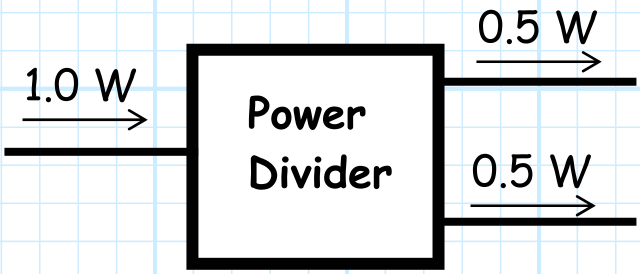
G. Power Dividers and Couplers

One of the most fundamental problems in microwave engineering is how to **divide** signal power.

This simplest microwave problem would **seemingly** be to divide a signal power in two:



This divider would seemingly need to be **matched** and **lossless**, and would likely be reciprocal.

However, building this **3-port** device is more difficult than you might think!

HO: The 3-Port Coupler

Q: *Is a matched, lossless, reciprocal 4-port coupler a possiblity?*

A: HO: The 4-Port Coupler

We will study **two** popular microwave couplers:

a) HO: The Wilkinson Power Divider

b) HO: The Directional Coupler

HO: The Directional Coupler Spec Sheet

Two **other** couplers that are very useful are the **quadrature** hybrid coupler and the **180-degree** hybrid coupler. I'm providing **these** handouts as purely informational material—this **won't** appear on the exam!

c) HO: The Quadrature Hybrid Coupler

d) HO: The 180-Degree Hybrid Coupler

The 3-Port Coupler

Say we desire a **matched** and **lossless** 3-port Coupler. Such a device would have a scattering matrix :

$$\mathcal{S} = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$$

Assuming the device is passive and made of simple (isotropic) materials, the device will be **reciprocal,** so that:

$$S_{21} = S_{12} \qquad S_{31} = S_{13} \qquad S_{23} = S_{32}$$

Likewise, if it is **matched,** we know that:

$$S_{11} = S_{22} = S_{33} = 0$$

As a result, a **lossless**, **reciprocal** coupler would have a scattering matrix of the form:

$$\mathcal{S} = \begin{bmatrix} 0 & S_{21} & S_{31} \\ S_{21} & 0 & S_{32} \\ S_{31} & S_{32} & 0 \end{bmatrix}$$

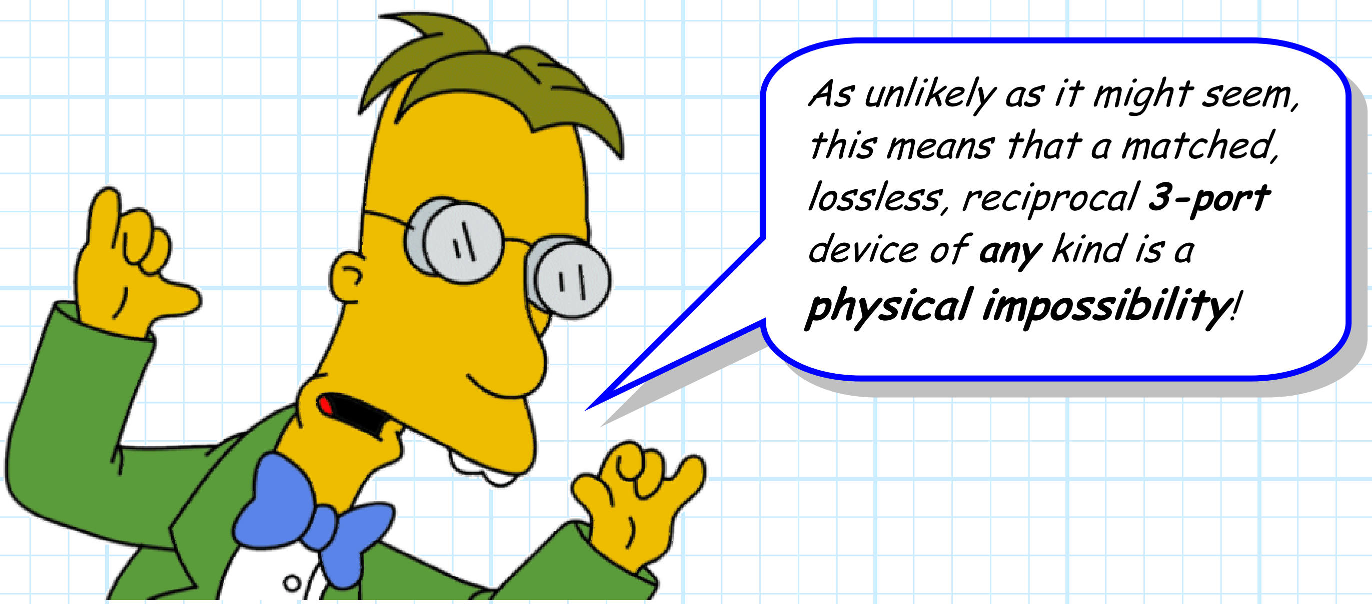
Just **3** non-zero scattering parameters define the **entire** matrix!

