H. Filters

Recall that the electromagnetic spectrum is generally **full** of signals at all different **frequencies**.

Somehow, we have to **reject** all those signals—except for the **one** signal we are interested in!

Q: *Yikes! How can we possibly do that?*

A: Each signal has its own "IP address"—its **carrier frequency** ω_0! We can build devices that **select** or **reject** signals based on this carrier frequency.

We call these devices **filters**.

HO: Filters

HO: The Filter Bandwidth

HO: The Filter Phase Function

Q: *Why do we give a darn about **phase** function $\angle S_{21}(\omega)$? After all, phase **doesn't** matter.*

A: Phase doesn't matter!?! A typical **rookie mistake**!

HO: Filter Dispersion

HO: The Linear Phase Filter

Q: *So how do we **specify** a microwave filter? How close to an **ideal** filter can we build?*

A: HO: Microwave Filter Design

Q: How do **I** decide what filter **type** and **order** to use?

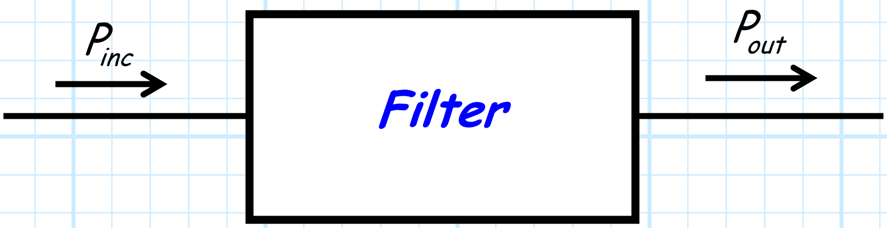
A: HO: The Filter Design Worksheet

Let's **summarize** what we've learned!

HO: The Microwave Filter Spec Sheet

Filters

A RF/microwave **filter** is (typically) a passive, reciprocal, 2-port linear device.



If port 2 of this device is terminated in a **matched** load, then we can relate the incident and output power as:

$$P_{out} = |S_{21}|^2 \, P_{inc}$$

We define this power transmission through a filter in terms of the **power transmission coefficient T**:

$$\mathbf{T} \doteq \frac{P_{out}}{P_{inc}} = |S_{21}|^2$$

Since microwave filters are typically **passive**, we find that:

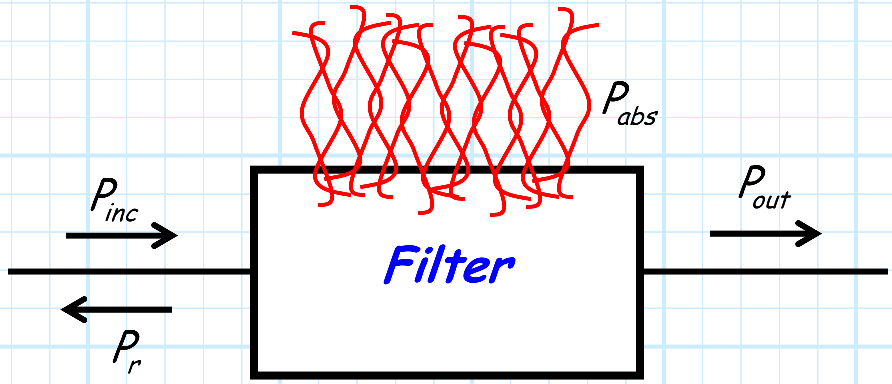
$$0 \le \mathbf{T} \le 1$$

in other words, $P_{out} \le P_{inc}$.

Q: *What happens to the "missing" power $P_{inc} - P_{out}$?*

A: **Two** possibilities: the power is either **absorbed** (P_{abs}) by the filter (converted to heat), or is **reflected** (P_r) at the input port.

I.E.:



Thus, by conservation of energy:

$$P_{inc} = P_r + P_{abs} + P_{out}$$

Now **ideally,** a microwave filter is lossless, therefore $P_{abs} = 0$ and:

$$P_{inc} = P_r + P_{out}$$

which **alternatively** can be written as:

$$\frac{P_{inc}}{P_{inc}} = \frac{P_r + P_{out}}{P_{inc}}$$

$$1 = \frac{P_r}{P_{inc}} + \frac{P_{out}}{P_{inc}}$$

Recall that $P_{out}/P_{inc} = \mathbf{T}$, and we can likewise **define** P_r/P_{inc} as the **power reflection coefficient**:

$$\boldsymbol{\Gamma} \doteq \frac{P_r}{P_{inc}} = |S_{11}|^2$$

We again emphasize that the filter output port is terminated in a **matched** load.

Thus, we can conclude that for a **lossless** filter:

$$1 = \boldsymbol{\Gamma} + \mathbf{T}$$

Which is simply **another** way of saying for a lossless device that $1 = |S_{11}|^2 + |S_{21}|^2$.

Now, **here's** the important part!

For a microwave **filter**, the coefficients $\boldsymbol{\Gamma}$ and $\mathbf{T}$ are **functions of frequency**! I.E.,:

$$\boldsymbol{\Gamma}(\omega) \quad \text{and} \quad \mathbf{T}(\omega)$$

The **behavior** of a microwave filter is described by these **functions**!

We find that for most signal frequencies ω_s, these functions will have a value equal to one of **two** different **approximate** values.

Either:

$$\Gamma(\omega = \omega_s) \approx 0 \qquad \text{and} \qquad T(\omega = \omega_s) \approx 1$$

or

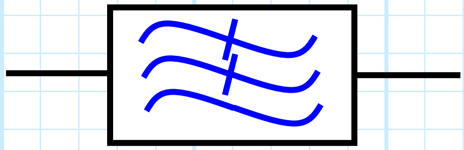
$$\Gamma(\omega = \omega_s) \approx 1 \qquad \text{and} \qquad T(\omega = \omega_s) \approx 0$$

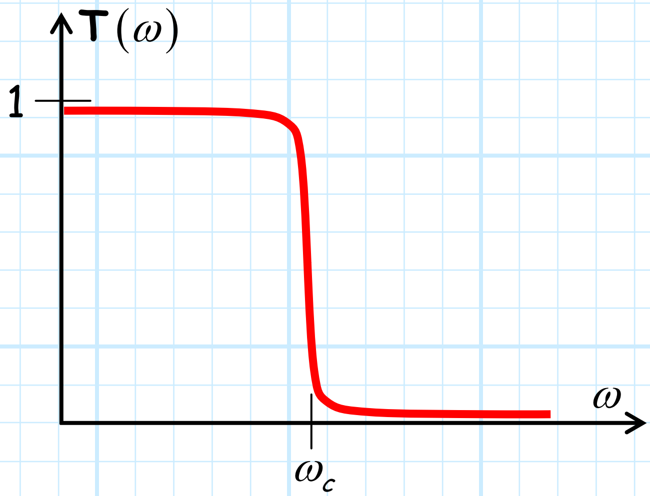
In the **first** case, the signal frequency ω_s is said to lie in the **pass-band** of the filter. Almost all of the incident signal power will **pass through** the filter.

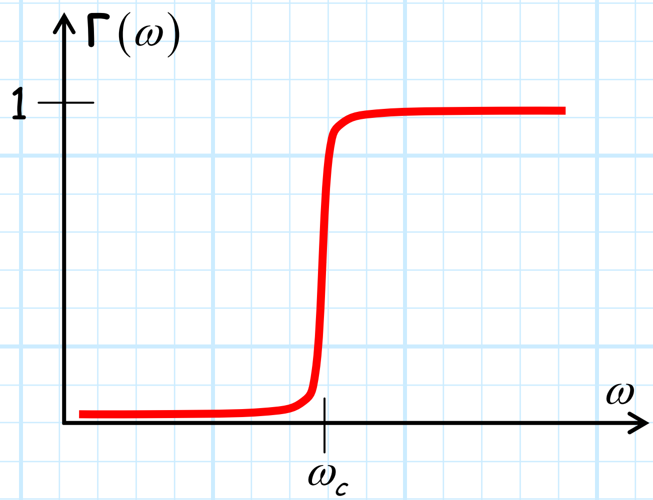
In the **second** case, the signal frequency ω_s is said to lie in the **stop-band** of the filter. Almost all of the incident signal power will be **reflected** at the input—almost no power will appear at the filter output.

Consider then these **four types** of functions of $\Gamma(\omega)$ and $T(\omega)$:

1. Low-Pass Filter







Note for this filter:

$$T(\omega) = \begin{cases} \approx 1 & \omega < \omega_c \\ \approx 0 & \omega > \omega_c \end{cases} \qquad \Gamma(\omega) = \begin{cases} \approx 0 & \omega < \omega_c \\ \approx 1 & \omega > \omega_c \end{cases}$$

This filter is a **low-pass** type, as it **"passes"** signals with frequencies **less** than ω_c, while **"rejecting"** signals at frequencies **greater** than ω_c.

Q: *This frequency ω_c seems to be very important! What is it?*

A: Frequency ω_c is a filter parameter known as the **cutoff frequency**; a value that **approximately** defines the frequency region where the filter pass-band **transitions** into the filter stop band.

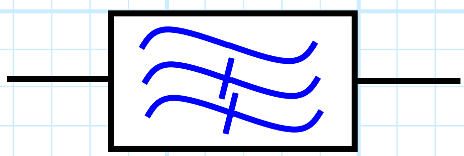
According, this frequency is defined as the frequency where the power **transmission** coefficient is equal to ½:

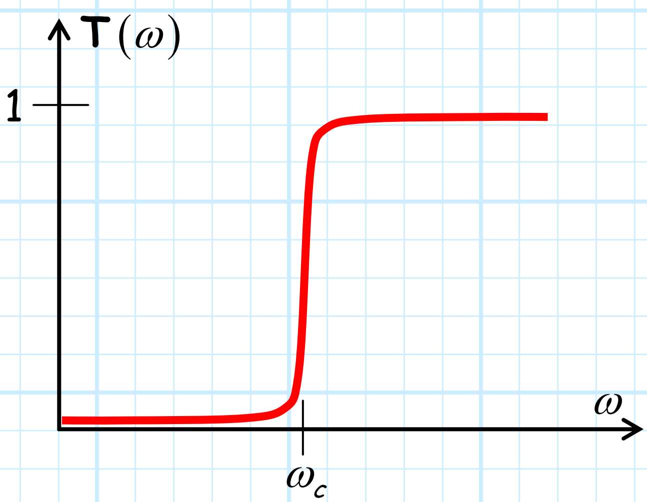
$$\mathbf{T}(\omega = \omega_c) = 0.5$$

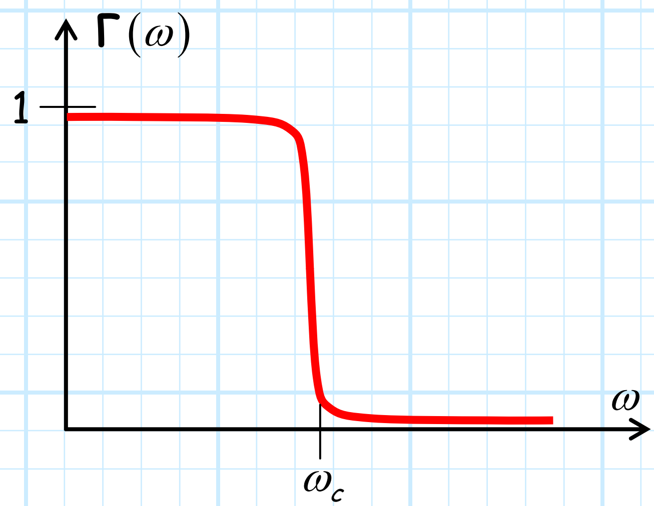
Note for a lossless filter, the cutoff frequency is **likewise** the value where the power **reflection** coefficient is ½:

