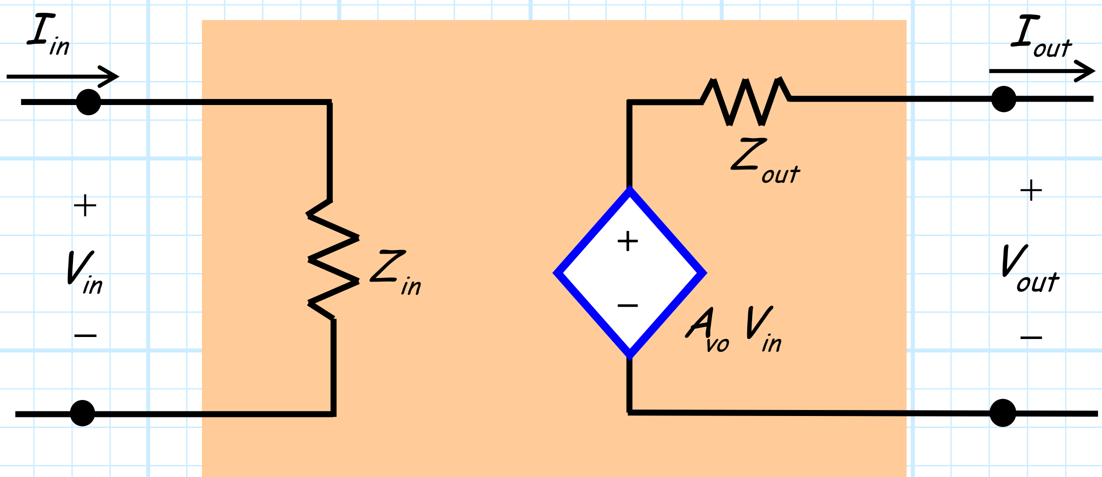
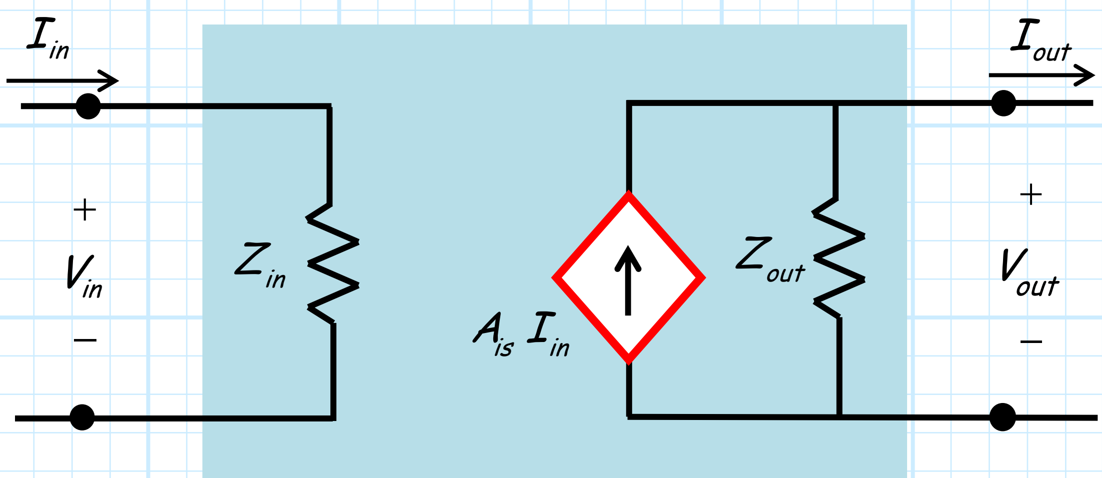
Circuit Models for Amplifiers

The two most important amplifier circuit models explicitly use the **open-circuit voltage gain** A_{vo}:



And the **short-circuit current gain** A_{is}:



Just three values describe all!