Likewise, if we wish for this coupler to be **lossless**, the scattering matrix must be **unitary**, and therefore:

$$|S_{21}|^2 + |S_{31}|^2 = 1 \qquad S_{31}^* S_{32} = 0$$
$$|S_{21}|^2 + |S_{32}|^2 = 1 \qquad S_{21}^* S_{32} = 0$$
$$|S_{31}|^2 + |S_{32}|^2 = 1 \qquad S_{21}^* S_{31} = 0$$

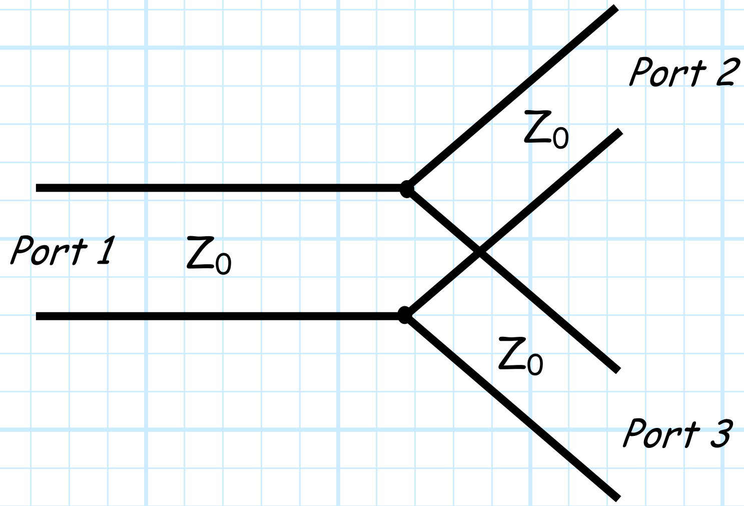
Since each complex value S is represented by **two real numbers** (i.e., real and imaginary parts), the equations above result in **9** real equations. The problem is, the 3 complex values S_{21}, S_{31} and S_{32} are represented by only **6** real unknowns.

We have **over constrained** our problem ! There are **no solutions** to these equations !



For example, the following 3 port coupler is lossless, but not matched:

$$\mathcal{S} = \begin{bmatrix} -1/3 & 2/3 & 2/3 \\ 2/3 & -1/3 & 2/3 \\ 2/3 & 2/3 & -1/3 \end{bmatrix}$$



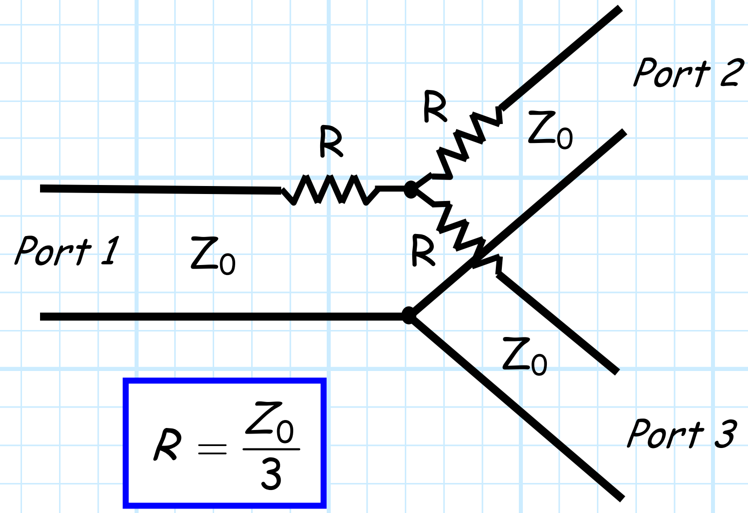
Since:

$$S_{11} = S_{22} = S_{33} = -1/3 \neq 0$$

the coupler is **not matched**! However, the matrix is unitary, and therefore this design **is lossless**.

Alternatively, we might try **this** 3-port device:

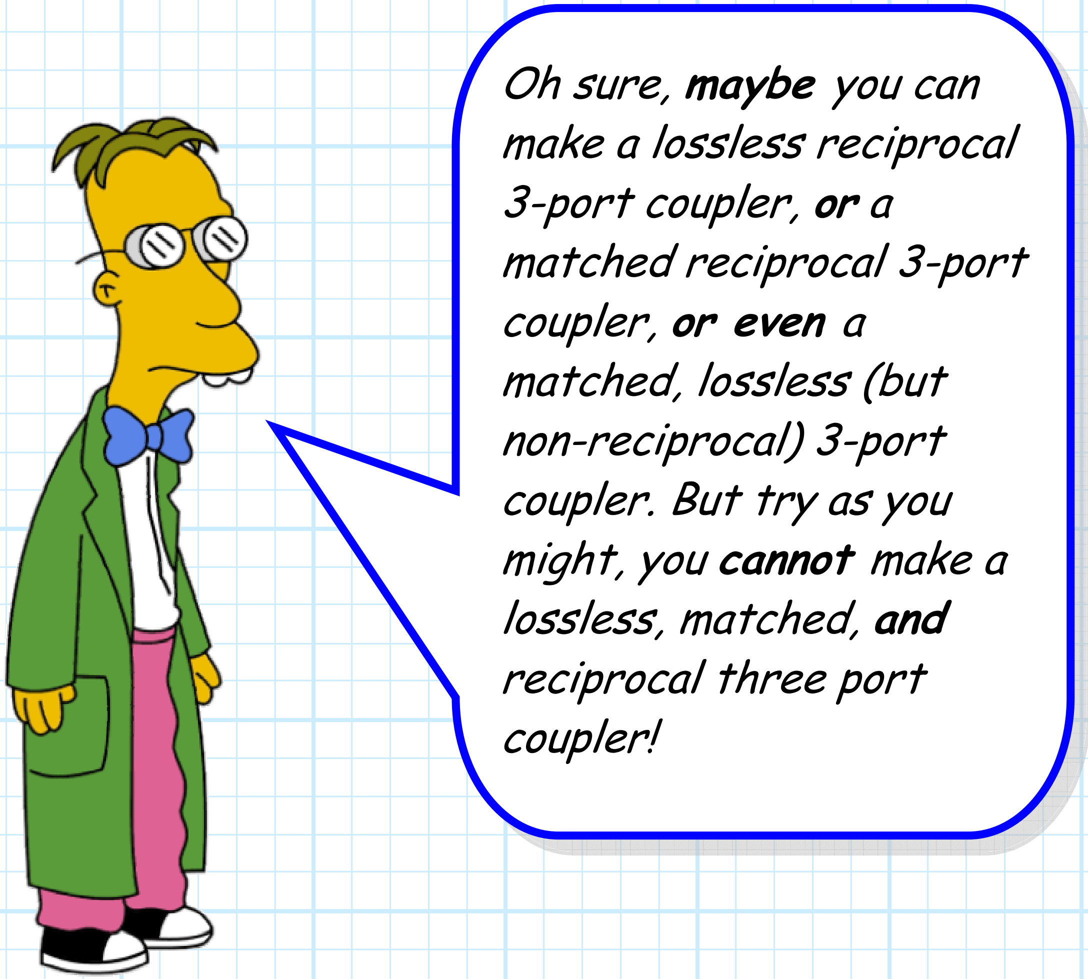
$$\mathcal{S} = \begin{bmatrix} 0 & 3/5 & 3/5 \\ 3/5 & 0 & 3/5 \\ 3/5 & 3/5 & 0 \end{bmatrix}$$