$$\Gamma(\omega = \omega_c) = 0.5$$

2. High-Pass Filter





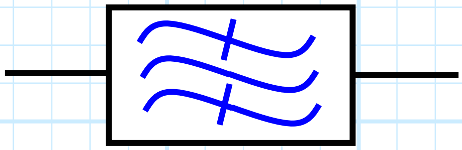


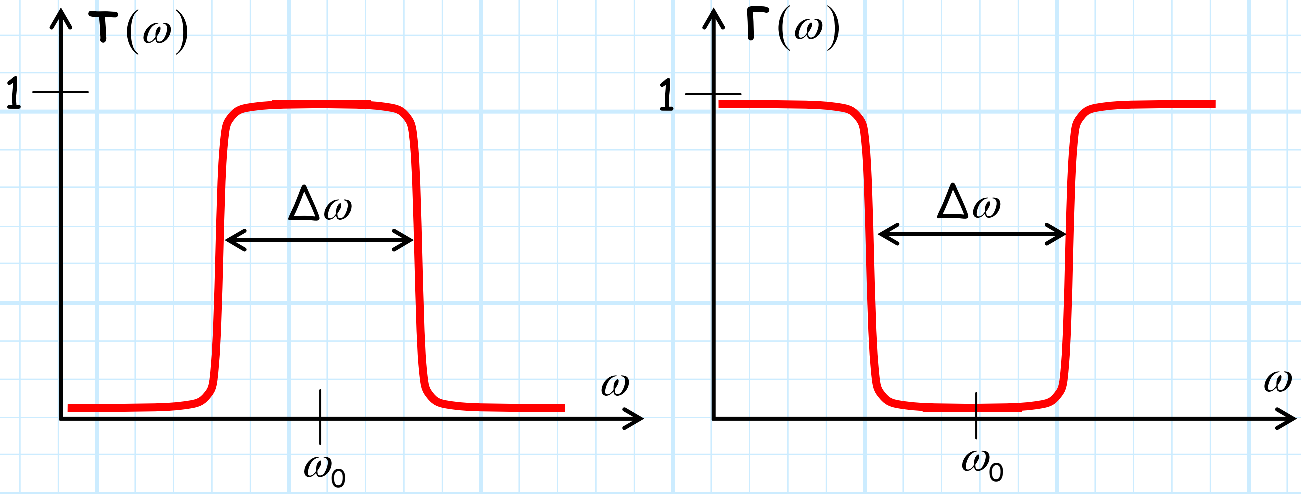
Note for this filter:

$$\mathbf{T}(\omega) = \begin{cases} \approx 0 & \omega < \omega_c \\ \approx 1 & \omega > \omega_c \end{cases} \qquad \Gamma(\omega) = \begin{cases} \approx 1 & \omega < \omega_c \\ \approx 0 & \omega > \omega_c \end{cases}$$

This filter is a **high-pass** type, as it **"passes"** signals with frequencies **greater** than ω_c, while **"rejecting"** signals at frequencies **less** than ω_c.

3. Band-Pass Filter





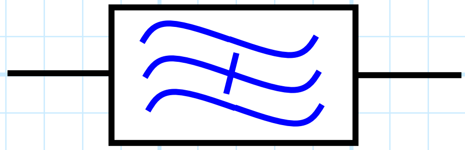
Note for this filter:

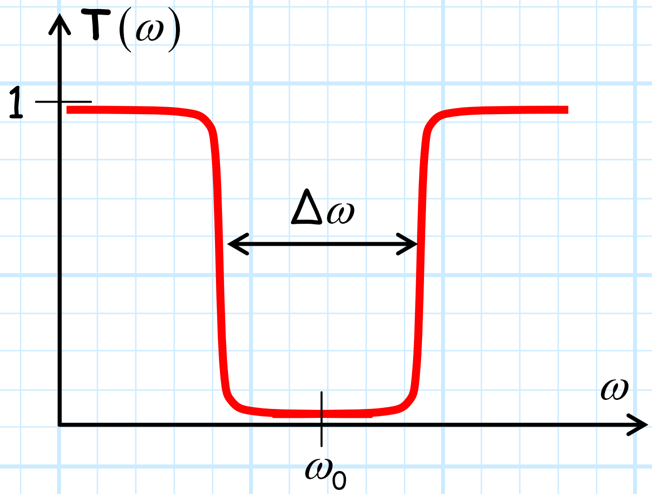
$$\mathbf{T}(\omega) = \begin{cases} \approx 1 & |\omega - \omega_0| < \Delta\omega/2 \\ \approx 0 & |\omega - \omega_0| > \Delta\omega/2 \end{cases} \qquad \Gamma(\omega) = \begin{cases} \approx 0 & |\omega - \omega_0| < \Delta\omega/2 \\ \approx 1 & |\omega - \omega_0| < \Delta\omega/2 \end{cases}$$

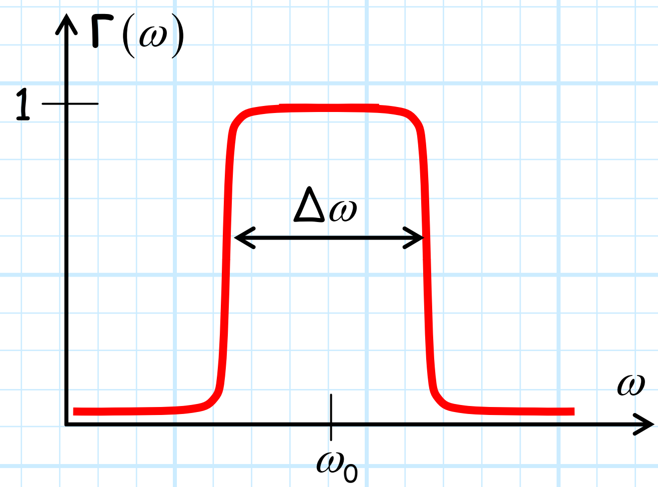
This filter is a **band-pass** type, as it **"passes"** signals within a frequency bandwidth $\Delta\omega$, while **"rejecting"** signals at all frequencies **outside this bandwidth**.

In addition to filter bandwidth $\Delta\omega$, a fundamental parameter of bandpass filters is ω_0, which defines the **center frequency** of the filter bandwidth.

3. Band-Stop Filter







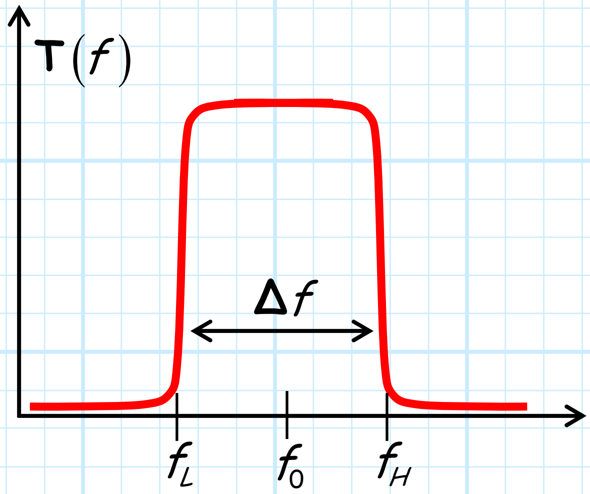
Note for this filter:

$$\mathbf{T}(\omega) = \begin{cases} \approx 0 & |\omega - \omega_0| < \Delta\omega/2 \\ \approx 1 & |\omega - \omega_0| > \Delta\omega/2 \end{cases} \qquad \Gamma(\omega) = \begin{cases} \approx 1 & |\omega - \omega_0| < \Delta\omega/2 \\ \approx 0 & |\omega - \omega_0| < \Delta\omega/2 \end{cases}$$

This filter is a **band-stop** type, as it **"rejects"** signals within a frequency bandwidth $\Delta\omega$, while **"passing"** signals at all frequencies **outside this bandwidth**.

Filter Bandwidth

We find that the vast majority of filters in microwave receivers are of the **bandpass** variety.



Some of these bandpass filters will likely have a fairly **wide-bandwidth**, a bandwidth specified by the **lowest frequency** that resides within the passband (f_L) and the **highest frequency** that resides within the passband (f_H).

Other bandpass filters in a microwave receiver will almost certainly be **narrow-band** filters. These filters are typically described by **bandwidth** $\Delta f = f_H - f_L$, and **center frequency** f_0.

The center frequency can be defined as **either** an arithmetic average:

$$f_0 \doteq \frac{f_H + f_L}{2}$$

or as a geometric average:

$$f_0 \doteq \sqrt{f_H \, f_L}$$

Another very important bandwidth parameter is **percent bandwidth**, defined as:

$$\%BW \doteq \frac{\Delta f}{f_0} \times 100\,\%$$

Q: *How can I distinguish between a "narrow-band" filter and a "wide-band" filter?*

A: The distinction is of course quite **subjective**, but generally it depends on **two things**:

1) **The percent bandwidth** - **I** would consider a filter will a bandwidth of—say—**less than 10%** a narrow-band filter, but other engineers might use a **different** percentage value.

2) **The technology used to construct the filters** – **Wideband** microwave filters are generally constructed using **lumped and/or distributed elements** (i.e., Ls and Cs!). However, filters with **small** percentage bandwidths (e.g., < 3%) are **difficult** to make using these elements.

For **very narrow-band filters**, engineers must use **high-Q** resonant elements, such as dielectric cavities, crystals, or SAWs (the same resonators we used in oscillators!).

Q: *Is there some **limit** on how **small** the percentage bandwidth can be?*

A: Absolutely! It is **exceedingly** difficult to build a filter whose %BW is less than (roughly) 0.1%, even with a high-Q resonator!

Note this chart provided by K&L microwave. The "cavity filters" are high-Q filters, and thus can provide a narrow percentage bandwidth (see furthest right column).