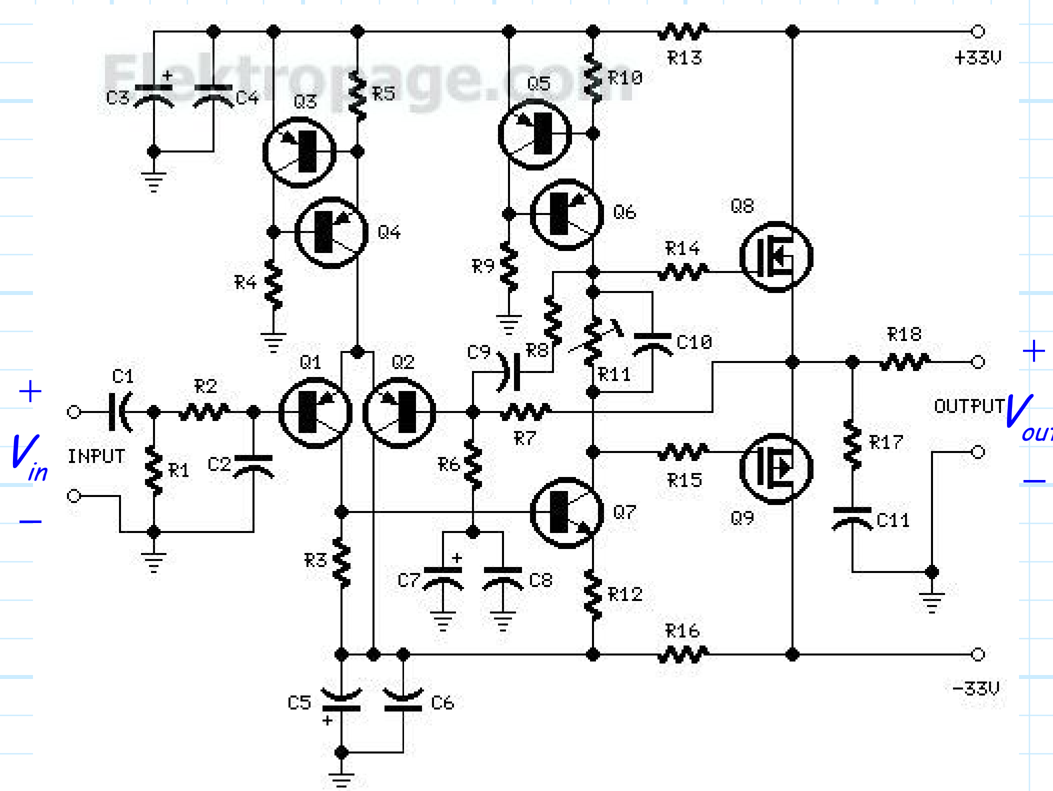
In addition, each equivalent circuit model uses the **same** two **impedance** values—the **input** impedance Z_{in} and **output** impedance Z_{out} .

Q: *So what are these models* ***good for?***

A: Say we wish to analyze a circuit in which an amplifier is but **one** component.