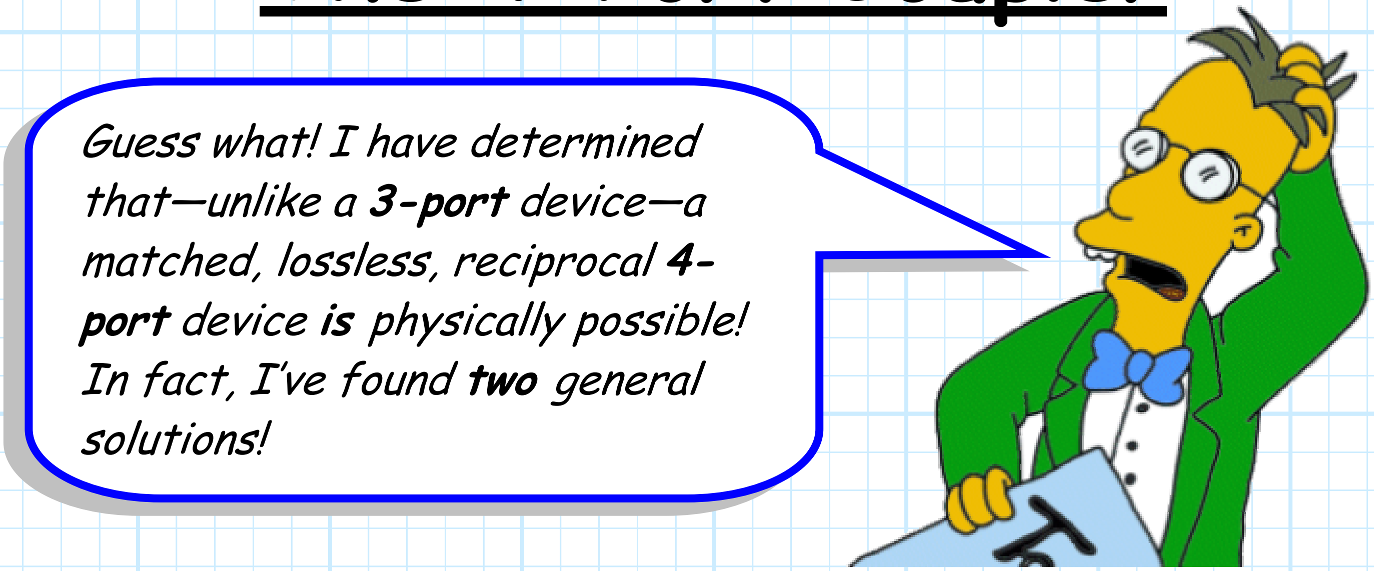
$$R = \frac{Z_0}{3}$$

For this design, the ports **are matched**! However, the resistors make the device **lossy**:

$$|S_{11}|^2 + |S_{21}|^2 + |S_{31}|^2 = 0 + \frac{9}{25} + \frac{9}{25} = \frac{18}{25} < 1$$



The 4-Port Coupler



The first solution is referred to as the **symmetric** solution:

$$\mathcal{S} = \begin{bmatrix} 0 & \alpha & j\beta & 0 \\ \alpha & 0 & 0 & j\beta \\ j\beta & 0 & 0 & \alpha \\ 0 & j\beta & \alpha & 0 \end{bmatrix}$$

Note for this symmetric solution, every row and every column of the scattering matrix has the **same** four values (i.e., α, $j\beta$, and two zeros)!