Topology	Frequency	(GHz)	% BW
Tubular			
B250 Series		.20 — 6	4 — 40
B120 Series		.10 — 2.5	
B340 Series		.08 — 1.7	
B110 Series		.06 — 1.0	
Cavity			
C20 Series		.03 — .14	.2 — 3.5
C30 Series		.14 — .5	
C40 Series		.501 — 2	
C42 Series		.8 — 2.5	
C45 Series		1 — 3	
C50 Series		2 — 10	.2 — 3
C52 Series		8 — 12	
C60 Series		6 — 22	.1 — .8
Interdigital			
ED10 Series		7 — 18	*5 — 50
ED20 Series		4 — 10	
ED30 Series		3 — 8	
ED40 Series		2 — 5	
ED50 Series		.5 — 2	
Combline			
FV10 Series		7 — 18	*3 — 20
FV20 Series		4 — 10	
FV30 Series		3 — 8	
FV40 Series		2 — 5	
FV50 Series		.5 — 2	
Microminiature IB Series		.01 — 6	3 — 70
Lumped Component LB Series		.0005 — .2	1 — 200
Miniature Cavity MC Series		.1 — 6	1 — 70
LC Commercial KCB Series		.010 — 3	5 — 7
LC Military MMB Series		.2 — 3	3 — 20
Dielectric Resonators DR Series		.5 — 3	1 — 10
Ceramic KeL-fil Series		.27 — 3	1 — 30
Waveguide WP Series		5 — 38	1 — 20
Tunable BT Series		.03 — 2.6	5
Suspended Substrate SB Series		2 — 18	50 — 100

0 .01 .1 1 10 20 30 40
(GHz)

* Varies with package and frequency

The Filter Phase Function

Recall that the power transmission coefficient $\mathsf{T}(\omega)$ can be determined from the **scattering parameter** $S_{21}(\omega)$:

$$\mathsf{T}(\omega) = \left|S_{21}(\omega)\right|^2$$

Q: *I see, we only care about the* ***magnitude*** *of complex function $S_{21}(\omega)$ when using microwave filters !?*

A: Hardly! Since $S_{21}(\omega)$ is complex, it can be expressed in terms of its magnitude and **phase**:

$$\begin{aligned} S_{21}(\omega) &= \text{Re}\{S_{21}(\omega)\} + j\,\text{Im}\{S_{21}(\omega)\} \\ &= \left|S_{21}(\omega)\right| e^{j\angle S_{21}(\omega)} \end{aligned}$$

where the phase is denoted as $\angle S_{21}(\omega)$:

$$\angle S_{21}(\omega) = \tan^{-1}\left[\frac{\text{Im}\{S_{21}(\omega)\}}{\text{Re}\{S_{21}(\omega)\}}\right]$$

We likewise care **very** much about this phase function!

Q: *Just what does this phase tell us?*

A: It describes the relative phase **between** the wave incident on the input to the filter, and the wave exiting the output of the filter (given the output port is matched).

In other words, if the **incident** wave is:

$$V_1^+(z_1) = V_{01}^+ \, e^{-j\beta z}$$

Then the exiting (output) wave will be:

$$\begin{aligned} V_2^-(z_2) &= V_{02}^- \, e^{+j\beta z_2} \\ &= S_{21} \, V_{01}^- \, e^{+j\beta z_2} \\ &= \left|S_{21}\right| V_{01}^- \, e^{+j(\beta z + \angle S_{21})} \end{aligned}$$

We say that there has been a "**phase shift**" of $\angle S_{21}$ between the input and output waves.

Q: *What **causes** this phase shift?*

A: Propagation **delay.** It takes some non-zero amount of **time** for signal energy to propagate from the input of the filter to the output.

Q: *Can we tell from $\angle S_{21}(\omega)$ how **long** this delay is?*

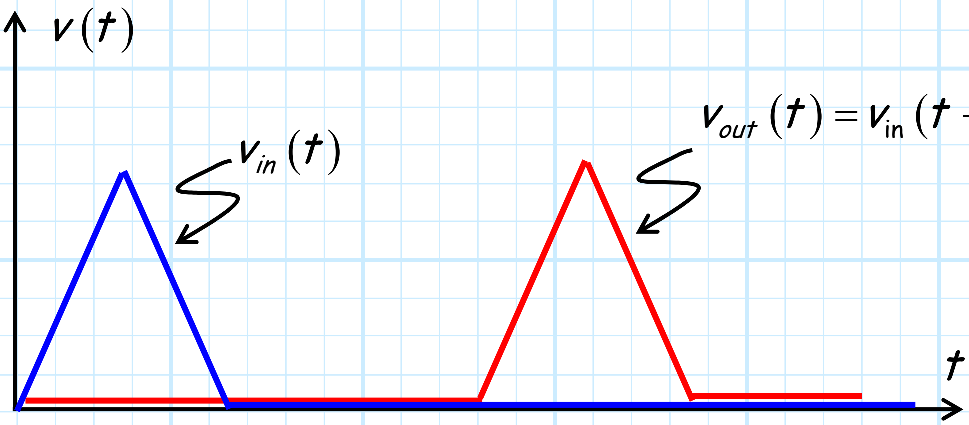
A: Yes!

To see how, consider an **example** two-port network with the impulse response:

$$h(t) = \delta(t-\tau)$$

We determined earlier that this device would merely **delay** and input signal by some amount τ:

$$\begin{aligned} v_{out}(t) &= \int_{-\infty}^{\infty} h(t-t')\, v_{in}(t')\, dt' \\ &= \int_{-\infty}^{\infty} \delta(t-t'-\tau) v_{in}(t')\, dt' \\ &= v_{in}(t'-\tau) \end{aligned}$$



Taking the **Fourier transform** of this impulse response, we find the **frequency response** of this two-port network is:

$$\begin{aligned} H(\omega) &= \int_{-\infty}^{\infty} h(t)\, e^{-j\omega t} dt \\ &= \int_{-\infty}^{\infty} \delta(t-\tau)\, e^{-j\omega t} dt \\ &= e^{-j\omega\tau} \end{aligned}$$

In other words:

$$|H(\omega)| = 1 \qquad \text{and} \qquad \angle H(\omega) = -\omega\tau$$

The interesting result here is the **phase** $\angle H(\omega)$. The result means that a delay of τ seconds results in an output "phase shift" of $-\omega\tau$ radians!

Note that although the **delay** of device is a **constant** τ, the **phase shift** is a **function** of ω --in fact, it is directly proportional to frequency ω.

Note if the **input** signal for this device was of the form:

$$v_{in}(t) = \cos\omega t$$

Then the output would be:

$$\begin{aligned} v_{out}(t) &= \cos\omega(t-\tau) \\ &= \cos(\omega t - \omega\tau) \\ &= |H(\omega)|\cos(\omega t + \angle H(\omega)) \end{aligned}$$

Thus, we could **either** view the signal $v_{in}(t) = \cos\omega t$ as being delayed by an amount τ seconds, **or** phase shifted by an amount $-\omega\tau$ radians.

Q: *So, by **measuring** the output signal phase shift $\angle H(\omega)$, we could determine the delay τ through the device with the equation:*

$$\tau = -\frac{\angle H(\omega)}{\omega}$$

right?

A: Not exactly. The problem is that we cannot **unambiguously** determine the phase shift $\angle H(\omega) = -\omega\tau$ by **looking** at the output signal!

The reason is that $\cos(\omega t + \angle H(\omega))$ = $\cos(\omega t + \angle H(\omega) + 2\pi)$ = $\cos(\omega t + \angle H(\omega) - 4\pi)$, etc. More specifically:

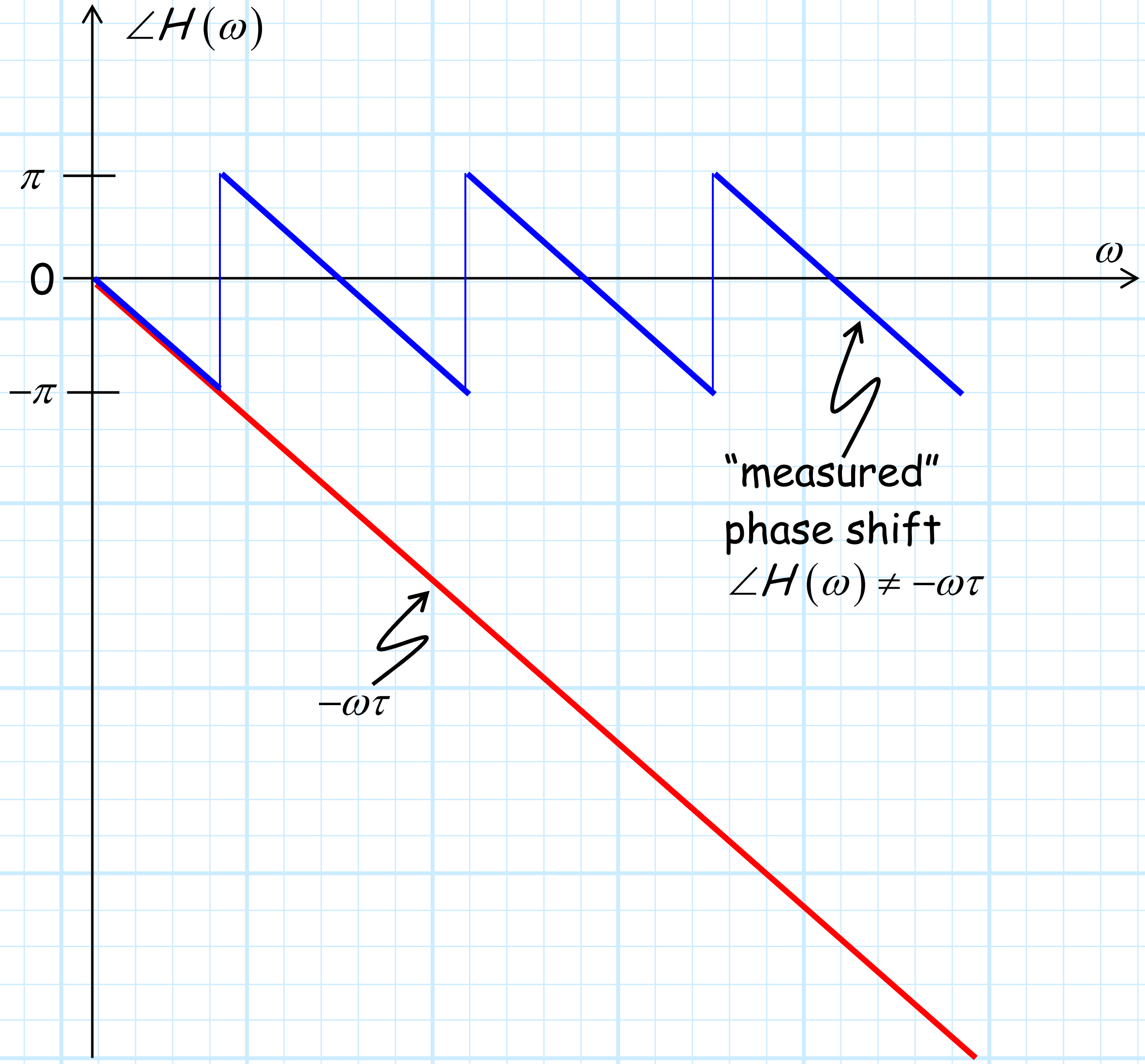
$$\cos(\omega t + \angle H(\omega)) = \cos(\omega t + \angle H(\omega) + n2\pi)$$

where n is **any integer**—positive or negative. We can't tell **which** of these output signal we are looking at!

Thus, any phase shift **measurement** has an inherent **ambiguity**. Typically, we interpret a phase measurement (in radians) such that:

$$-\pi < \angle H(\omega) \leq \pi \qquad \text{or} \qquad 0 \leq \angle H(\omega) < 2\pi$$

But almost certainly the actual value of $\angle H(\omega) = -\omega\tau$ is **nowhere** near these interpretations!



Clearly, using the equation:

$$\tau = -\frac{\angle H(\omega)}{\omega}$$

would **not** get us the correct result in this case—after all, there will be **several** frequencies ω with exactly the **same measured** phase $\angle H(\omega)$!