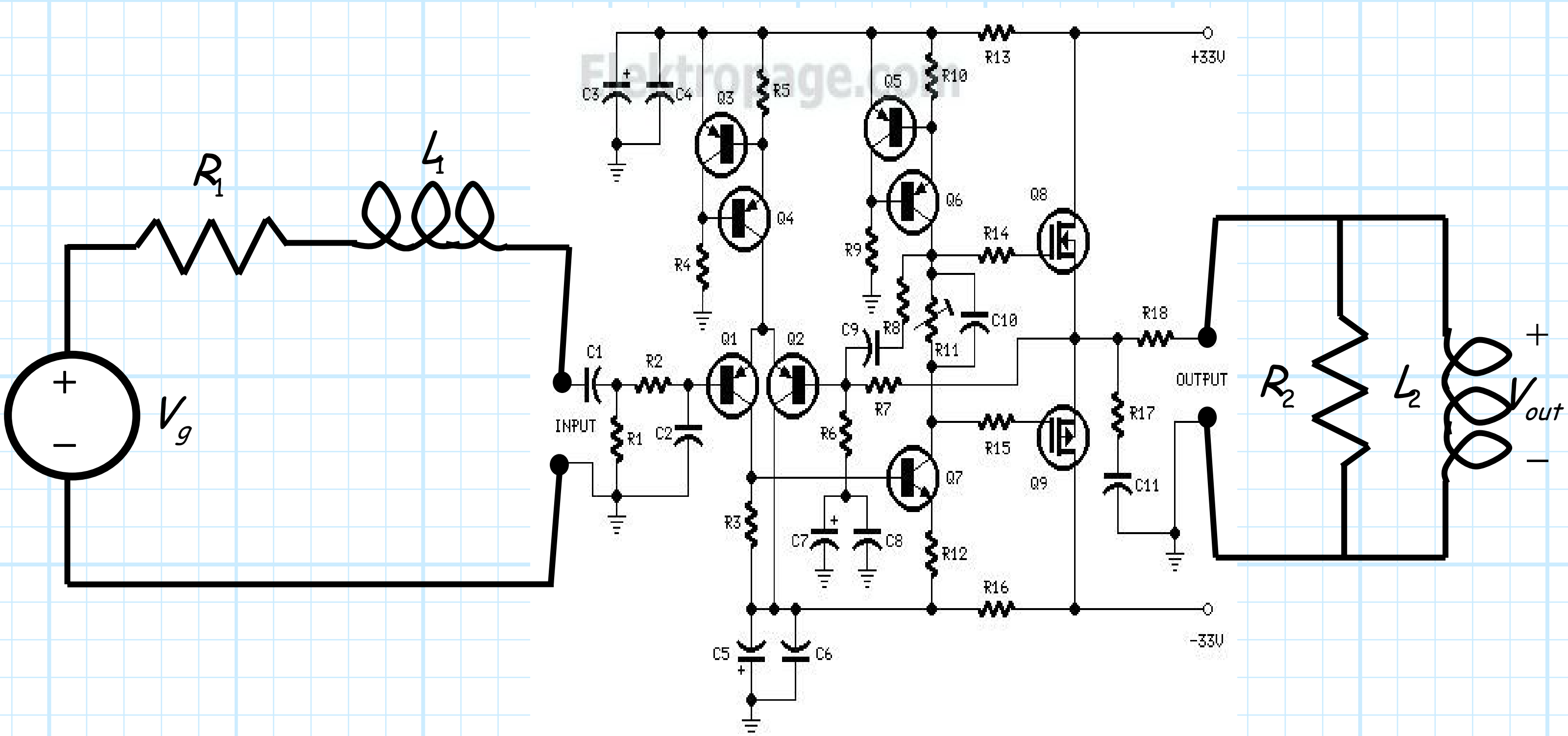
Instead of needing to analyze the **entire** amplifier circuit, we can analyze the circuit using the (far) **simpler** equivalent circuit model.

For **example**, consider **this** audio amplifier design:



This might be on the final

Say we wish to connect a **source** (e.g., microphone) to its **input**, and a **load** (e.g., speaker) to its **output**:



Let's say on the **EECS 412 final**, I ask **you** to determine V_{out} in the circuit above.

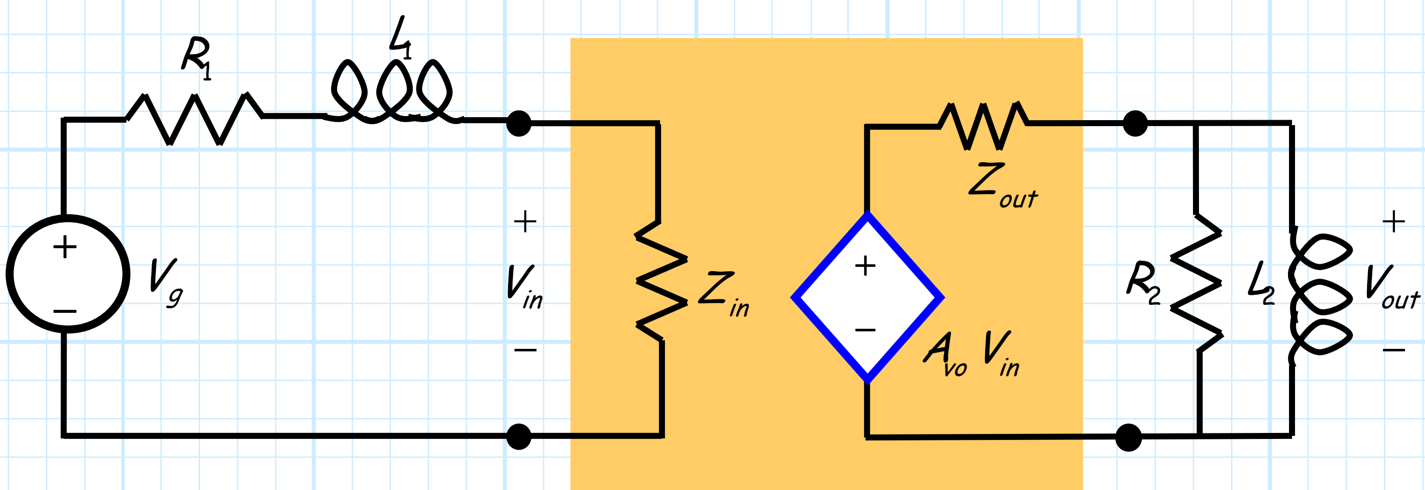
I'm not *quite* the jerk I appear to be!