The second solution is referred to as the **anti-symmetric** solution:

$$\mathcal{S} = \begin{bmatrix} 0 & \alpha & \beta & 0 \\ \alpha & 0 & 0 & -\beta \\ \beta & 0 & 0 & \alpha \\ 0 & -\beta & \alpha & 0 \end{bmatrix}$$

Note that for this anti-symmetric solution, **two** rows and **two** columns have the same four values (i.e., α, β, and two zeros), while the **other** two row and columns have (slightly) **different** values (α, $-\beta$, and two zeros)

It is **quite** evident that each of these solutions are **matched** and **reciprocal**. However, to ensure that the solutions are indeed **lossless**, we must place an **additional** constraint on the values of α, β. Recall that a **necessary** condition for a lossless device is:

$$\sum_{m=1}^{N} \left| S_{mn} \right|^2 = 1 \quad \text{for all } n$$

Applying this to the **symmetric** case, we find:

$$|\alpha|^2 + |\beta|^2 = 1$$

Likewise, for the **anti-symmetric** case, we also get

$$|\alpha|^2 + |\beta|^2 = 1$$

It is evident that if the scattering matrix is **unitary** (i.e., lossless), the values α and β **cannot** be independent, but must **related** as:

$$|\alpha|^2 + |\beta|^2 = 1$$

Generally speaking, we will find that $|\alpha| \geq |\beta|$. Given the constraint on these two values, we can thus conclude that:

$$0 \leq |\beta| \leq \frac{1}{\sqrt{2}} \quad \text{and} \quad \frac{1}{\sqrt{2}} \leq |\alpha| \leq 1$$

The Wilkinson Power Divider

The **Wilkinson power divider** is a 3-port device with a scattering matrix of:

$$\mathcal{S} = \begin{bmatrix} 0 & -j/\sqrt{2} & -j/\sqrt{2} \\ -j/\sqrt{2} & 0 & 0 \\ -j/\sqrt{2} & 0 & 0 \end{bmatrix}$$

Note this device is **matched**, but it is **lossy.** What makes this device interesting is the behavior of **port 1** (i.e., column 1).

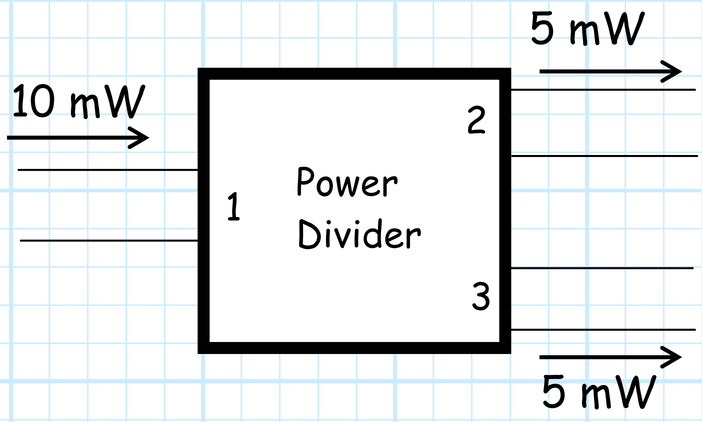
Say that a signal (P_1^+) is incident on port 1 **only**. Provided that all ports are all terminated in matched loads, we find of course that **no power** is reflected at port 1 :

$$P_1^- = |S_{11}|^2 P_1^+ = 0$$

Instead, all the incident power is **evenly divided** between the outputs of port 2 and port 3:

$$P_2^- = |S_{21}|^2 P_1^+ = \frac{P_1^+}{2} \qquad P_3^- = |S_{31}|^2 P_1^+ = \frac{P_1^+}{2}$$

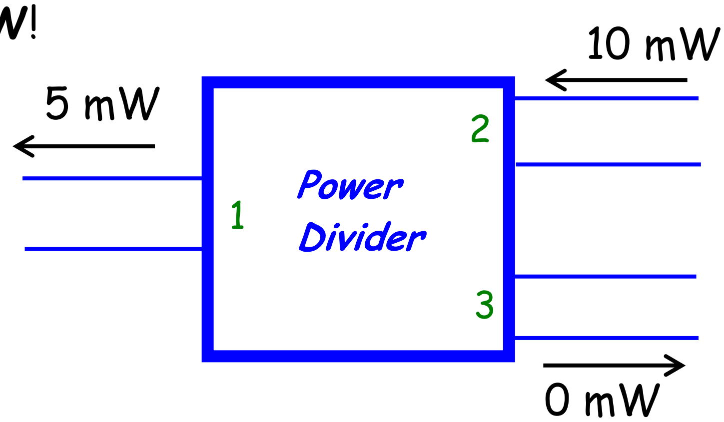
In other words, if 10 mW of signal power flows **into** port 1, then 5 mW will flow **out of** ports 2 and 3.



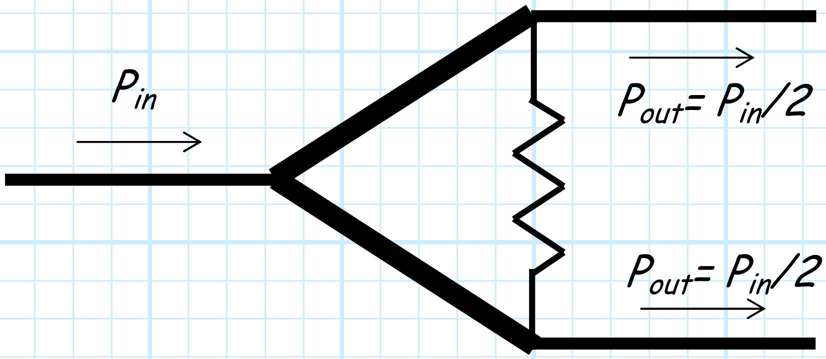
Q: *Hey! This device appears to be **lossless**! I thought you said it is **lossy**?*

A: Yes, from the standpoint of port 1, it **does** appear to be lossless. That is why the Wilkinson power divider is **so useful**.