Q: *So determining the delay τ is **impossible**?*

A: NO! It is **entirely** possible—we simply must find the correct **method**.

Looking at the plot on the previous page, this method should become **apparent**. Not that although the measured phase (blue curve) is definitely **not** equal to the phase function $-\omega\tau$ (red curve), the **slope** of the two are **identical** at every point!

Q: *What good is knowing the **slope** of these functions?*

A: Just look! Recall that we can determine the slope by taking the first **derivative**:

$$\frac{\partial\left(-\omega\tau\right)}{\partial\omega} = -\tau$$

The slope directly tells us the **propagation delay**!

Thus, we can determine the propagation delay of this device by:

$$\tau = -\frac{\partial\angle H(\omega)}{\partial\omega}$$

where $\angle H(\omega)$ can be the **measured** phase. Of course, the method requires us to **measure** $\angle H(\omega)$ as a **function** of frequency (i.e., to make measurements at **many** signal frequencies).

Q: *Now I see! If we wish to **determine** the propagation delay τ through some **filter**, we simply need to take the derivative of $\angle S_{21}(\omega)$ with respect to frequency. **Right**?*

A: Well, sort of.

Recall for the **example** case that $h(t) = \delta(t-\tau)$ and $\angle H(\omega) = -\omega\tau$, where τ is a **constant**. For a microwave filter, **neither** of these conditions are true.

Specifically, the phase function $\angle S_{21}(\omega)$ will typically be some arbitrary **function** of frequency ($\angle S_{21}(\omega) \neq -\omega\tau$).

Q: *How could this be true? I thought you said that phase shift was **due** to filter delay τ!*

A: Phase shift **is** due to device delay, it's just that the propagation delay of most devices (such as filters) is **not a constant**, but instead depends on the **frequency** of the signal propagating through it!

In other words, the propagation delay of a filter is typically some arbitrary **function** of frequency (i.e., $\tau(\omega)$). That's why the phase $\angle S_{21}(\omega)$ is **likewise** an arbitrary function of frequency.

Q: *Yikes! Is there **any** way to determine the relationship between these two arbitrary functions?*

A: **Yes** there is! Just as before, the two can be related by a first **derivative**:

$$\tau(\omega) = -\frac{\partial \angle S_{21}(\omega)}{\partial \omega}$$

This result $\tau(\omega)$ is also know as **phase delay**, and is a **very** important function to consider when designing/specifying/ selecting a microwave **filter**.

Q: *Why; what might happen?*

A: If you get a filter with the wrong $\tau(\omega)$, your **output** signal could be horribly **distorted**—distorted by the evil effects of **signal dispersion**!

Filter Dispersion

Any signal that carries significant **information** must have some non-zero **bandwidth**. In other words, the signal energy (as well as the information it carries) is **spread** across many frequencies.

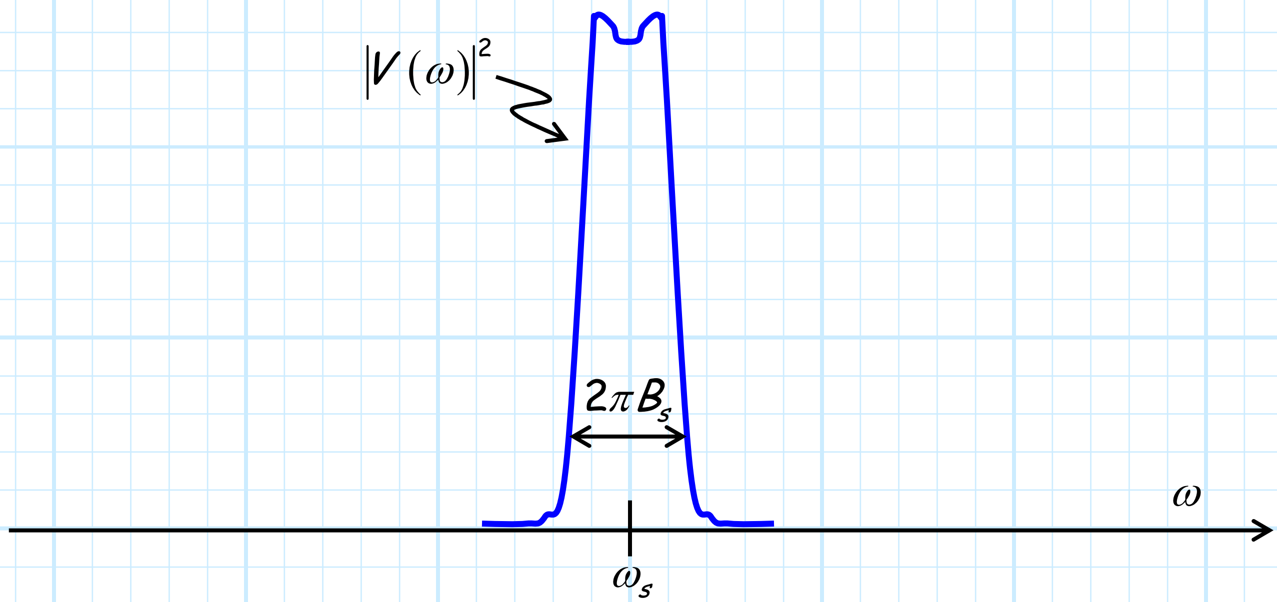
If the different frequencies that comprise a signal propagate at different velocities through a microwave filter (i.e., each signal frequency has a different delay τ), the output signal will be **distorted**. We call this phenomenon signal **dispersion**.

Q: *I see! The phase delay $\tau(\omega)$ of a filter **must** be a constant with respect to frequency—otherwise signal dispersion (and thus signal distortion) will result. Right?*

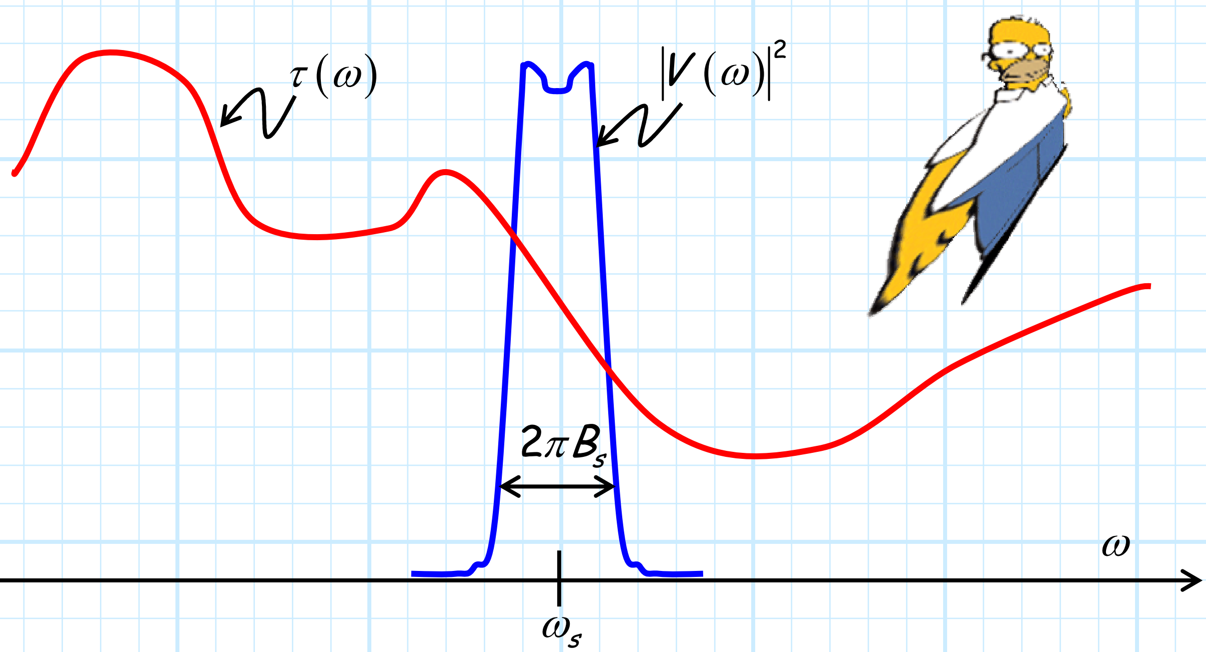
A: Not necessarily! Although a constant phase delay will **insure** that the output signal is not distorted, it is **not** strictly a requirement for that happy event to occur.

This is a **good** thing, for as we shall latter see, building a good filter with a constant phase delay is **very** difficult!

For example, consider a modulated signal with the following frequency **spectrum**, exhibiting a **bandwidth** of B_s Hertz.



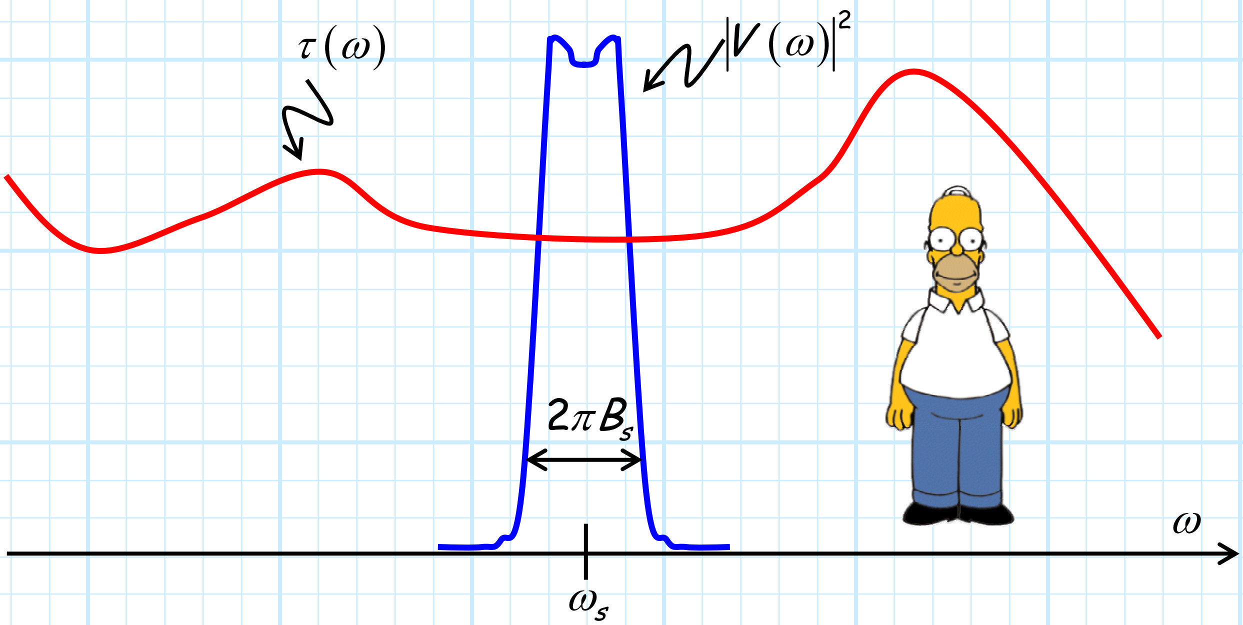
Now, let's likewise plot the **phase delay** function $\tau(\omega)$ of some filter:



Note that for this case the filter phase delay is **nowhere** near a constant with respect to frequency.