Q: *Yikes! How could we **possibly** analyze this circuit on an exam—it would take way **too** much time (not to mention way **too** many pages of work)?*

A: Perhaps, but let's say that I also provide you with the amplifier **input impedance** Z_{in}, **output impedance** Z_{out}, and **open-circuit voltage gain** A_{vo}.

You thus know **everything** there is to know about the amplifier!

Just replace the amplifier with its **equivalent circuit**:



The relationship between input and output voltages

From **input** circuit, we can conclude (with a little help from voltage division):

$$V_{in} = V_g \left(\frac{Z_{in}}{R_1 + j\omega L_1 + Z_{in}} \right)$$

And the **output** circuit is likewise:

$$V_{out} = A_{vo} V_{in} \left(\frac{R_2 \| j\omega L_2}{Z_{out} + R_2 \| j\omega L_2} \right)$$

where:

$$R_2 \| j\omega L_2 = \frac{j\omega R_2 L_2}{R_2 + j\omega L_2}$$

The output is not open-circuited!

Q: *Wait! I **thought** we could determine the output voltage from the input voltage by simply multiplying by the voltage gain A_{vo}. I am **certain** that you told us:*

$$V_{out}^{oc} = A_{vo} V_{in}$$

A: I **did** tell you that! And this expression is **exactly correct**.

However, the voltage V_{out}^{oc} is the **open-circuit** output voltage of the amplifier—in **this** circuit (like most amplifier circuits!), the output is **not open**!

Hence $V_{out} \neq V_{out}^{oc}$, and so :

$$\begin{aligned} V_{out} &= A_{vo} V_{in} \left(\frac{R_2 \| j\omega L_2}{Z_{out} + R_2 \| j\omega L_2} \right) \\ &= V_{out}^{oc} \left(\frac{R_2 \| j\omega L_2}{Z_{out} + R_2 \| j\omega L_2} \right) \\ &\neq V_{out}^{oc} \end{aligned}$$

We can define a voltage gain

Now, combining the two expressions, we have our **answer**:

$$V_{out} = V_g\, A_{vo} \left(\frac{Z_{in}}{R_1 + j\omega L_1 + Z_{in}} \right) \left(\frac{R_2 \| j\omega L_2}{Z_{out} + R_2 \| j\omega L_2} \right)$$

$$= A_{vo} V_g \left(\frac{Z_{in}}{R_1 + j\omega L_1 + Z_{in}} \right) \left(\frac{j\omega R_2 L_2}{Z_{out} \left(R_2 + j\omega L_2 \right) + j\omega R_2 L_2} \right)$$

Now, be aware that we can (and often do!) **define** a voltage gain A_v, a value that is different from the **open-circuit** voltage gain of the **amplifier**.

For instance, in the above circuit example we could **define** a voltage gain as the ratio of the input voltage V_{in} and the output voltage V_{out}:

$$A_v \doteq \frac{V_{out}}{V_{in}} = A_{vo} \left(\frac{R_2 \| j\omega L_2}{Z_{out} + R_2 \| j\omega L_2} \right) = A_{vo} \left(\frac{j\omega R_2 L_2}{Z_{out} \left(R_2 + j\omega L_2 \right) + j\omega R_2 L_2} \right)$$

Or we can define a different gain

Or, we could alternatively **define** voltage gain as the ratio of the source voltage V_g and the output voltage V_{out}:

$$A_v \doteq \frac{V_{out}}{V_g} = A_{vo}\left(\frac{Z_{in}}{R_1 + j\omega L_1 + Z_{in}}\right)\left(\frac{j\omega R_2 L_2}{Z_{out}\left(R_2 + j\omega L_2\right) + j\omega R_2 L_2}\right)$$

Q: *Yikes! **Which** result is correct; which voltage gain is **"the"** voltage gain?*

A: Both are!