However, the device is **clearly** lossy, as if we put 10 mW in either port 2 or port 3, then 5 mW will leave port 1, but **no power** will leave the other port—we've **lost 5mW**!



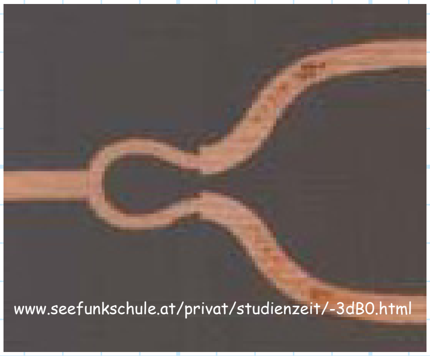
The **Wilkinson power divider** is therefore a useful device for **dividing** signal power into two **equal** parts. **No** power is lost, either due to **reflection** at the input port or **absorption** by the device!



The Wilkinson Power Divider

We often refer to this device as a **3 dB power divider**, as:

$$10 \log_{10}\left[\frac{P_{out}}{P_{in}}\right] = 10 \log_{10}\left[\frac{1}{2}\right] = -3 \text{ dB}$$



The Directional Coupler

A lossless, reciprocal, matched 4-port **directional** coupler will have a scattering matrix of the form:

$$\mathcal{S} = \begin{bmatrix} 0 & \alpha & j\beta & 0 \\ \alpha & 0 & 0 & j\beta \\ j\beta & 0 & 0 & \alpha \\ 0 & j\beta & \alpha & 0 \end{bmatrix}$$

This ideal coupler is **completely** characterized by the **coupling coefficient** c, where we find:

$$\mathcal{S} = \begin{bmatrix} 0 & \sqrt{1-c^2} & jc & 0 \\ \sqrt{1-c^2} & 0 & 0 & jc \\ jc & 0 & 0 & \sqrt{1-c^2} \\ 0 & jc & \sqrt{1-c^2} & 0 \end{bmatrix}$$

In other words:

$$\beta = c \qquad \text{and} \qquad \alpha = \sqrt{1-\beta^2} = \sqrt{1-c^2}$$

Additionally, for a directional coupler, the coupling coefficient c will be less than $1/\sqrt{2}$ **always**. Therefore, we find that:

$$0 \le c \le \frac{1}{\sqrt{2}} \qquad \text{and} \qquad \frac{1}{\sqrt{2}} \le \sqrt{1-c^2} \le 1$$

Lets see what this means in terms of the **physical behavior** of a directional coupler. First, consider the case where some signal is incident on **port 1**, with power P_1^+.

If all other ports are matched, we find that the power flowing out of **port 1** is:

$$P_1^- = |S_{11}|^2 P_1^+ = 0$$

while the power out of **port 2** is:

$$P_2^- = |S_{21}|^2 P_1^+ = \left(1 - c^2\right) P_1^+$$

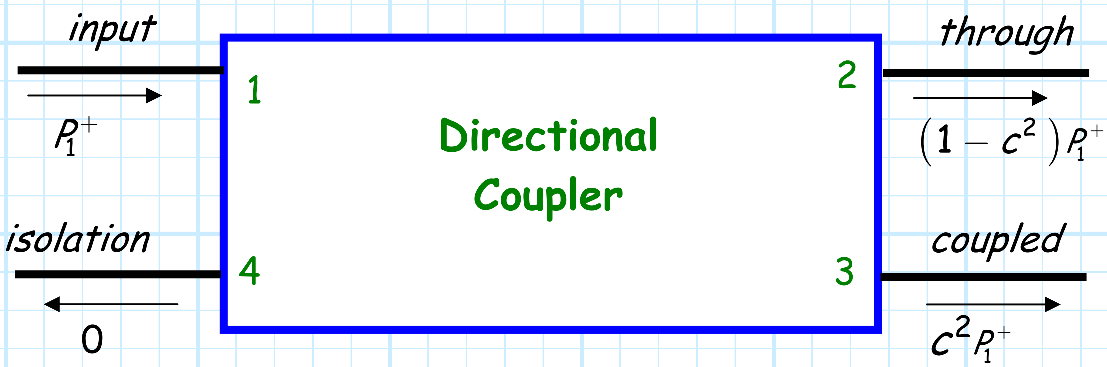
and the power out of **port 3** is:

$$P_3^- = |S_{31}|^2 P_1^+ = c^2 P_1^+$$

Finally, we find there is **no power** flowing out of **port 4**:

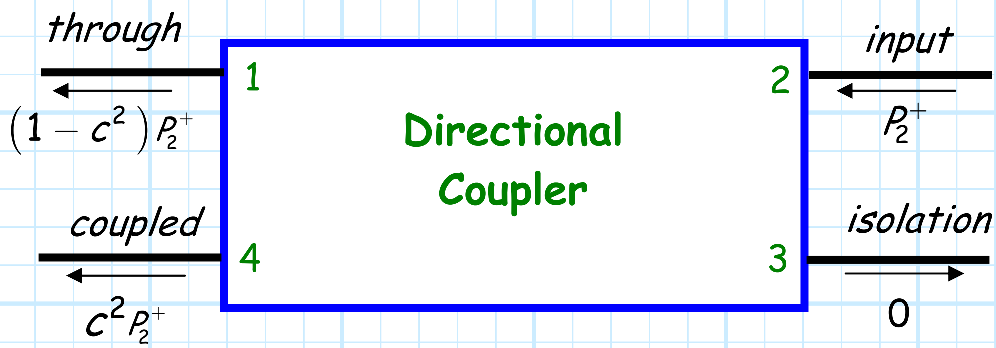
$$P_4^- = |S_{41}|^2 P_1^+ = 0$$

In the terminology of the directional coupler, we say that port 1 is the **input** port, port 2 is the **through** port, port 3 is the **coupled** port, and port 4 is the **isolation** port.