However, this fact alone does **not** necessarily mean that our signal would suffer from **dispersion** if it passed through this filter. Indeed, the signal in this case **would** be distorted, but **only** because the phase delay $\tau(\omega)$ changes significantly across the **bandwidth** B_s of the signal.

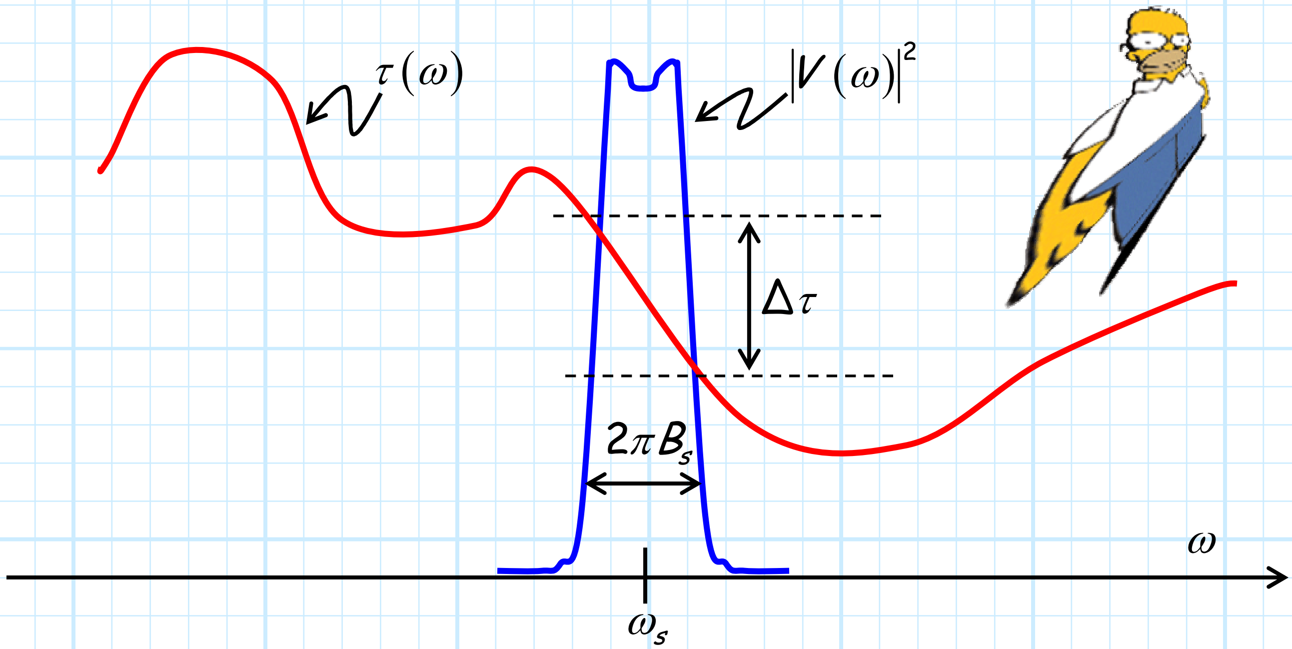
Conversely, consider this **phase delay**:



As with the previous case, the phase delay of the filter is **not** a constant. Yet, if this signal were to pass through this filter, it would **not** be distorted!

The reason for this is that the phase delay across the **signal bandwidth** is approximately constant—each frequency component of the **signal** will be delayed by the **same** amount.

Compare this to the **previous** case, where the phase delay changes by a precipitous value $\Delta\tau$ across signal bandwidth B_s:



Now **this** is a case where dispersion **will** result!

Q: *So does $\Delta\tau$ need to be **precisely** zero for no signal distortion to occur, or is there some **minimum** amount $\Delta\tau$ that is acceptable?*

A: Mathematically, we find that dispersion will be **insignificant** if:

$$B_s \, \Delta\tau \ll 1$$

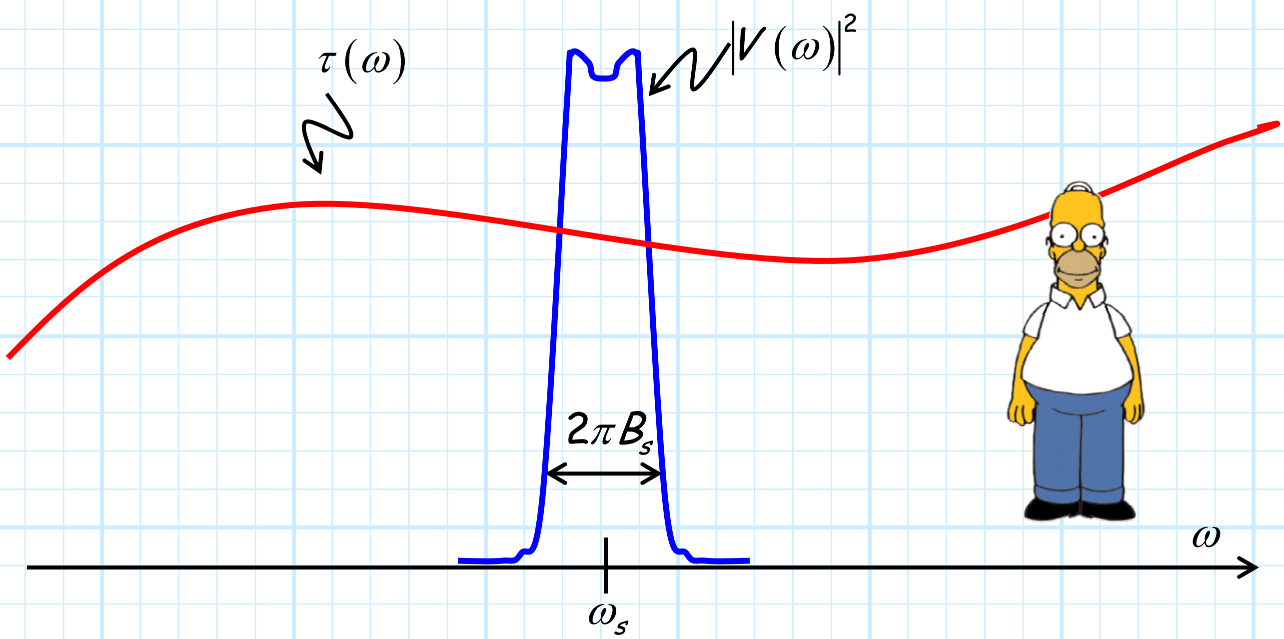
A more specific (but **subjective**) "rule of thumb" is:

$$B_s \, \Delta\tau < 0.1$$

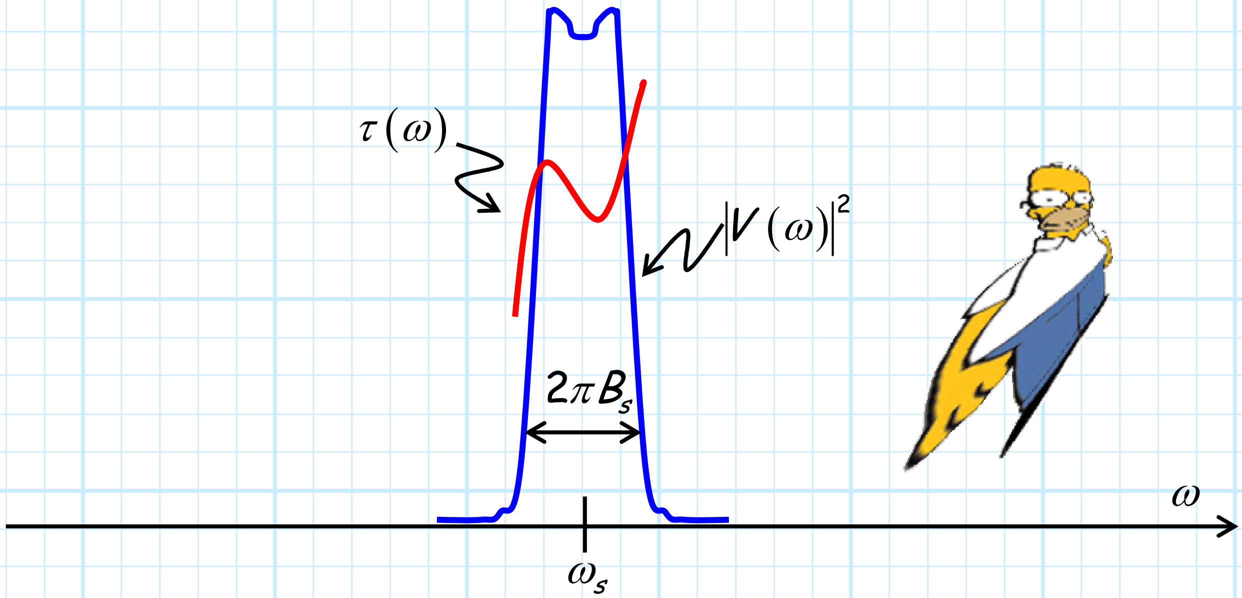
Generally speaking, we find for **wideband** filters—where filter bandwidth B is much greater than the signal bandwidth (i.e., $B \gg B_s$)—the above criteria is **easily** satisfied. In other words, signal dispersion is **not** typically a problem for wide band filters (e.g., preselector filters).

This is **not** to say that $\tau(\omega)$ is a constant for wide band filters. Instead, the phase delay can change **significantly** across the wide **filter** bandwith.

What we typically find however, is that the function $\tau(\omega)$ does not change very **rapidly** across the wide filter bandwidth. As a result, the phase delay will be **approximately** constant across the relatively narrow signal bandwidth B_s.



Conversely, a **narrowband** filter—where filter bandwidth B is approximately **equal** to the signal bandwidth (i.e., $B_s \approx B$)—can (if we're not careful!) exhibit a phase delay which likewise changes **significantly** over **filter** bandwidth B. This means of course that it **also** changes significantly over the **signal** bandwidth B_s!



Thus, a **narrowband** filter (e.g., IF filter) must exhibit a **near constant** phase delay $\tau(\omega)$ in order to **avoid** distortion due to signal dispersion!

The Linear Phase Filter

Q: *So, narrowband filters should exhibit a* ***constant*** *phase delay $\tau(\omega)$. What should the phase function $\angle S_{21}(\omega)$ be for this* ***dispersionless*** *case?*

A: We can express this problem mathematically as requiring:

$$\tau(\omega) = \tau_c$$

where τ_c is some **constant**.

Recall that the definition of **phase delay** is:

$$\tau(\omega) = -\frac{\partial \angle S_{21}(\omega)}{\partial \omega}$$

and thus **combining** these two equations, we find ourselves with a **differential equation**:

$$-\frac{\partial \angle S_{21}(\omega)}{\partial \omega} = \tau_c$$

The **solution** to this differential equation provides us with the necessary phase function $\angle S_{21}(\omega)$ for a **constant** phase delay τ_c.

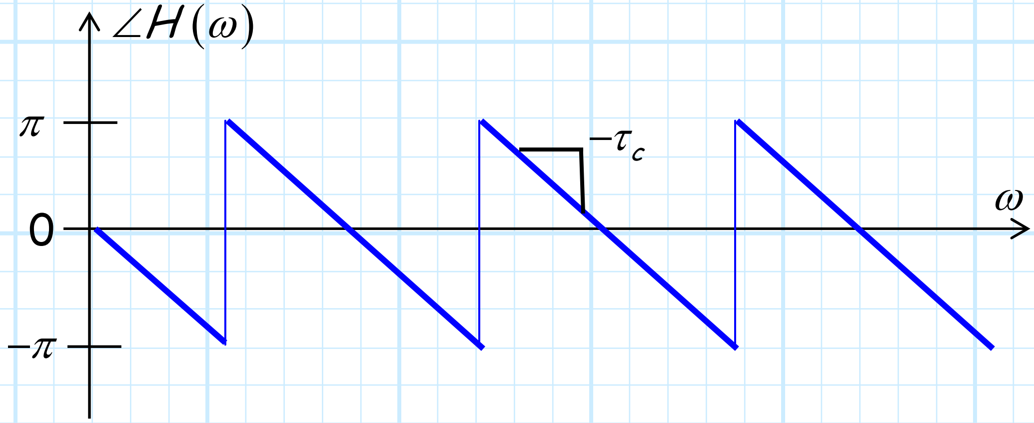
Fortunately, this differential equation is **easily** solved!

The solution is:

$$\angle S_{21}(\omega) = -\omega\,\tau_c + \phi_c$$

where ϕ_c is an arbitrary **constant**.

Plotting this phase function (with $\phi_c = 0$):



As **you** likely expected, this phase function is **linear**, such that it has a **constant slope** ($-\tau_c$).

Filters with this phase response are called **linear phase filters**, and have the desirable trait that they cause **no** dispersion distortion.

Microwave Filter Design

Recall that a **lossless** filter can be described in terms of either its power transmission coefficient $T(\omega)$ **or** its power reflection coefficient $\Gamma(\omega)$, as the two values are completely **dependent**:

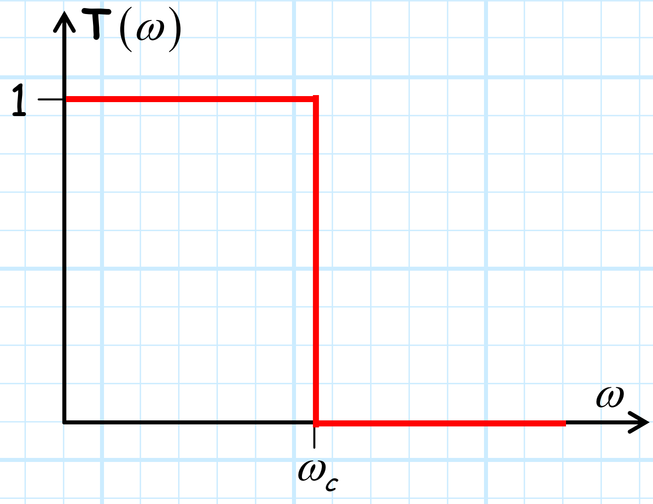
$$\Gamma(\omega) = 1 - T(\omega)$$

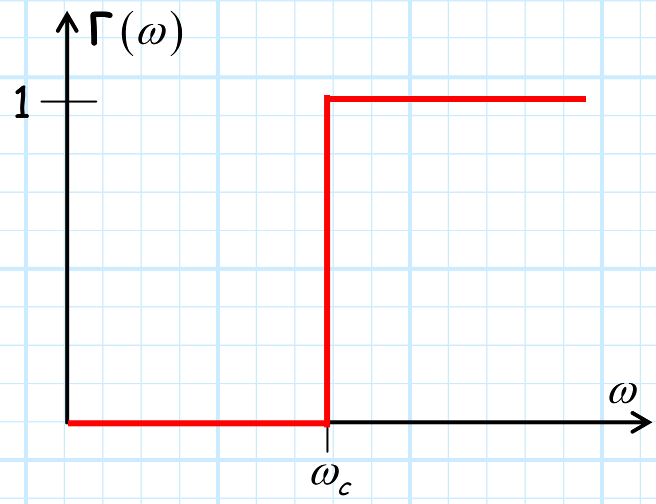
Ideally, these functions would be quite **simple**:

1. $T(\omega) = 1$ and $\Gamma(\omega) = 0$ for **all** frequencies within the **pass**-band.

2. $T(\omega) = 0$ and $\Gamma(\omega) = 1$ for **all** frequencies within the **stop**-band.

For example, the **ideal** low-pass filter would be:





Add to this a **linear phase** response, and you have the **perfect** microwave filter!

There's just one small problem with this **perfect** filter→ It's **impossible** to build!

Now, if we consider only possible (i.e., **realizable**) filters, we must limit ourselves to filter functions that can be expressed as **finite polynomials** of the form:

$$\mathbf{T}(\omega) = \frac{a_0 + a_1\,\omega + a_2\,\omega^2 + \cdots}{b_0 + b_1\,\omega + b_2\,\omega^2 + \cdots + b_N\,\omega^N}$$

The **order** N of the (denominator) polynomial is likewise the **order** of the filter.

There are many different **types** of polynomials that result in good filter responses, and each type has its own set of **characteristics**.

The type of **polynomial** likewise describes the type of microwave **filter**. Let's consider **three** of the most popular types:

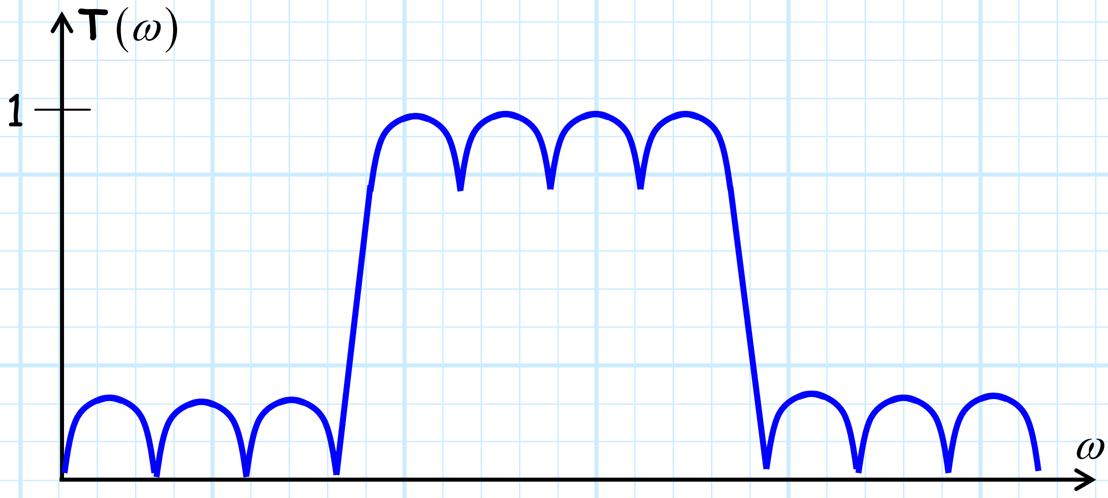
1. Elliptical

Elliptical filters have three primary characteristics:

a) They exhibit very **steep "roll-off"**, meaning that the transition from pass-band to stop-band is very rapid.

b) They exhibit **ripple** in the **pass**-band, meaning that the value of **T** will vary slightly within the pass-band.

c) They exhibit ripple in the **stop**-band, meaning that the value of **T** will vary slightly within the stop-band.



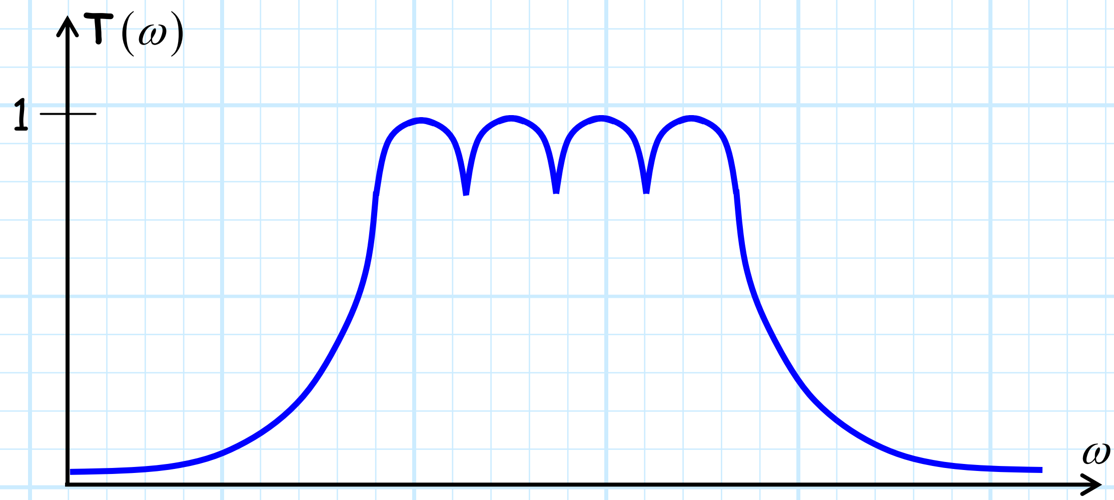
We find that we can make the roll-off **steeper** by accepting more **ripple**.

2. Chebychev

Chebychev filters are also known as **equal-ripple** filters, and have two primary characteristics

a) **Steep** roll-off (but not as steep as Elliptical).

b) Pass-band **ripple** (but not stop-band ripple).



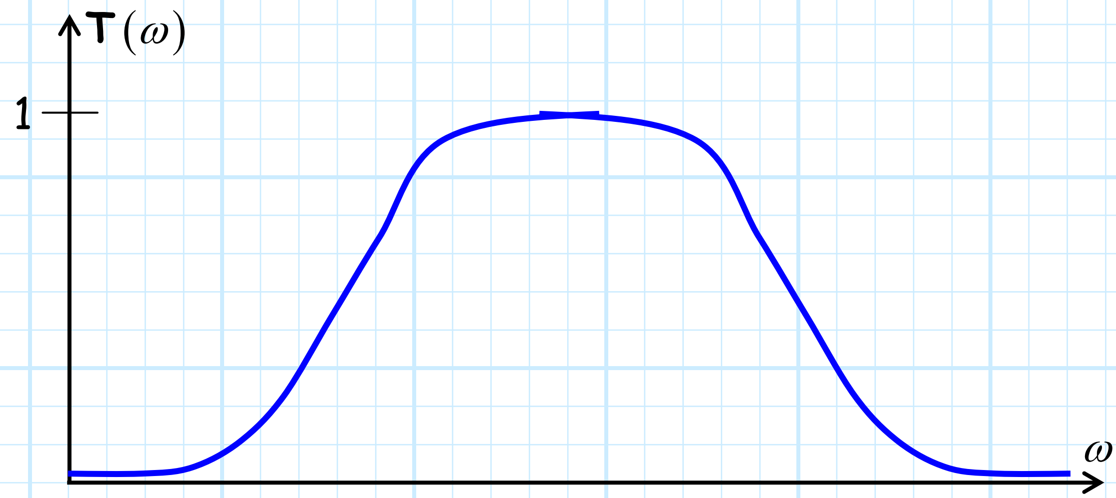
We likewise find that the roll-off can be made steeper by **accepting** more ripple.

3. Butterworth

Also known as **maximally flat** filters, they have two primary characteristics

a) **Gradual** roll-off .

b) **No ripple**—not anywhere.