We can **define** a voltage gain A_v in **any** manner that is **useful** to us. However, we must make this definition explicit—**precisely** what two voltages are involved in the definition?

➔ **No** voltage gain A_v is **"the"** voltage gain!

Note that the open-circuit voltage gain A_{vo} is a parameter of the **amplifier**—and of the amplifier **only**!

The open-circuit gain is the *amplifier* gain

Contrast A_{vo} to the two voltage gains defined above (i.e., V_{out}/V_{in} and V_{out}/V_g).

In each case, the result—of course—depends on **amplifier** parameters $(A_{vo}, Z_{in}, Z_{out})$.

However, the results **likewise** depend on the devices (source and load) **attached** to the amplifier (e.g., L_1, R_1, L_2, R_2).

→ The only **amplifier** voltage gain is its **open-circuit** voltage gain A_{vo}!

The low-frequency model

Now, let's switch gears and consider **low-frequency** (e.g., audio and video) applications.

At these frequencies, parasitic elements are typically **too small** to have any practical significance.

Additionally, low-frequency circuits **frequently** employ **no** reactive circuit elements (no capacitor or inductors).

As a result, we find that the input and output **impedances** exhibit almost **no imaginary** (i.e., reactive) components:

$$Z_{in}(\omega) \cong R_{in} + j\,0$$

$$Z_{out}(\omega) \cong R_{out} + j\,0$$

We can express this in the time domain

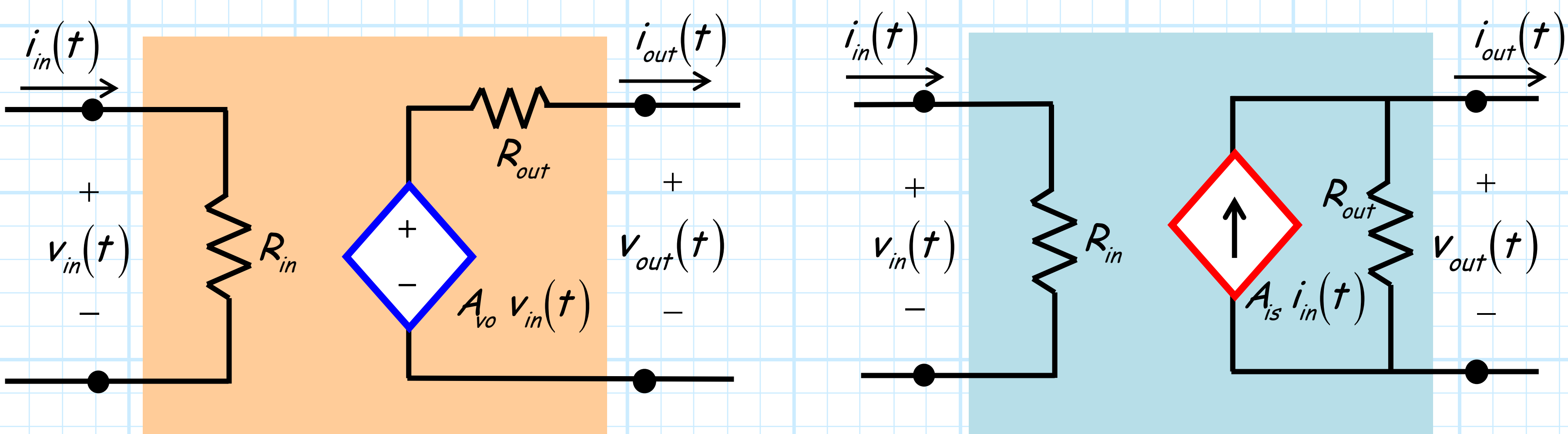
Likewise, the voltage and current **gains** of the amplifier are (almost) purely **real**:

$$A_{vo}(\omega) \cong A_{vo} + j0$$

$$A_{is}(\omega) \cong A_{is} + j0$$

Note that these real values can be **positive** or **negative**.