Note however, that **any** of the coupler ports can be an input, with a different through, coupled and isolation port for each case.

For example, **if** a signal is incident on **port 2**, while all other ports are matched, we find that:



Thus, from the scattering matrix of a directional coupler, we can form the following table:

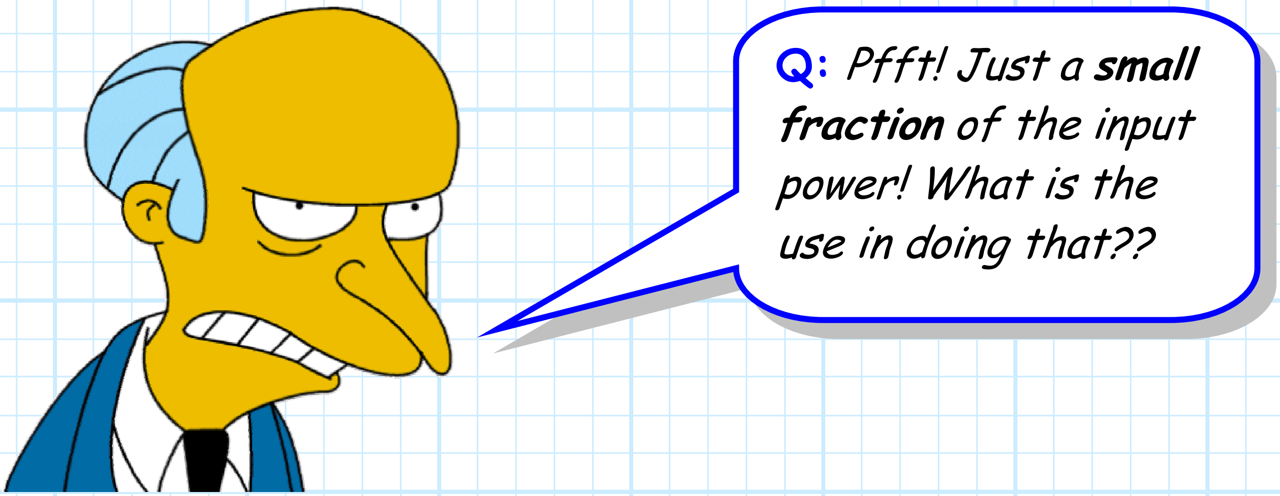
Input	*Through*	*Coupled*	*Isolation*
Port 1	Port 2	Port 3	Port 4
Port 2	Port 1	Port 4	Port 3
Port 3	Port 4	Port 1	Port 2
Port 4	Port 3	Port 2	Port 1

Typically, the coupling coefficients for a directional coupler are in the range of approximately:

$$0.25 > c^2 > 0.0001$$

As a result, we find that $\sqrt{1-c^2} \approx 1$. What this means is that the power out of the **through** port is just **slightly smaller** (typically) than the power incident on the input port.

Likewise, the power out of the **coupling** port is typically a **small fraction** of the power incident on the input port.



A: A directional coupler is often used for **sampling** a small portion of the signal power. For example, we might **measure** the output power of the **coupled** port (e.g., P_3^-) and then we can determine the amount of signal power flowing through the device (e.g., $P_1^+ = P_3^- / c^2$)

Unfortunately, the **ideal** directional coupler **cannot** be built! For example, the input match is never **perfect**, so that the diagonal elements of the scattering matrix, although **very small**, are not zero.

Likewise, the isolation port is never **perfectly** isolated, so that the values S_{41}, S_{32}, S_{23} and S_{14} are also non-zero—some **small** amount of power leaks out!

As a result, the through port will be **slightly less** than the value $\sqrt{1-c^2}$. The scattering matrix for a **non-ideal coupler** would therefore be:

$$\mathcal{S} = \begin{bmatrix} S_{11} & S_{21} & jc & S_{41} \\ S_{21} & S_{11} & S_{41} & jc \\ jc & S_{41} & S_{11} & S_{21} \\ S_{41} & jc & S_{21} & S_{11} \end{bmatrix}$$

From **this** scattering matrix, we can extract **some important parameters** about directional couplers:

Coupling C

The coupling value is the ratio of the coupled output power to the input power, in dB:

$$C\,(dB) = 10\log_{10}\left[\frac{P_1^+}{P_3^-}\right] = -10\log_{10}|jc|^2$$

This is the **primary** specification of a directional coupler!

Directivity *D*

The directivity is the ratio of the power out of the coupling port to the power out of the isolation port, in dB. This value indicates how effective the device is in "**directing**" the coupled energy into the correct port. The **higher** the directivity, the better.

$$D(dB) = 10\log_{10}\left[\frac{P_3^-}{P_4^-}\right] = 10\log_{10}\left[\frac{|jc|^2}{|S_{41}|^2}\right]$$

Isolation *I*

Isolation is the ratio of the input power to the power out of the isolation port, in dB. This value indicates how "isolated" the isolation port actually is. The **higher** the isolation, the better.

$$I(dB) = 10\log_{10}\left[\frac{P_1^+}{P_4^-}\right] = -10\log_{10}\left[|S_{41}|^2\right]$$

Note that isolation, directivity, and coupling are **not** independent values! **You** should be able to quickly show that:

$$I(dB) = C(dB) + D(dB)$$

Mainline Loss *ML*

The mainline loss is the ratio of the input power to the power out of the through port, in dB. It indicates how much power the signal **loses** as it travels from the input to the through port.

$$ML(dB) = 10\log_{10}\left[\frac{P_1^+}{P_2^-}\right] = -10\log_{10}\left[|S_{21}|^2\right]$$

Coupling Loss *ML*

The coupling loss indicates the portion of the mainline loss that is due to coupling some of the input power into the coupling port. Conservation of energy makes this loss is **unavoidable.**

$$CL(dB) = -10\log_{10}\left[1-|jc|^2\right]$$

Insertion Loss *IL*

The coupling loss indicates the portion of the mainline loss that is **not** due to coupling some of the input power into the coupling port. This loss **is** avoidable, and thus the **smaller** the insertion loss, the better.

$$IL(dB) = ML(dB) - CL(dB)$$

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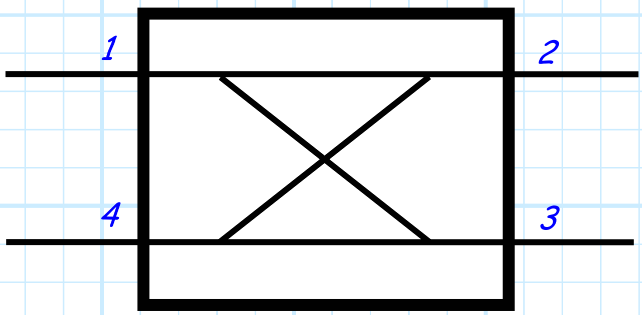
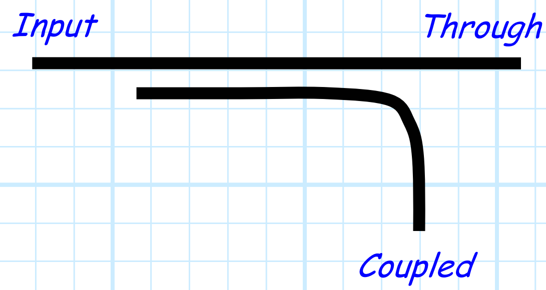
The Directional Coupler Specification Sheet





Here are some typical parameters used to specify **technical** the performance of **directional couplers**!

Bandwidth (Hz)

A directional coupler, like all other devices, can effectively operate only within a **finite bandwidth**. Generally, bandwidth is defined as the frequency range where the **coupling** is that of the specified value, within some minimum deviation (e.g., 3 dB).

Port Impedance (Γ, return loss, VSWR, S_{11}, Z_{in})

A parameter that specifies the match of the input ports. Can (and will) be specified **any number** of ways.

Input power (Watts)

The maximum input power the coupler can handle before it will be **damaged**.

Coupling (dB)

Coupling typically ranges from 6 dB (**a lot** of coupling) to 40 dB (very **little** coupling).

Directivity (dB)

Typically ranges from 15dB (**bad**) to 35dB (**good**).

Isolation (dB)

Remember, isolation is a value that is directly **dependent** on Directivity and Coupling!

Insertion Loss (dB)

Typically ranges from 0.2 dB (**good**) to 1.5 dB (**bad**).

Coupling Loss (dB)

Recall that this is completely **dependent** on the Coupling of the device—more coupling likewise means more coupling **loss** (conservation of energy says you **can't** get something for nothing!).

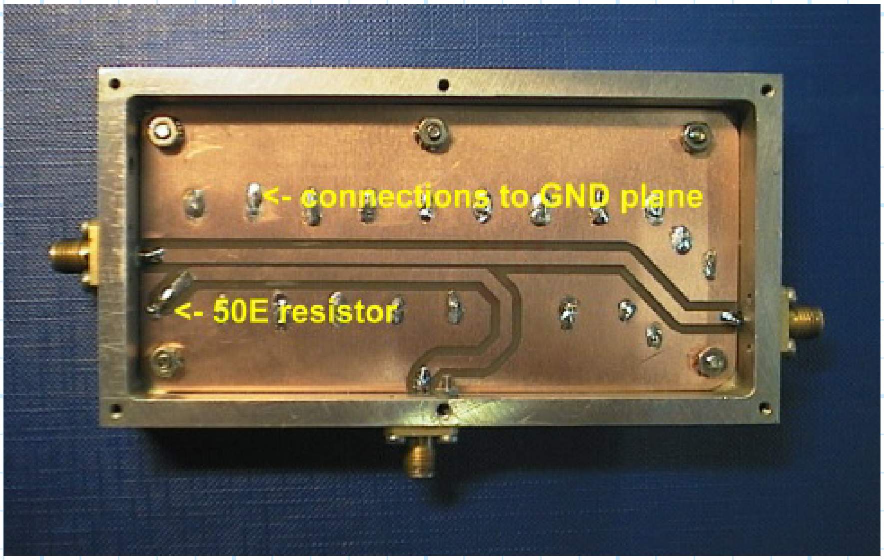
Thus, the Coupling Loss for devices with very small coupling (e.g., 20 dB or more) is likewise **very** small (a small fraction of 1 dB).

Mainline Loss (dB)

Recall the quantity is completely **dependent** on Insertion Loss and Coupling Loss. The Mainline Loss for devices with very small coupling (e.g., 20 dB or more) is **approximately equal** to the Insertion Loss of the device.

Coupling Flatness (dB)

This parameter specifies how much the coupling varies over the bandwidth of the device. **Typically** this value is 2.0 dB or less.



The 90° Hybrid Coupler

The 90° Hybrid Coupler, otherwise known as the **Quadrature Coupler**, has the **same** symmetric form as the directional coupler:

$$\mathcal{S} = \begin{bmatrix} 0 & \alpha & j\beta & 0 \\ \alpha & 0 & 0 & j\beta \\ j\beta & 0 & 0 & \alpha \\ 0 & j\beta & \alpha & 0 \end{bmatrix}$$

However, for **this** coupler we find that

$$\alpha = \beta = \frac{1}{\sqrt{2}}$$

Therefore, the scattering matrix of a quadrature coupler is:

$$\mathcal{S} = \begin{bmatrix} 0 & 1/\sqrt{2} & j/\sqrt{2} & 0 \\ 1/\sqrt{2} & 0 & 0 & j/\sqrt{2} \\ j/\sqrt{2} & 0 & 0 & 1/\sqrt{2} \\ 0 & j/\sqrt{2} & 1/\sqrt{2} & 0 \end{bmatrix}$$

It is evident that, just as with the directional coupler, the ports are **matched** and the device is **lossless**. Note also, that if a signal is incident on one port only, then there will be a port from which **no** power will exit (i.e., an **isolation** port).

Unlike the directional coupler, the power that is flows into the input port will be **evenly** divided between the two non-isolated ports.

For example, if 10 mW is incident on port 3 (and all other ports are matched), then 5 mW will flow out of **both** port 1 and port 4, while no power will exit port 2 (the isolated port).

Note however, that the although the **magnitudes** of the signals leaving ports 1 and 4 are **equal**, the relative **phase** of the two signals are separated by **90 degrees** ($e^{j\pi/2} = j$).

We find, therefore, that if in **real** terms the voltage out of port 1 is:

$$v_1(z,t) = \frac{|V_{03}^-|}{\sqrt{2}} \cos(\omega_0 t + \beta z)$$

then the signal form port 4 will be:

$$v_4(z,t) = \frac{|V_{03}^-|}{\sqrt{2}} \sin(\omega_0 t + \beta z)$$

There are **many** useful applications where we require both the **sine** and **cosine** of a signal!

The 180° Hybrid Coupler

The 180° **Hybrid Coupler** (sometimes know as the "ring" or "rat-race" hybrid) is a lossless, matched and reciprocal 4-port device, with a scattering matrix of the **anti-symmetric** form:

$$\mathcal{S} = \begin{bmatrix} 0 & \alpha & \beta & 0 \\ \alpha & 0 & 0 & -\beta \\ \beta & 0 & 0 & \alpha \\ 0 & -\beta & \alpha & 0 \end{bmatrix}$$

Just like the quadrature coupler, however, we find that:

$$\alpha = \beta = \frac{1}{\sqrt{2}}$$

So that the scattering matrix for this device is:

$$\mathcal{S} = \begin{bmatrix} 0 & 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 0 & 0 & -1/\sqrt{2} \\ 1/\sqrt{2} & 0 & 0 & 1/\sqrt{2} \\ 0 & -1/\sqrt{2} & 1/\sqrt{2} & 0 \end{bmatrix}$$

Hence, this coupler is likewise a **3dB coupler**—the power into a given port (with all other ports matched) is equally divided between two of the three output ports.

Note the relative phase between the outputs, however, is **dependent** on which port is the input.

For example, if the **input** is port 1 or port 3, the two signals will be **in phase**—no difference in their relative phase!

However, if the input is port 2 or port 4, the output signals will be 180° **out of phase** ($e^{j\pi} = -1$)!

An interesting application of this coupler can be seen if we place **two input signals** into the device, at ports 2 and 3. Note the signal out of port 1 would therefore be:

$$\begin{aligned} V_1^-(z) &= S_{12} V_2^+(z) + S_{13} V_3^+(z) \\ &= \frac{1}{\sqrt{2}}\left(V_3^+(\;) + V_2^+(\;)\right) \end{aligned}$$

while the signal out of port 4 is:

$$\begin{aligned} V_4^-(z) &= S_{42} V_2^+(z) + S_{43} V_3^+(z) \\ &= \frac{1}{\sqrt{2}}\left(V_3^+(z) - V_2^+(z)\right) \end{aligned}$$

Note that the output of port 1 is proportional to the **sum** of the two inputs. Port 1 of a 180° Hybrid Coupler is thus often referred to as the **sum** (Σ) port.

Likewise, port 4 is proportional to the **difference** between the two inputs. Port 4 a 180° Hybrid Coupler is thus often referred to as the **delta** (Δ) port.

There are **many** applications where we wish to take the sum and/or difference between two signals!

The 180° Hybrid Coupler can likewise be used in the **opposite** manner. If we have **both** the sum and difference of two signals available, we can use this device to separate the signals into their separate components!