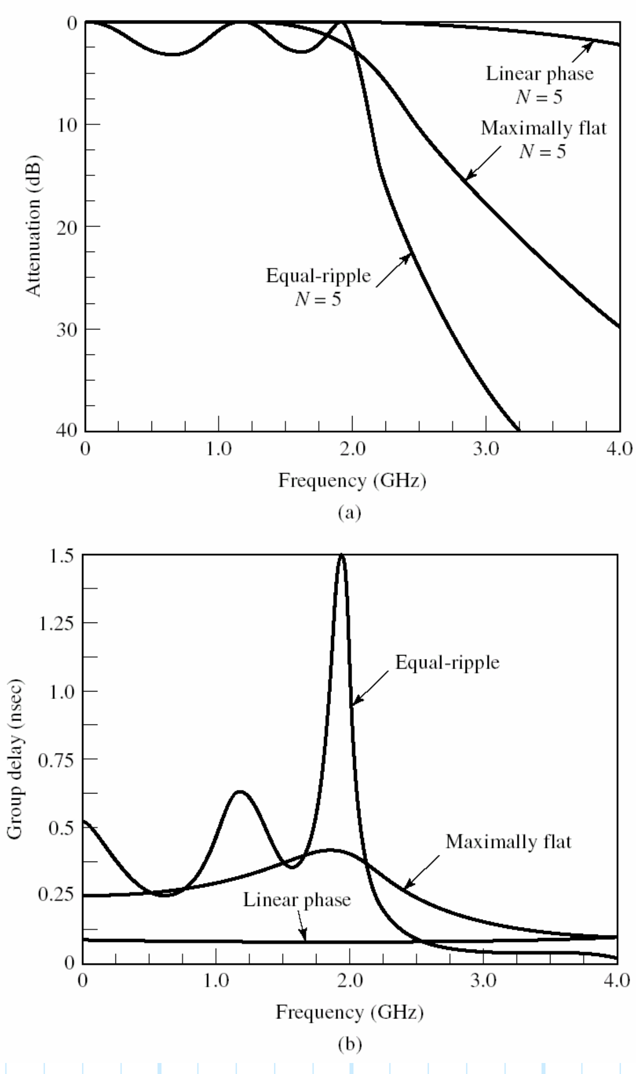
Q: *So we always chose **elliptical** filters; since they have the steepest roll-off, they are **closest** to ideal—**right**?*

A: Ooops! I forgot to talk about the **phase response** $\angle S_{21}(\omega)$ of these filters. Let's examine $\angle S_{21}(\omega)$ for each filter type **before** we pass judgment.

Butterworth $\angle S_{21}(\omega)$ → **Close** to linear phase.

Chebychev $\angle S_{21}(\omega)$ → **Not** very linear.

Elliptical $\angle S_{21}(\omega)$ → A big non-linear **mess**!



Thus, it is apparent that as a filter roll-off **improves**, the phase response gets **worse** (watch out for **dispersion!**).

→ There is **no** such thing as the "**best**" filter type!

Q: *So, a filter with **perfectly** linear phase is impossible to construct?*

A: No, it **is** possible to construct a filter with **near** perfect linear phase—but it will exhibit a **horribly** poor roll-off!

Now, for any **type** of filter, we can **improve** roll-off (i.e., increase stop-band attenuation) by **increasing the filter order** N. However, be aware that increasing the filter order likewise has these **deleterious** effects:

1. It makes **phase response** $\angle S_{21}(\omega)$ worse (i.e., more non-linear).

2. It increases filter **cost**, **weight**, and **size**.

3. It increases filter **insertion loss** (this is bad).

4. It makes filter performance more **sensitive** to temperature, aging, etc.

From a **practical** viewpoint, the **order** of a filter should typically be kept to $\boldsymbol{N} < \mathbf{10}$.

Q: *So exactly what* ***are*** *these filter polynomials* $\mathbf{T}(\omega)$*? How do we* ***determine*** *them?*

A: Fortunately, **radio engineers** do not need to determine specific filter polynomials in order to **specify** (to filter manufacturers) what they want built.

Instead, radio engineers simply can specify the **type** and **order** of a filter, saying things like:

*"I need a **3rd-order Chebychev** filter!"*

or

*"Get me a **5th-order Butterworth** filter!"*

or

*" I wish I'd paid **more** attention in EECS 622!"*

Thus, the most **important** filter specifications are:

1. Filter bandwidth and center frequency

2. Filter type and order.

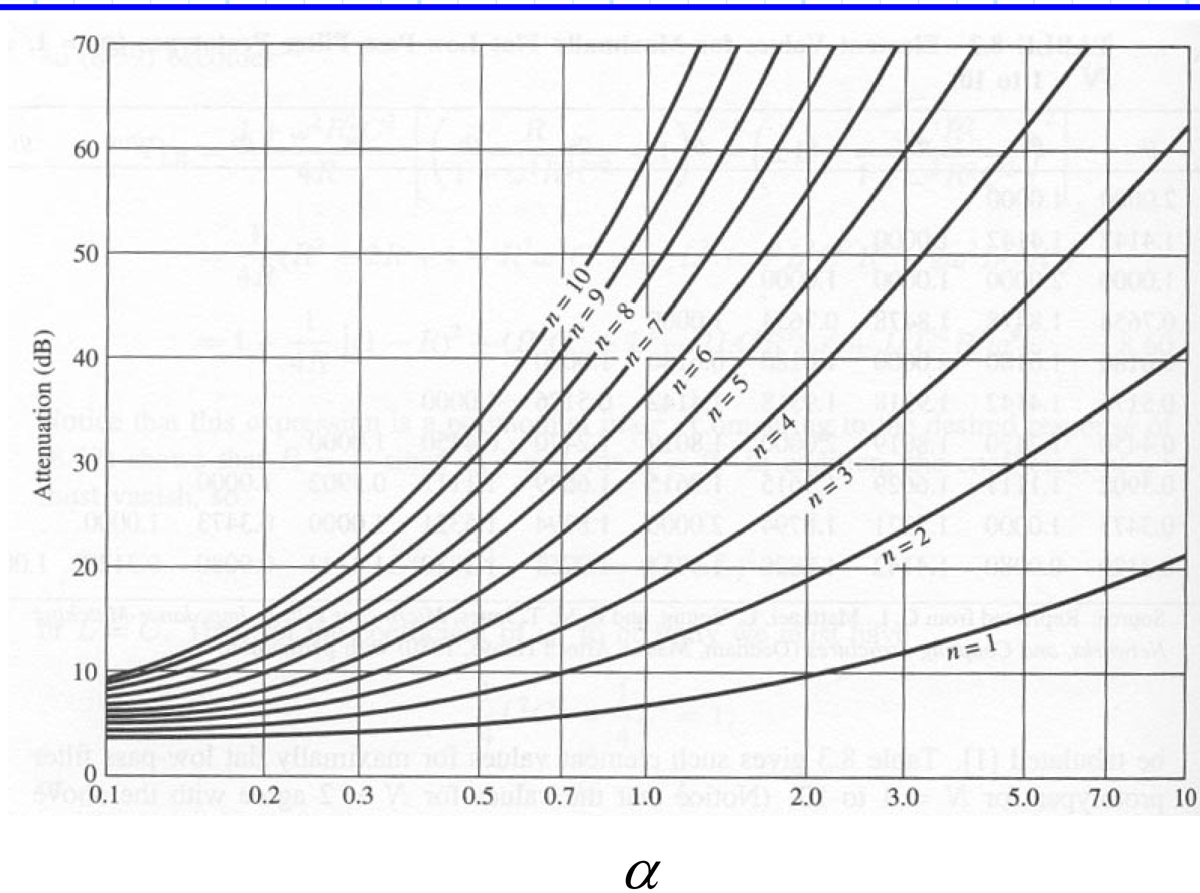
However, there are **many more** important filter specifications!

Filter Design Worksheet

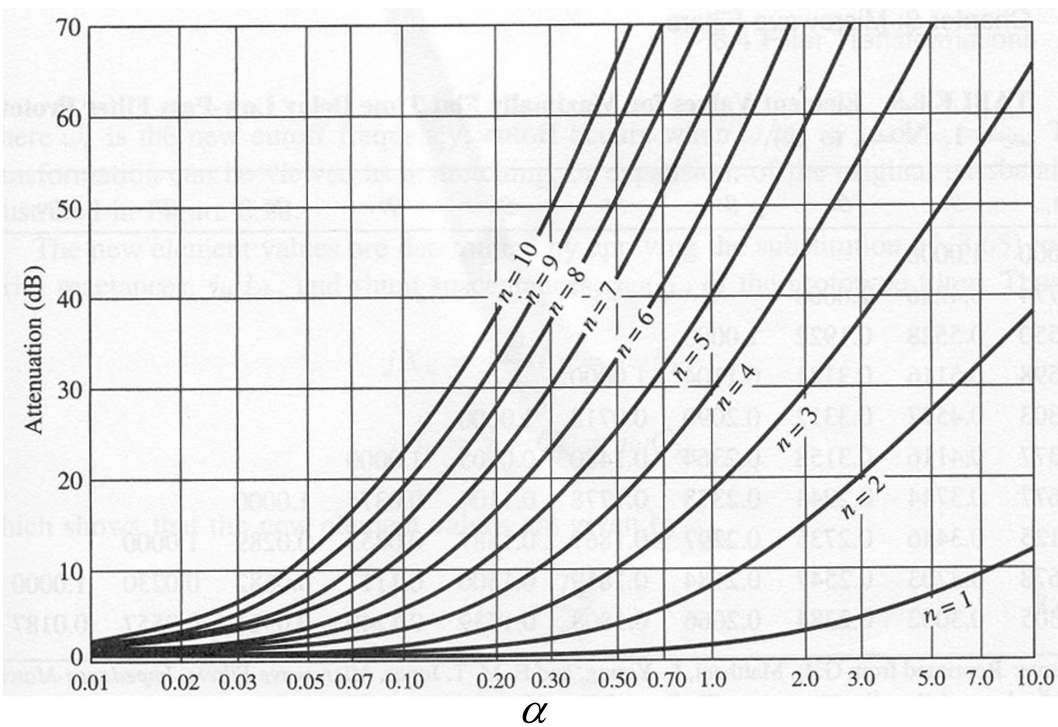
Q: *Given the order of a Butterworth or Chebychev bandpass filter, what will my stop band attenuation be? Or stated another way, what should the order of my filter be, in order to achieve a desired amount of stop-band attenuation* $\left(-10\log_{10} T(\omega)\right)$ *??*

A: Consult the **normalized attenuation charts** (They're in your book)!

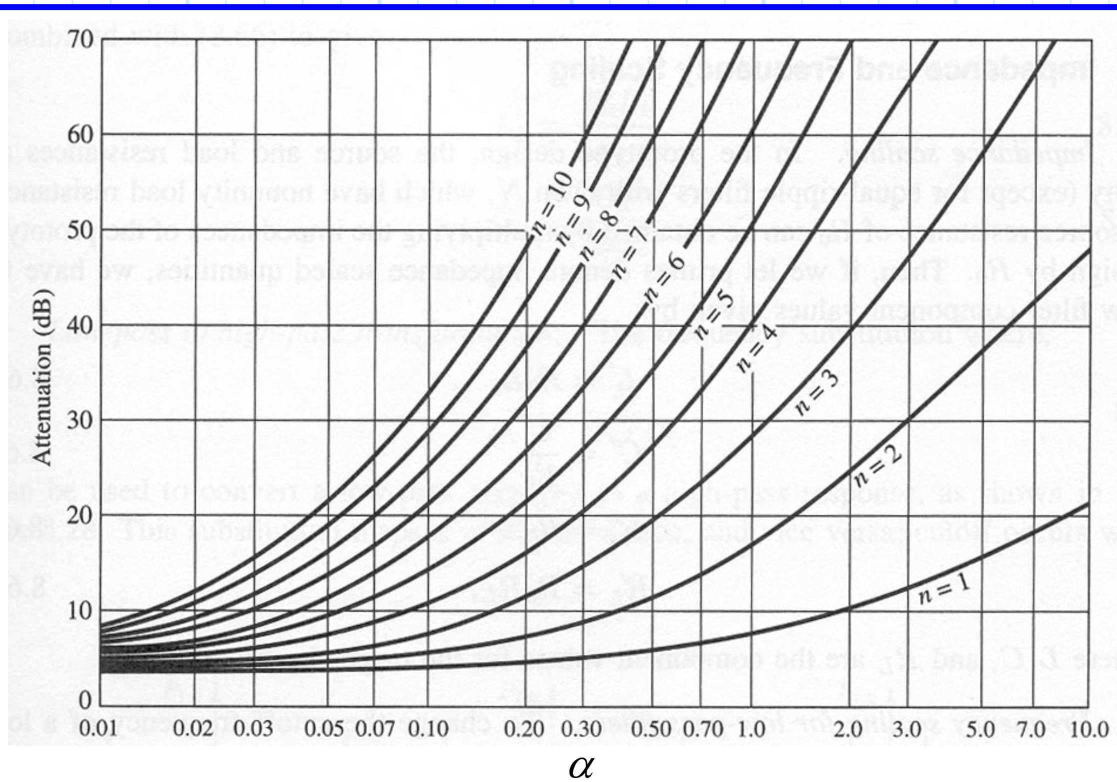
For example, the normalized attenuation chart for a **Butterworth** filter is:



While the normalized attenuation chart for a **Chebychev** with **0.5 dB** of passband ripple is:



And the normalized attenuation chart for a **Chebychev** with **3.0 dB** of passband ripple is:



Q: *Great, how the heck do I use* ***these*** *??*

A: The variable α is a **normalized** frequency variable. The plots show attenuation ($-10\log_{10}\mathbf{T}(\omega)$) versus frequency for a filter of **order** *n.*

Say we have a **bandpass filter**, whose (3 dB) passband extends from f_1 to f_2 ($f_2 > f_1$). The bandwidth of this filter would therefore be $f_2 - f_1$.

Using these values, we can define a **normalized frequency** α as:

$$\alpha = \left| \frac{1}{\Delta}\left(\frac{f}{f_0} - \frac{f_0}{f} \right) \right| - 1$$

where:

$$f_0 = \sqrt{f_1 f_2}$$

$$\Delta = \frac{f_2 - f_1}{f_0}$$

Thus, given a frequency f, we can calculate a value α.

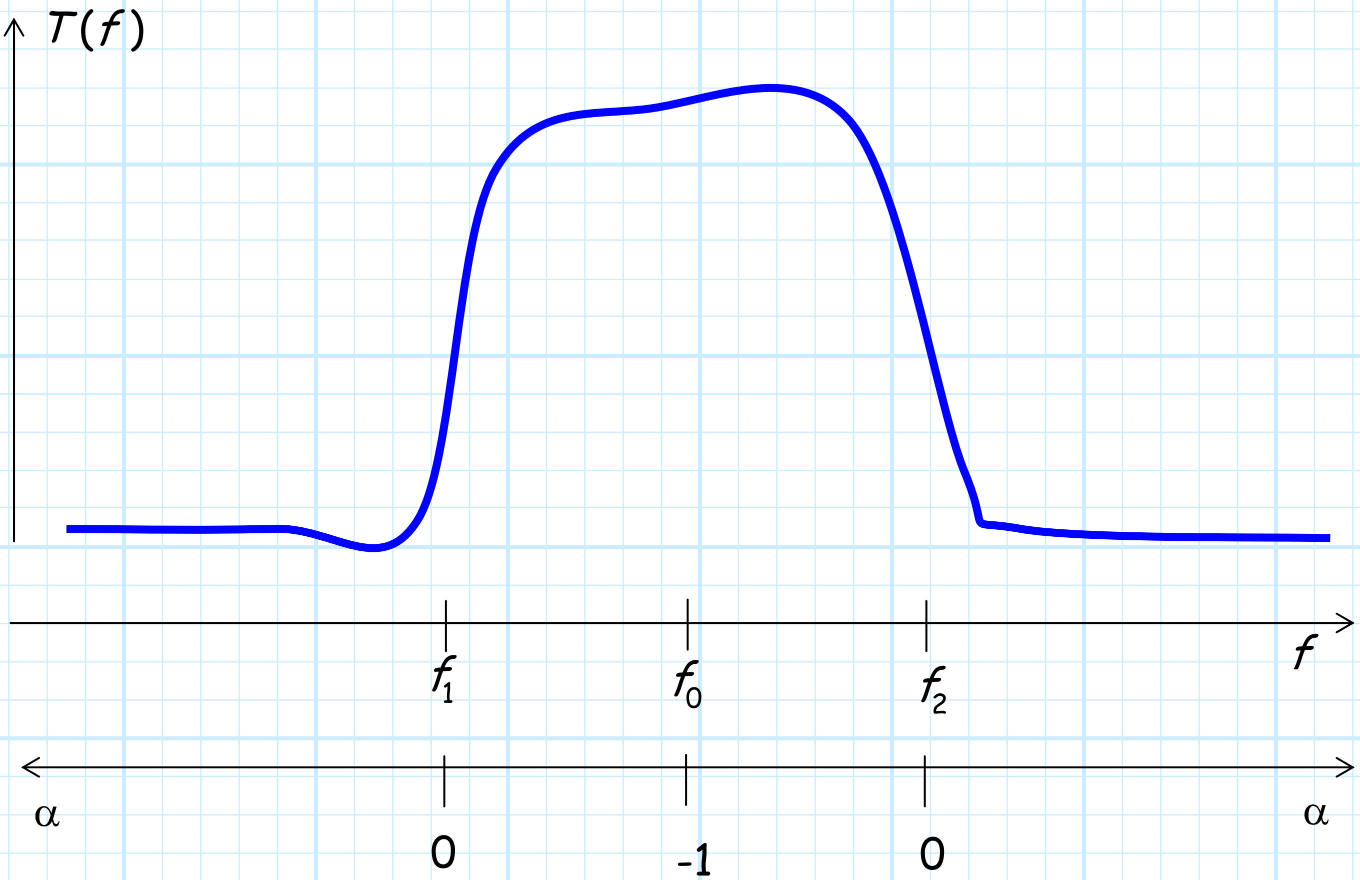
* It turns out that all frequencies f **outside** the pass band of the filter will have **positive** values of α, while frequencies **within** the pass band will result in **negative** values of α.

* Accordingly, if $f = f_1$ or $f = f_2$, the value of α will be **zero** (try it!).

* As a result, the attenuation charts give answers for **positive** values of α only, corresponding to frequencies in the **stop band**.

* In other words, the attenuation charts provide information about the stop band **attenuation** only. Note as α gets **larger**, the attenuation for all filter orders **increases**.

* This makes since, as an increasing α corresponds to a frequency f either greater than f_2 and increasing, or a frequency f less than f_1 and decreasing.



For **example**, consider a Butterworth bandpass filter whose passband extends from 1 GHz to 4 GHz.

Therefore, $f_1 = 1$ GHz and $f_2 = 4$ GHz, resulting in $f_0 = 2$ GHz and $\Delta = 1.5$.

Q1: *By how **much** is a 500 MHz signal attenuated if the filter has order n=6 ?*

For $f = 0.5$ GHz:

$$\begin{aligned}\alpha &= \left|\frac{1}{\Delta}\left(\frac{f}{f_0} - \frac{f_0}{f}\right)\right| - 1 \\ &= \left|\frac{1}{1.5}\left(\frac{0.5}{2.0} - \frac{2.0}{0.5}\right)\right| - 1 \\ &= 1.5\end{aligned}$$

It appears from the **attenuation chart** that this filter attenuates a 500 MHz signal approximately 50 dB.

Q2: *What should the filter order n be, if we need to attenuate signals at 6.6 GHz by at least 40 dB?*

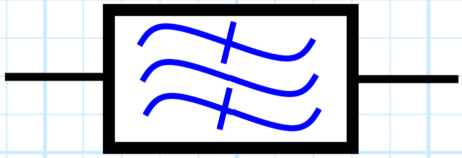
For f = 8 GHz:

$$\begin{aligned}\alpha &= \left|\frac{1}{\Delta}\left(\frac{f}{f_0} - \frac{f_0}{f}\right)\right| - 1 \\ &= \left|\frac{1}{1.5}\left(\frac{6.6}{2.0} - \frac{2.0}{6.6}\right)\right| - 1 \\ &= 1.0\end{aligned}$$

Again from the chart, we find at α = 1.0, a filter with order n =7 (or higher) will attenuate a 6.6 GHz signal by more than 40 dB.

Now **you** too can determine filter attenuation and /or order. I hope you've been **paying attention** !!

Filter Spec Sheet



Kind

Low-pass, high-pass, band-pass, stop-band.

Bandwidth (Hz)

For wideband filters, usually expressed as the **minimum** and **maximum** frequencies with the pass band (i.e., 2GHz to 6 GHz). For narrowband filters, the bandwidth is expressed as the **difference** of the two (e.g. $\Delta f = 1.0 MHz$).

But be careful! Bandwidth is defined with respect to some maximum insertion loss. Typically this is **3.0dB**, but sometimes **other** values are used (e.g., 1.0 dB).

Center Frequency (Hz)

Relevant **only** for band-pass and stop-band, and generally used only for "narrow-band" varieties of each.

Type

Chebychev, Butterworth, Elliptical, Bessel, etc.

Order

An integer value that sometimes is referred to as the number of **sections** or number of **poles**.

Input/Output Impedance

This describes the input impedance for **pass-band** frequencies (remember, the filter is supposed to be mis-matched in the stopband!).

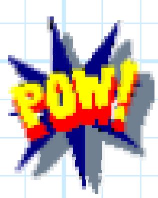
Insertion Loss (dB)

Insertion Loss is the value of $\mathbf{T}(\omega)$ **in the pass band**, expressed in decibels.

$$IL = -10\log_{10} \mathbf{T}(\omega)$$

Although ideally this would be 0 dB ($\mathbf{T}(\omega)=1$), we find that there is always a **little** bit of power **absorbed** by the filter, and thus $\mathbf{T}(\omega)$ is slightly less than one (again, **in the pass-band**). As a result, the insertion loss of most filters is **typically** 1 dB or less (e.g., 0.2 dB), but can approach 2 or 3 dB for filters of very **high order** N.

Maximum Input Power (Watts)



You can only put so much signal **power** into a passive filter! Exceeding this spec will typically result in filter **destruction**.