The amplifier **circuit models** can thus be **simplified**—to the point that we can easily consider arbitrary **time-domain** signals (e.g., $v_{in}(t)$ or $i_{out}(t)$):



All real-valued

For this case, we find that the (approximate) relationships between the input and output are that of an **ideal** amplifier:

$$v_{out}^{oc}(t) = \int_{-\infty}^{t} A_{vo}\,\delta(t-t')\,v_{in}(t') = A_{vo}\;v_{in}(t)$$

$$i_{out}^{sc}(t) = \int_{-\infty}^{t} A_{is}\,\delta(t-t')\,i_{in}(t') = A_{is}\;i_{in}(t)$$

Specifically, we find that for these low-frequency **models**:

$$R_{in} = \frac{v_{in}(t)}{i_{in}(t)} \qquad R_{out} = \frac{v_{out}^{oc}(t)}{i_{out}^{sc}(t)}$$

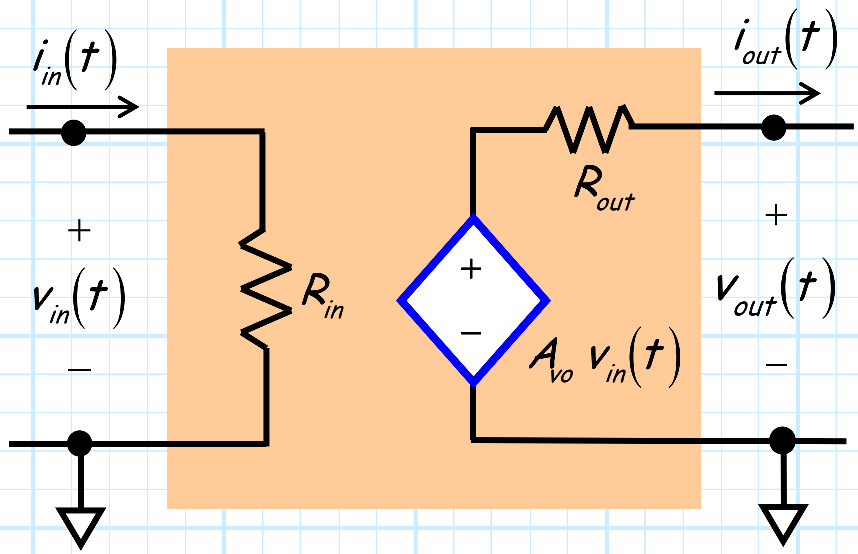
$$A_{vo} = \frac{v_{out}^{oc}(t)}{v_{in}(t)} \qquad A_{is} = \frac{i_{out}^{sc}(t)}{i_{in}(t)}$$

One important **caveat** here; this "low-frequency" model is applicable only for **input signals** that are **likewise** low-frequency—the input signal spectrum must **not** extend beyond the amplifier **bandwidth**.

Voltage is referenced to ground potential

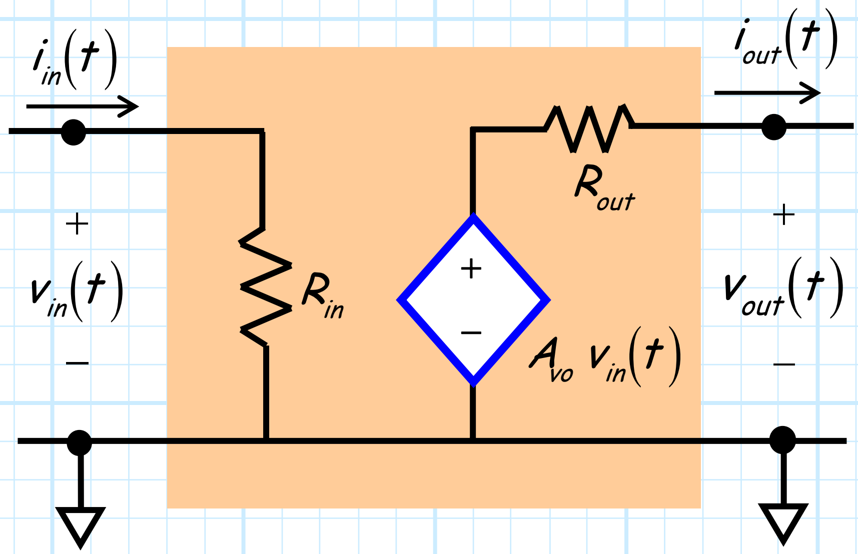
Now one **last topic**.

Frequently, both the input and output **voltages** are expressed with respect to **ground potential**, a situation expressed in the circuit **model** as:



You'll often see this notation

Now, two nodes at ground potential are two nodes that are **connected** together! Thus, an **equivalent** model to the one above is:



Which is **generally** simplified to this model: