C. Antenna Pattern

Radiation **Intensity** is dependent on **both** the antenna and the radiated power. We can **normalize** the Radiation Intensity function to construct a result that describes the **antenna** only. We call this normalized function the **Antenna Directivity Pattern.**

HO: Antenna Directivity

The antenna directivity function essentially describes the **antenna pattern**, from which we can ascertain fundamental antenna parameters such as (maximum) **directivity**, **beamwidth**, and **sidelobe** level.

HO: The Antenna Pattern

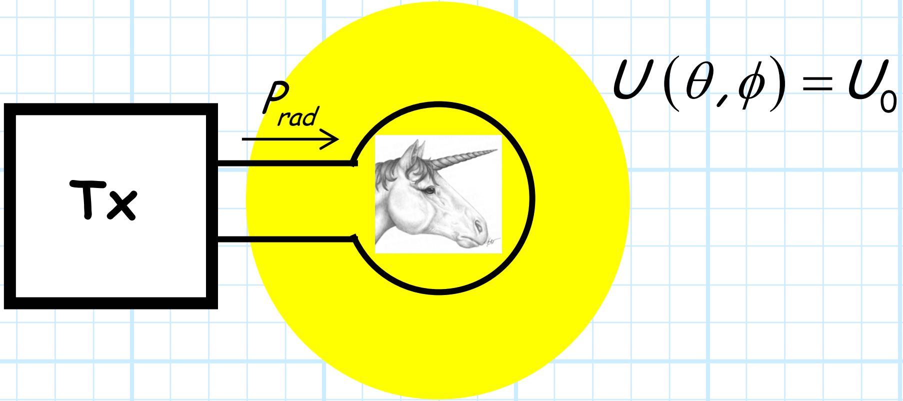
We find that conservation of energy requires a **tradeoff** between antenna (maximum) **directivity** and **beamwidth**—we increase one, we decrease the other.

HO: Beamwidth and Directivity

Antenna Directivity

Recall the **intensity** of the E.M. wave produced by the mythical **isotropic** radiator (i.e., an antenna that radiates **equally** in all directions) is:

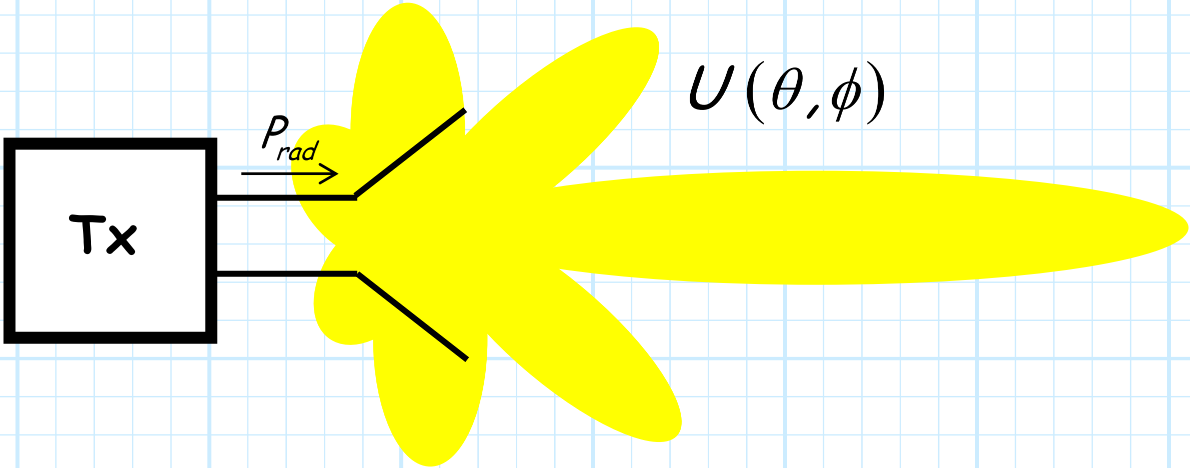
$$U_0 = \frac{P_{rad}}{4\pi}$$



But remember, and isotropic radiator is actually a physical **impossibility**!

If the electromagnetic energy is **monochromatic**—that is, it is a sinusoidal function of time, oscillating at a **one** specific frequency ω—then an antenna **cannot** distribute energy uniformly in all directions.

The intensity function $U(\theta,\phi)$ thus describes this **uneven** distribution of radiated power as a function of direction, a function that is dependent on the design and construction of the **antenna** itself.



Q: *But doesn't the radiation intensity* ***also*** *depend on the power delivered to the antenna by* ***transmitter****?*

A: That's right! **If** the transmitter delivers **no power** to the antenna, then the resulting radiation intensity will likewise be **zero** (i.e., $U(\theta,\phi)=0$).

Q: *So is there some way to* ***remove*** *this dependence on the transmitter power? Is there some function that is dependent on the antenna* ***only****, and thus describes* ***antenna behavior*** *only?*

A: There sure is, and a **very important** function at that!

Will call this function $D(\theta,\phi)$—the **directivity pattern** of the antenna.

The directivity pattern is simply a **normalized** intensity function. It is the intensity function produce by an **antenna** and transmitter, normalized to the intensity pattern produced when the **same** transmitter is connected to an **isotropic** radiator.

$$\boxed{D(\theta,\phi)=\frac{U(\theta,\phi)}{U_0}=\frac{\text{intensity of antenna}}{\text{intensity of isotropic radiator}}}$$

Using $U_0 = P_{rad}/4\pi$, we can likewise express the directivity pattern as:

$$D(\theta,\phi)=\frac{4\pi\, U(\theta,\phi)}{P_{rad}}$$

Q: *Hey wait! I thought that this function was supposed to* ***remove*** *the dependence on transmitter power, but there* ***is*** P_{rad} *sitting* ***smack dab*** *in the middle of the* ***denominator.***

A: The value P_{rad} in the denominator is necessary to **normalize** the function. The reason of course is that $U(\theta,\phi)$ (in the **numerator**) is **likewise** proportional to the radiated power.

In other words, if P_{rad} doubles then **both** numerator and denominator increases by a factor of two—thus, the **ratio** remains **unchanged**, independent of the value P_{rad}.

$$D(\theta,\phi)=\frac{U(\theta,\phi)}{U_0}$$

Another indication that directivity pattern $D(\theta,\phi)$ is independent of the transmitter power are it **units**. Note that the directivity pattern is a **coefficient**—it is unitless!

Perhaps we can rearrange the above expression to make this all more clear:

Dependent on antenna **only**.

Dependent on Tx power **and** the antenna.

$$U(\theta,\phi) = \frac{P_{rad}}{4\pi} D(\theta,\phi)$$

Dependent on Tx power

Hopefully it is apparent that the value of this function $D(\theta,\phi)$ in some direction θ and ϕ describes the intensity in that direction **relative** to that of an isotropic radiator (when radiating the same power P_{rad}).

For **example**, if $D(\theta,\phi) = 10$ in some direction, then the intensity in that direction is **10 times** that produced by an isotropic radiator in that direction.

If in another direction we find $D(\theta,\phi) = 0.5$, we conclude that the intensity in that direction is **half** the value we would find if an isotropic radiator is used.

Q: *So, can the directivity function take **any** form? Are there any **restrictions** on the function $D(\theta,\phi)$?*

A: Absolutely! For example, let's **integrate** the directivity function over **all directions** (i.e., over 4π steradians).

$$
\begin{aligned}
\int_0^{2\pi}\int_0^{\pi} D(\theta,\phi)\, \sin\theta \, d\theta \, d\phi &= \int_0^{2\pi}\int_0^{\pi} \frac{U(\theta,\phi)}{U_0} \sin\theta \, d\theta \, d\phi \\
&= \frac{1}{U_0}\int_0^{2\pi}\int_0^{\pi} U(\theta,\phi)\, \sin\theta \, d\theta \, d\phi \\
&= \frac{4\pi}{P_{rad}}\int_0^{2\pi}\int_0^{\pi} U(\theta,\phi)\, \sin\theta \, d\theta \, d\phi \\
&= \frac{4\pi}{P_{rad}}\left(P_{rad}\right) \\
&= 4\pi
\end{aligned}
$$

Thus, we find that the directivity pattern $D(\theta,\phi)$ of **any** and **all** antenna must satisfy the equation:

$$\int_0^{2\pi}\int_0^{\pi} D(\theta,\phi)\, \sin\theta \, d\theta \, d\phi = 4\pi$$

We can slightly **rearrange** this integral to find:

$$\boxed{\frac{1}{4\pi}\int_0^{2\pi}\int_0^{\pi} D(\theta,\phi)\, \sin\theta \, d\theta \, d\phi = 1.0}$$

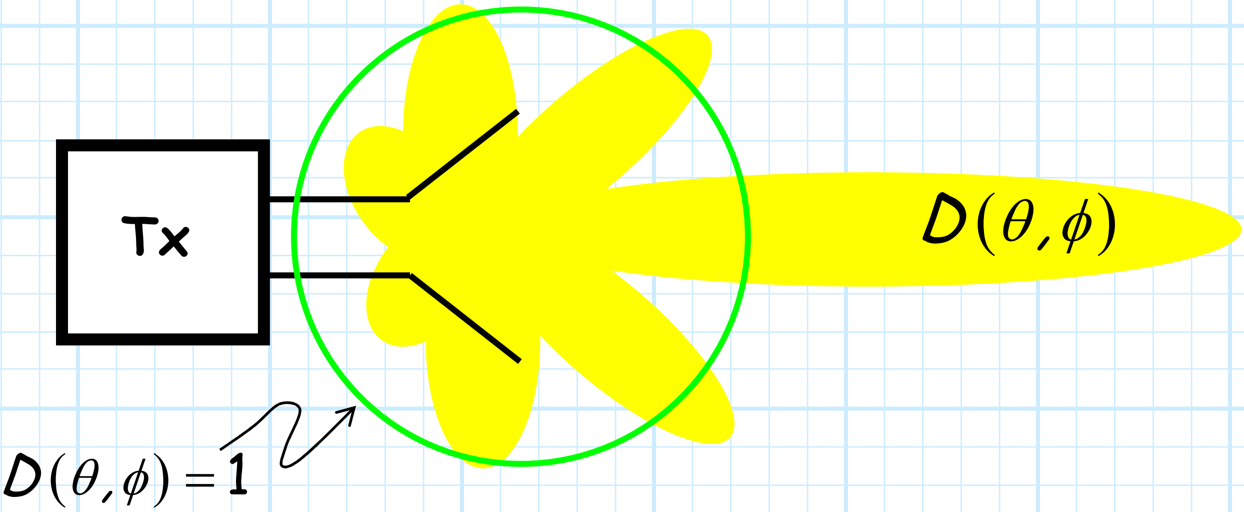
The left side of the equation is simply the **average** value of the directivity pattern (D_{ave}), when averaged over **all directions**—over 4π **steradians**!

The equation thus says that the **average** directivity of **any** and **all** antenna **must** be equal to **one**.

$$D_{ave} = 1.0$$

This means that—on average—the intensity created by an **antenna** will equal the intensity created by an **isotropic radiator**.

→ In some directions the intensity created by any and all antenna will be **greater** than that of an isotropic radiator (i.e., $D > 1$), while in other directions the intensity will be **less** than that of an isotropic resonator(i.e., $D < 1$).



Q: *Can the directivity pattern $D(\theta,\phi)$ equal one for **all directions** θ and ϕ? Can the directivity pattern be the **constant** function $D(\theta,\phi) = 1.0$?*

A: Nope! The directivity function **cannot** be isotropic.

In other words, since:

$$U(\theta,\phi) \neq U_0$$

then:

$$U(\theta,\phi) \neq U_0 \quad \Rightarrow \quad \frac{U(\theta,\phi)}{U_0} \neq \frac{U_0}{U_0} \quad \Rightarrow \quad D(\theta,\phi) \neq 1.0$$

Q: *Does this mean that there is* ***no*** *value of θ and ϕ for which $D(\theta,\phi)$ will equal 1.0?*

A: NO! There will be **many** values of θ and ϕ (i.e., directions) where the value of the directivity function will be equal to **one**!

Instead, when we say that:

$$D(\theta,\phi) \neq 1.0$$

we mean that the directivity function **cannot** be a **constant** (with value 1.0) with respect to θ and ϕ.

The Antenna Pattern

Another term for the directivity pattern $D(\theta,\phi)$ is the **antenna pattern**. Again, this function describes how a specific antenna distributes energy as a function of direction.

An **example** of this function is:

$$D(\theta,\phi) = c\,(1+cos\,\phi)^2\, sin^2\theta$$

where c is a constant that **must** be equal to:

$$c = \frac{4\pi}{\int_0^{2\pi}\int_0^{\pi}(1+cos\,\phi)^2\, sin^2\theta\, sin\,\theta\, d\theta\, d\phi}$$

Do **you** see why c must be equal to this value?

Q: *How can we **determine** the antenna pattern of given antenna? How do we **find** the explicit form of the function $D(\theta,\phi)$?*

A: There are **two ways** of determining the pattern of a given antenna

1. **By electromagnetic analysis** - Given the size, shape, structure, and material parameters of an antenna, we can use Maxwell's equations to determine the **function** $D(\theta,\phi)$.

However, this analysis often must resort to **approximations** or assumption of ideal conditions that can lead to some **error**.

$$\mathbf{E}^{\mathbf{s}}_{\mathbf{n}}(\bar{r}) =$$

$$\frac{(\epsilon_r-1)}{4\pi}\,f(z_n)\int_{-k\Delta\ell}^{k\Delta\ell}\int_0^{2\pi}\int_0^{ka}\frac{\hat{x}2}{(\epsilon_r+1)}\,\frac{e^{\imath\sqrt{k\rho'^2+(ku-k\delta)^2}}}{\sqrt{k\rho'^2+(ku-k\delta)^2}}\,dk\rho'\,d\phi'\,dk\delta'$$

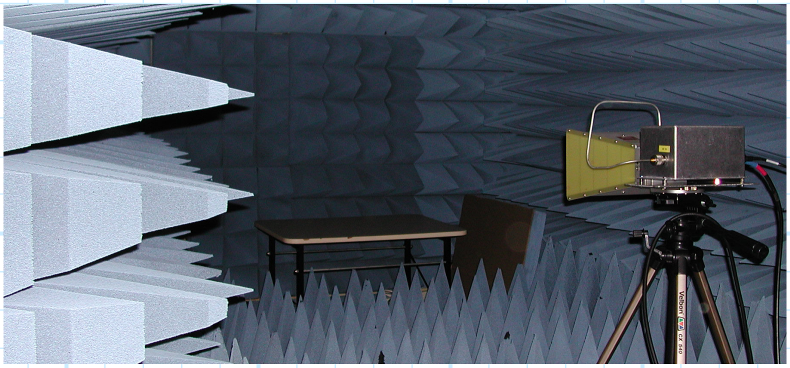
$$-\imath\frac{(\epsilon_r-1)}{4\pi}\,f(z_n)\int_{-k\Delta\ell}^{k\Delta\ell}\int_0^{2\pi}(\cos\phi'\hat{x}+\sin\phi'\hat{y})\cdot\frac{\hat{x}2}{(\epsilon_r+1)}\,\frac{e^{\imath\sqrt{k\rho'^2+(ku-k\delta)^2}}}{(\sqrt{k\rho'^2+(ku-k\delta)^2})^2}$$

$$\Big(-ka\cos\phi'\hat{x}-ka\sin\phi'\hat{y}+(ku-k\delta')\hat{z}\Big)\,ka\,d\phi'\,dk\delta'$$

$$+\frac{(\epsilon_r-1)}{4\pi}\,f(z_n)\int_{-k\Delta\ell}^{k\Delta\ell}\int_0^{2\pi}(\cos\phi'\hat{x}+\sin\phi'\hat{y})\cdot\frac{\hat{x}2}{(\epsilon_r+1)}\,\frac{e^{\imath\sqrt{k\rho'^2+(ku-k\delta)^3}}}{(\sqrt{k\rho'^2+(ku-k\delta)^2})^3}$$

$$\Big(-ka\cos\phi'\hat{x}-ka\sin\phi'\hat{y}+(ku-k\delta')\hat{z}\Big)\,ka\,d\phi'\,dk\delta'$$

2. **By direct measurement** - We can directly measure the antenna pattern in the laboratory. This has the advantage that it requires **no** assumptions or approximations, so it **may** be more accurate.

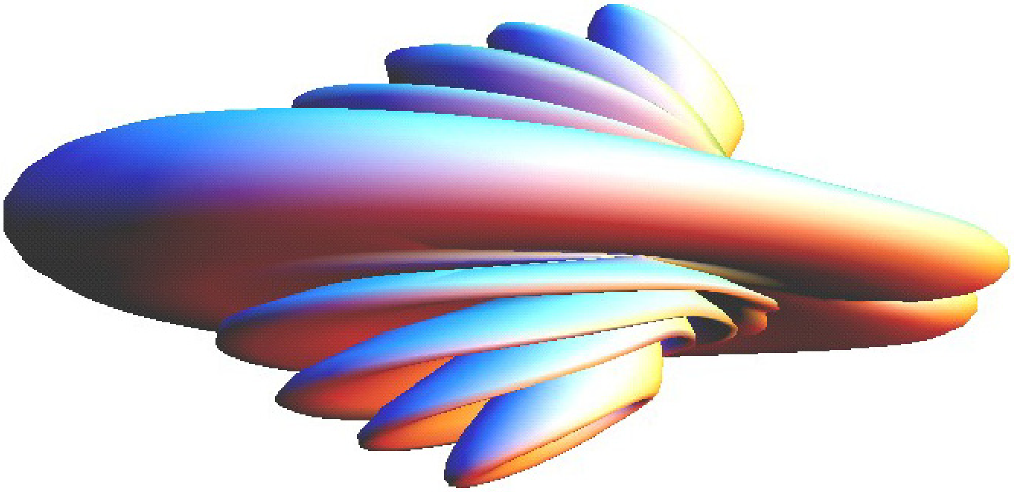


However, accuracy ultimately depends on the **precision** of your measurements, and the result $D(\theta,\phi)$ is provided as a **table** of measured data, as opposed to an explicitly mathematical function.

Q: *Functions and tables!? Isn't there some way to simply plot the antenna pattern $D(\theta,\phi)$?*

A: Yes, but it is a bit **tricky**.

Remember, the function $D(\theta,\phi)$ describes how an antenna distributes energy in **three** dimensions. As a result, it is difficult to plot this function on a **two-dimensional** sheet (e.g., a page of your notes!).

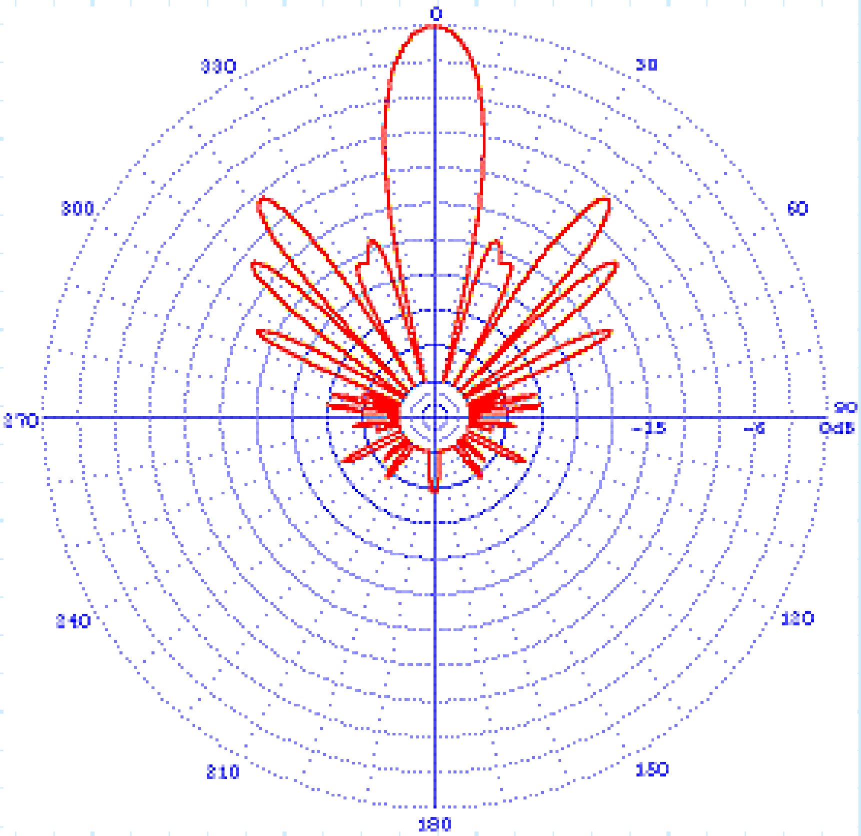


Antenna patterns are thus typically plotted as "**cuts**" in the antenna pattern—the value of $D(\theta,\phi)$ on a (two-dimensional) **plane**.

* For **example**, we might plot $D\left(\theta = 90^{\circ},\phi\right)$ as a function of ϕ. This would be a plot of $D(\theta,\phi)$ on the ***x-y* plane**.

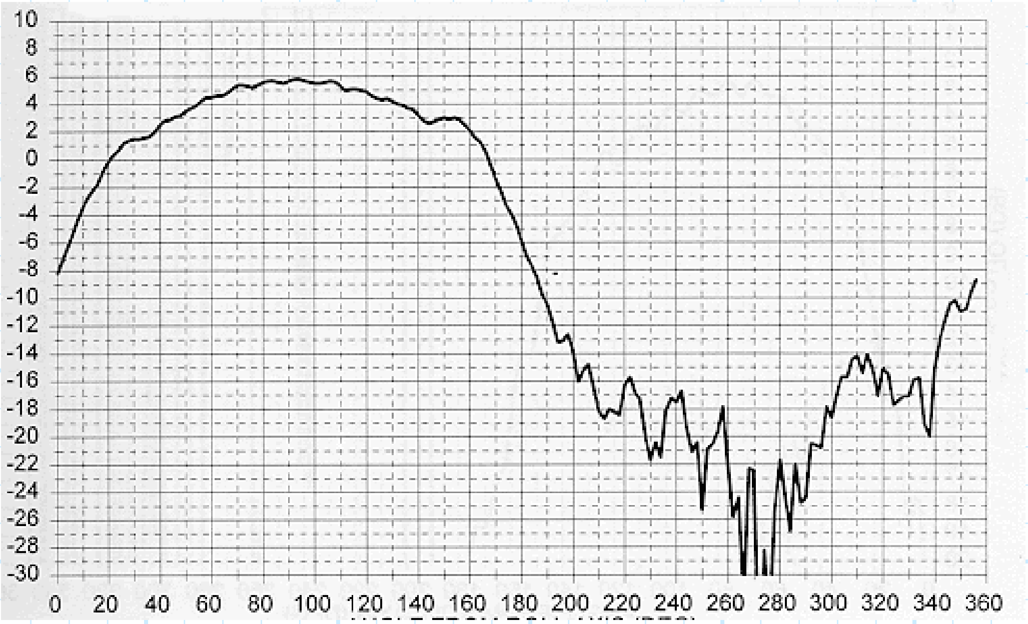
* Or, we might plot $D(\theta,\phi = 0)$ as a function of θ. This would be a plot of $D(\theta,\phi)$ along the ***x-z* plane**.

Sometimes these cuts are plotted in **polar** format, and other times in **Cartesian**.

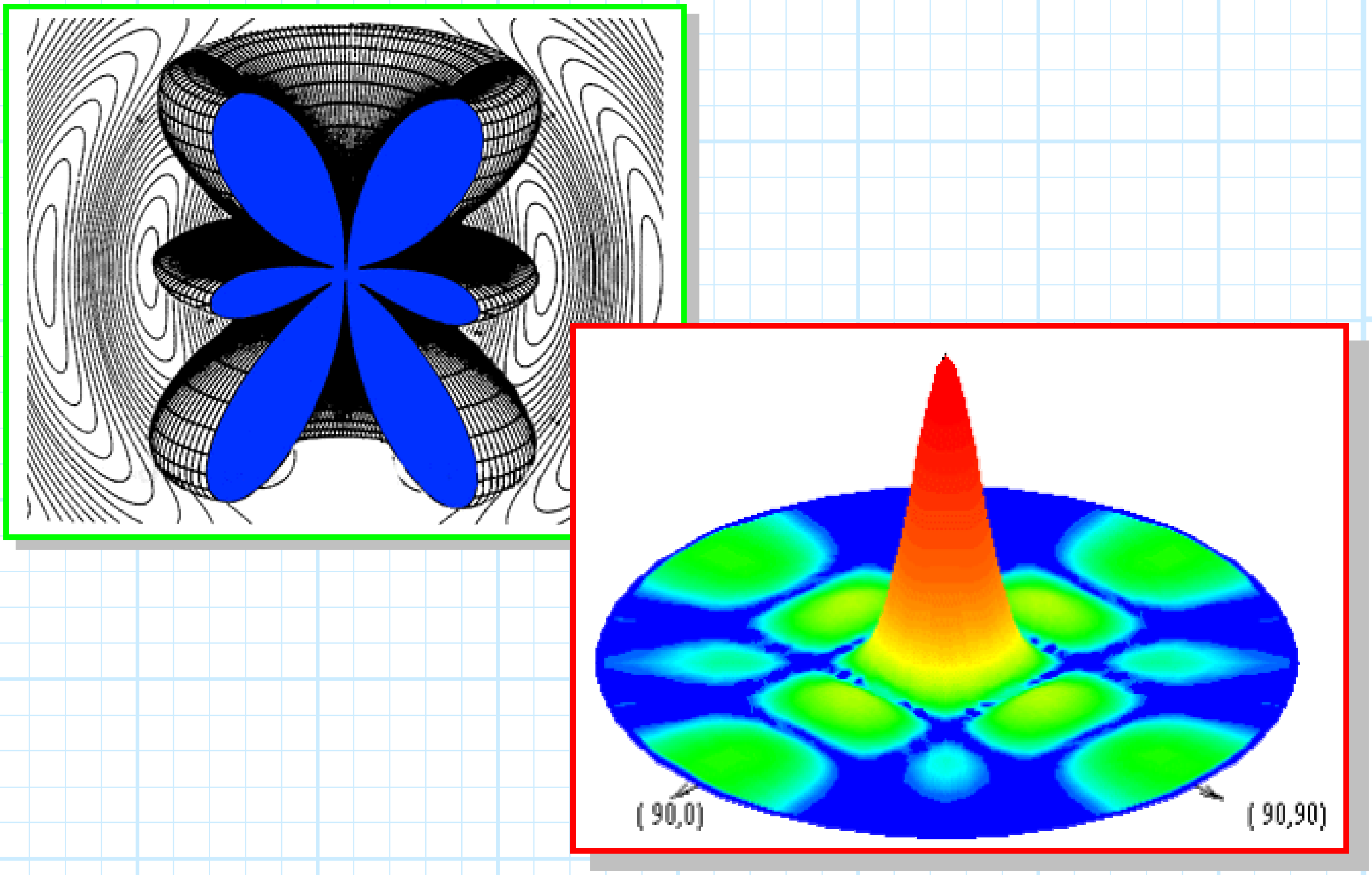


Polar plot of antenna cut $D\left(\theta = \pi/2, \phi\right)$ as a function of ϕ.

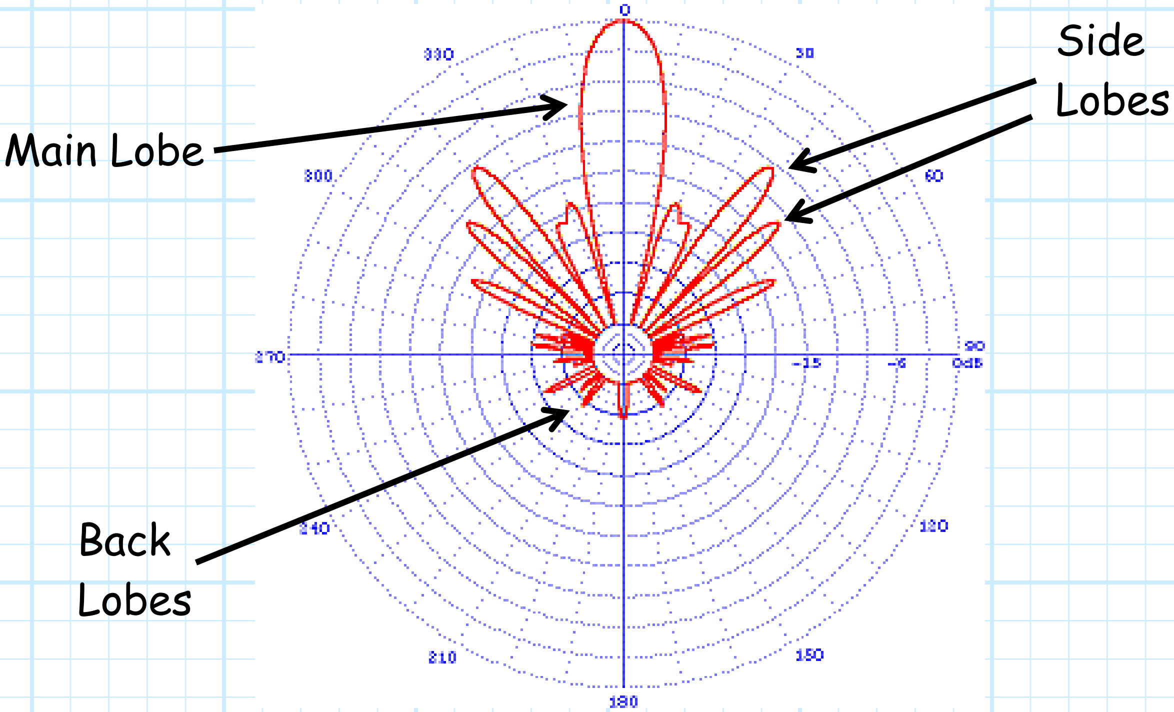
Cartesian plot of antenna cut $D\left(\theta = \pi/2, \phi\right)$ as a function of ϕ.



The entire function $D(\theta,\phi)$ can likewise be plotted in **3-D** for either polar or Cartesian (**if** you have the proper software!).



Note that the majority of antenna patterns consist of a number of “**lobes**”.



Note these lobes have both a **magnitude** (the largest value of $D(\theta,\phi)$ within the lobe), and a **width** (the size of the lobe in steradians).

* Note that every antenna pattern has a direction(s) where the function $D(\theta,\phi)$ is at its **peak** value. The lobe associated with this peak value (i.e., the lobe with the largest magnitude) is known as the antennas **Main Lobe.**

* The main lobe is typically surrounded by **smaller** (but significant) lobes called **Side Lobes**.

* There frequently are also **very small** lobes that appear in the pattern, usually in the opposite direction of the main lobe. We call these tiny lobes **Back Lobes**.

The important characteristics of an antenna are defined by the **main lobe**. Generally, side and back lobes are **nuisance** lobes—we ideally want them to be as **small as possible**!

Q: *These plots and functions describing antenna pattern $D(\theta,\phi)$ are very* ***complete*** *and helpful, but also a bit busy and complex. Are there some* ***set of values*** *that can be used to indicate the important* ***characteristics*** *of an antenna pattern?*

A: Yes there is! The **three** most important are:

1. Antenna Directivity D_0 .
2. Antenna Beamwidth .
3. Antenna Sidelobe level.

Directivity and Beamwidth

One of the most fundamental of antenna parameters is **antenna directivity**.

$$D_0 \doteq Directivity$$

Q: *Antenna directivity? Haven't we **already** studied this? Isn't directivity $D(\theta,\phi)$?*

A: NO! Recall that $D(\theta,\phi)$ is known as the directivity **pattern** (a.k.a. the **antenna pattern**). Unlike the directivity pattern $D(\theta,\phi)$, which is a **function** of coordinates θ and ϕ, antenna directivity is simply a **number** (e.g., 100 or 20 dB).

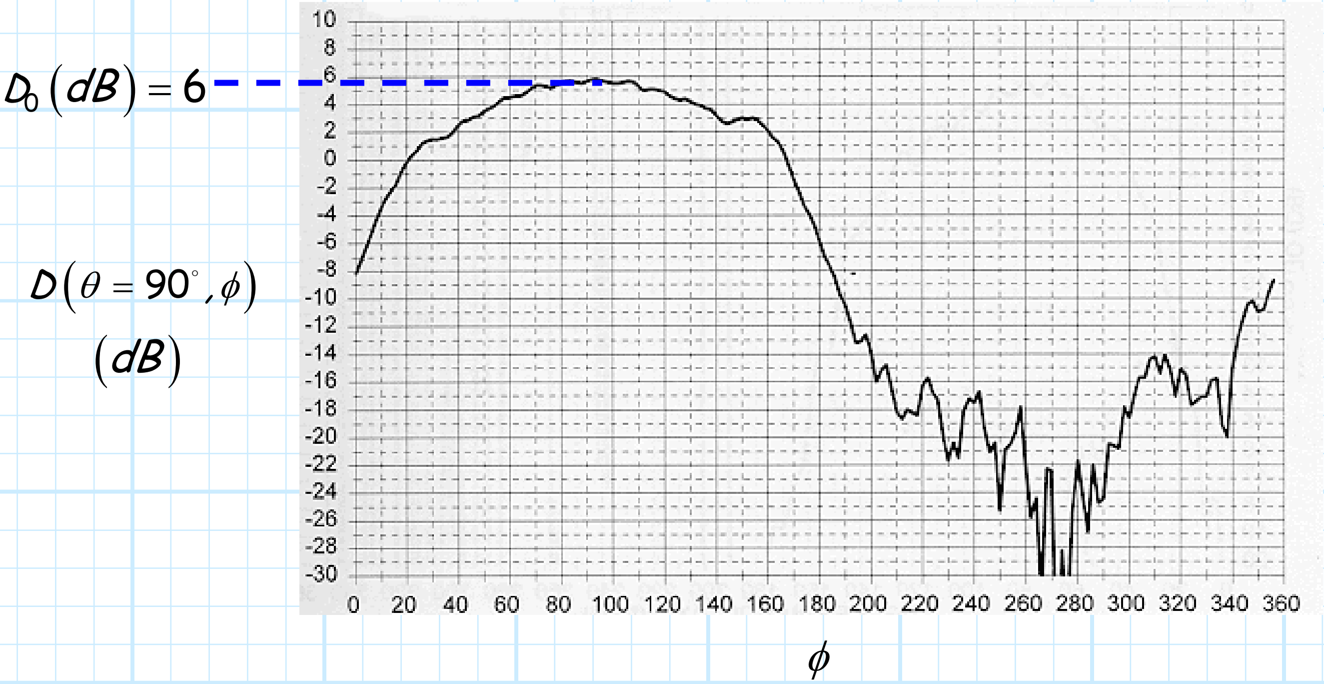
Q: *But isn't antenna directivity somehow **related** to antenna pattern $D(\theta,\phi)$?*

A: Most definitely!

The directivity of an antenna is simply equal to the **largest value** of the directivity pattern:

$$D_0 = \max_{\theta,\phi} \{D(\theta,\phi)\}$$

Thus, the directivity of an antenna is generally determined from the magnitude (i.e., peak) of the **main lobe**.



Note that directivity is likewise a **unitless** value, and thus is often **expressed in dB**.

Another fundamental antenna parameter is the **antenna beamwidth**.

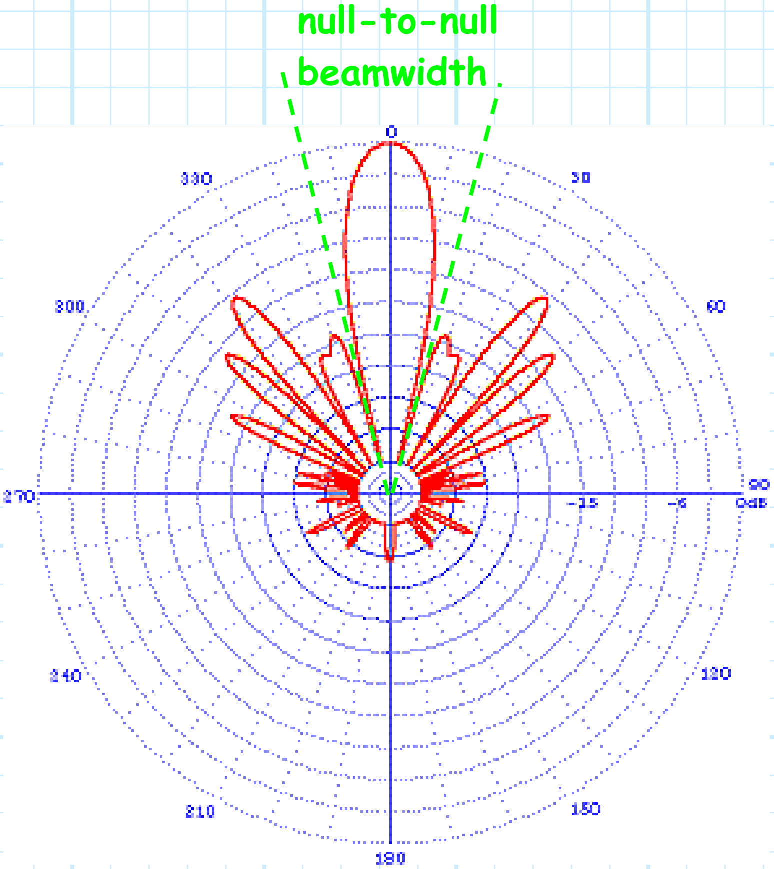
$$\Omega_A \doteq \text{beamwidth} \quad \left[\textit{steradians}\right]$$

Just like the "bandwidth" of a microwave device, antenna beamwidth is a **subjective** value. Ideally, we can say that the beamwidth is the size of the antenna **mainlobe**, expressed in steradians.

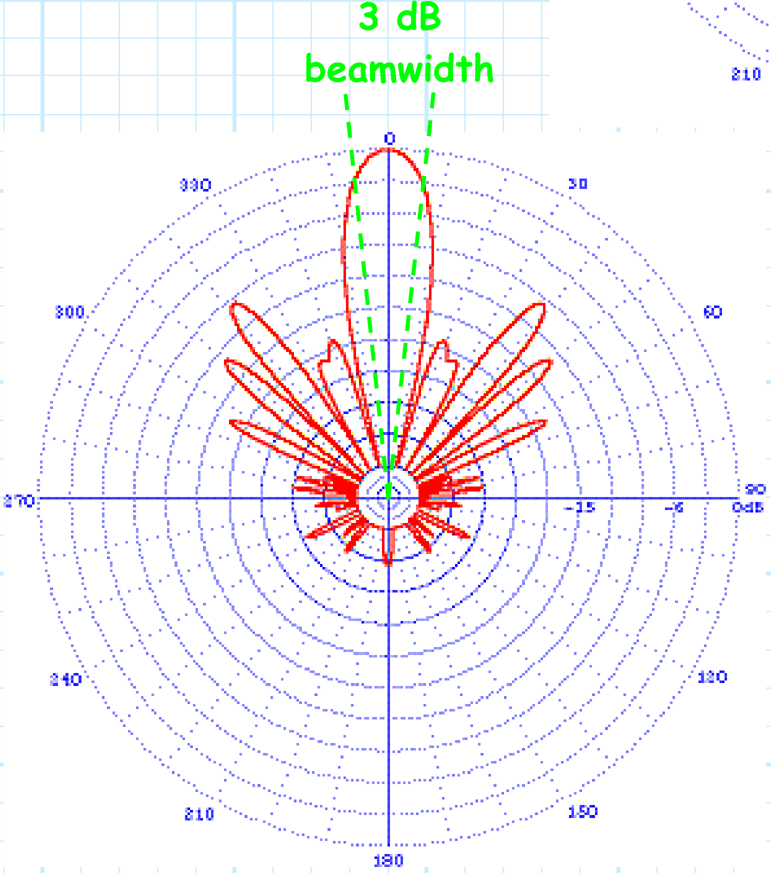
Q: *But how do we* ***define*** *the "size" of the mainlobe?*

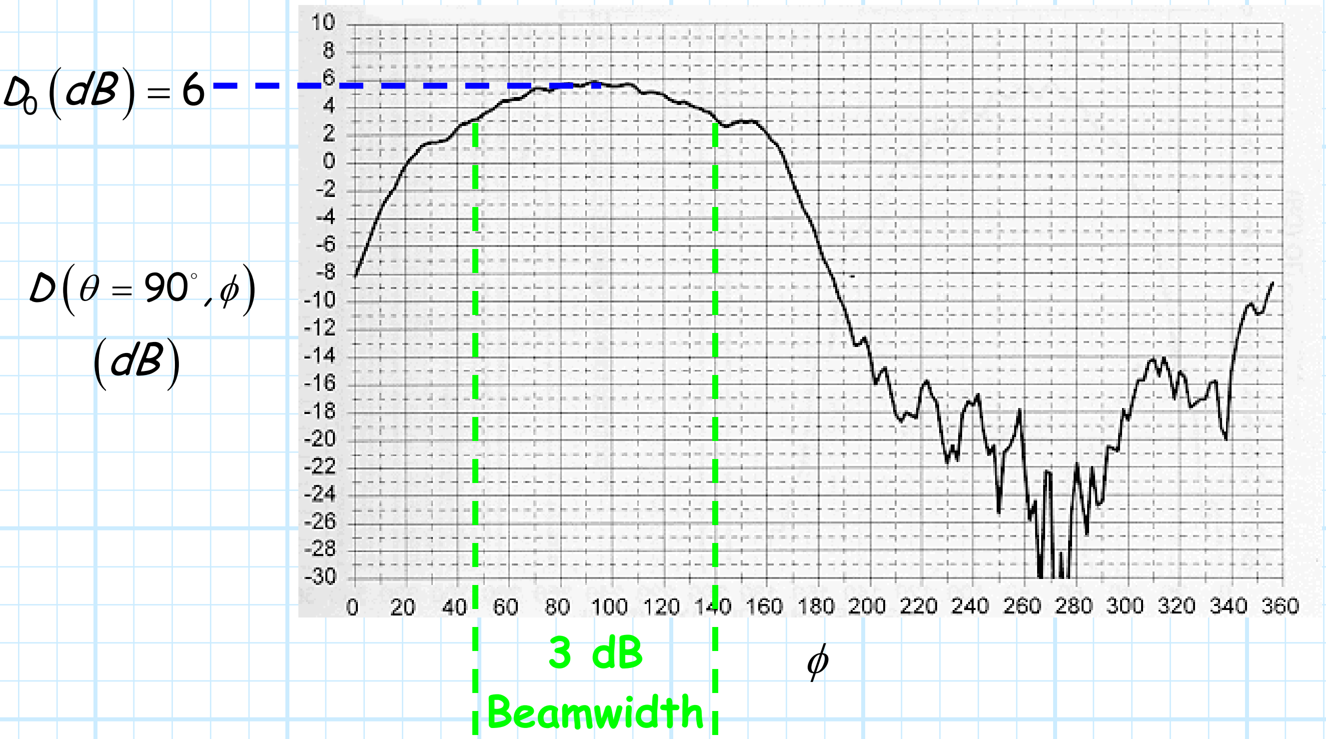
A: That's the **subjective** part!

Sometimes, we define beamwidth as the "**null-to-null**" beamwidth:



But much more common is the **3dB** beamwidth, defined by the points on the mainlobe where the directivity pattern $D(\theta,\phi)$ has a value of one half that of value directivity D_0 (i.e., 3 dB less than $D_0(dB)$):





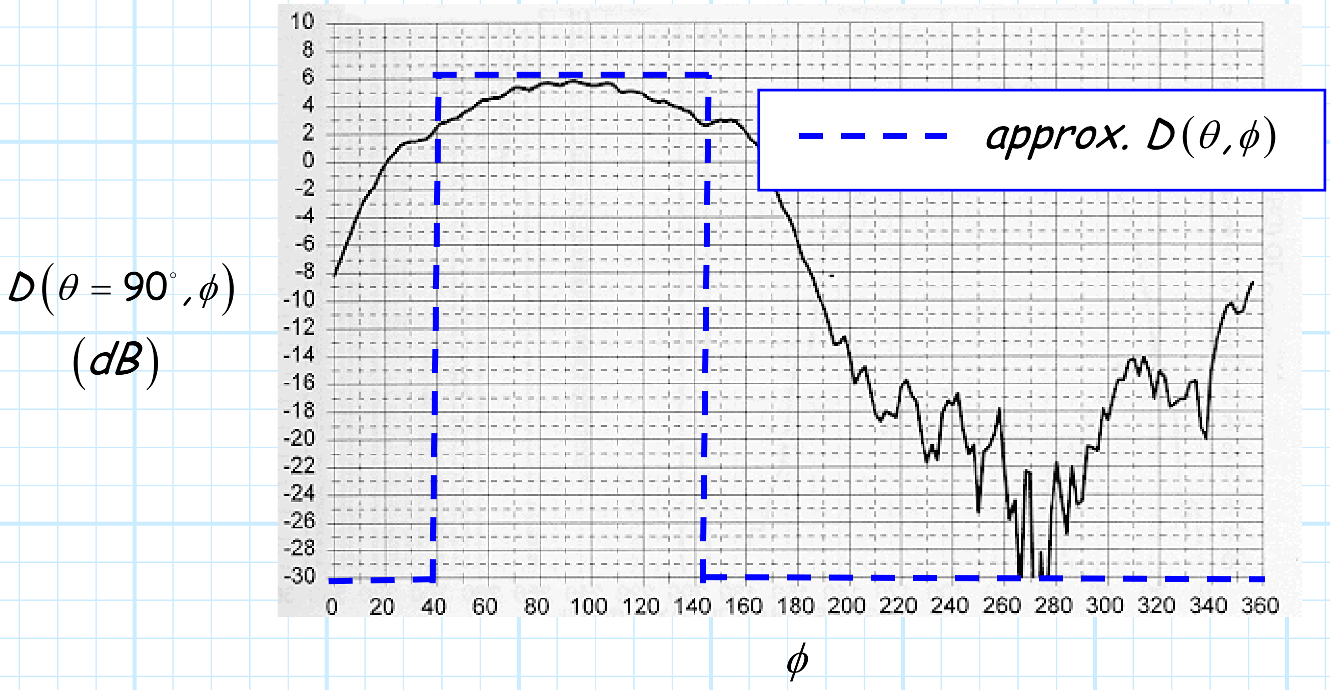
Q: *But how do we* ***determine*** *the antenna beamwidth* Ω_A *?*

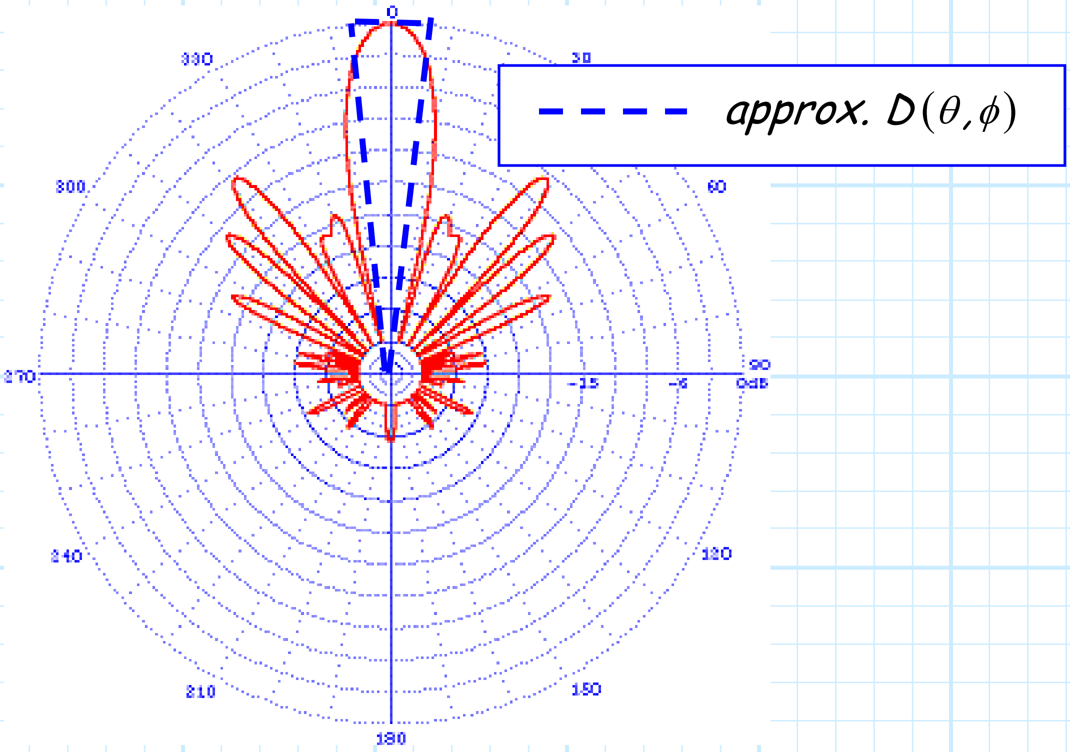
A: Theoretically, we can use either of the beamwidth definitions above and **integrate** over all directions θ and ϕ that lie within the mainlobe:

$$\Omega_A = \iint\limits_{\substack{main\\lobe}} \sin\theta \, d\theta \, d\phi$$

However, we more often use an **approximation** to determine the antenna beamwidth. **If** the sidelobes of an antenna are small, then we can **approximate** its directivity pattern as:

$$D(\theta,\phi) \approx \begin{cases} D_0 & \text{within the mainbeam} \\ 0 & \text{outside the mainbeam} \end{cases}$$





In other words, this approximation "says" that the antenna radiates its power **uniformly throughout the mainlobe**, but radiates **no energy** in any other direction.

This of course is a fairly **rough** approximation, but we can use it to determine (approximately) the antenna **beamwidth** Ω_A.

To see how, first **recall** that the **average** directivity of any antenna (averaged over 4π steradians) is:

$$1.0 = \frac{1}{4\pi} \int_0^{2\pi} \int_0^{\pi} D(\theta,\phi) \sin\theta \, d\theta \, d\phi$$

Inserting our approximation into this integral, we find:

$$\begin{aligned} 1.0 &= \frac{1}{4\pi} \iint_{\substack{main \\ lobe}} D(\theta,\phi) \sin\theta \, d\theta \, d\phi + \frac{1}{4\pi} \iint_{\substack{side \\ lobe}} D(\theta,\phi) \sin\theta \, d\theta \, d\phi \\ &= \frac{1}{4\pi} \iint_{\substack{main \\ lobe}} D_0 \sin\theta \, d\theta \, d\phi + \frac{1}{4\pi} \iint_{\substack{side \\ lobe}} 0 \sin\theta \, d\theta \, d\phi \\ &= \frac{D_0}{4\pi} \iint_{\substack{main \\ lobe}} \sin\theta \, d\theta \, d\phi \end{aligned}$$

Look! Recall the integral above is the **beamwidth** of the antenna:

$$\Omega_A = \iint_{\substack{main \\ lobe}} \sin\theta \, d\theta \, d\phi$$

And so:

$$1.0 = \frac{D_0}{4\pi} \iint_{\substack{main \\ lobe}} \sin\theta \, d\theta \, d\phi = \frac{D_0}{4\pi} \Omega_A$$

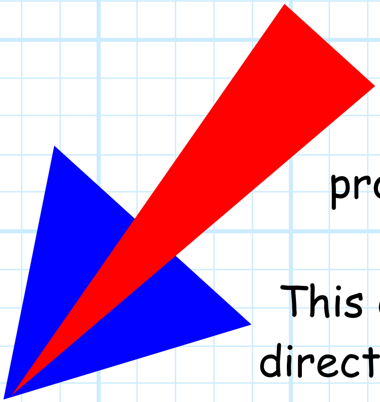
Rearranging, we find an important result:

$$\boxed{D_0 \, \Omega_A = 4\pi}$$

This says that the **product** of the antenna directivity and antenna beamwidth is a **constant** (i.e., 4π).

Q: *So what?*

A: This means—yet again—that we **cannot** "have our cake and eat it too"! If we **increase** the directivity of an antenna, then its beamwidth must **decrease**.



Conversely, if we increase antenna **beamwidth**, its **directivity** must diminish proportionately.

This of course makes sense; we can **increase** directivity only by "**crushing**" the available power into a **smaller** solid angle (i.e., the main lobe beamwidth Ω_A).

Moreover, the expression above allows us to **determine**—given beamwidth Ω_A—the (approximate) value of antenna **directivity**:

$$D_0 = \frac{4\pi}{\Omega_A}$$

Note from this equation we can "define" antenna directivity as the **ratio** of the **beamwidth** of an isotropic radiator (4π) to the **beamwidth** of the antenna (Ω_A)!

$$\boxed{D_0 = \frac{4\pi}{\Omega_A} = \frac{\text{beamwidth of isotropic radiator}}{\text{beamwidth of antenna}}}$$

Likewise, we can—**given** antenna directivity D_0—determine the antenna **beamwidth**:

$$\boxed{\Omega_A = \frac{4\pi}{D_0}}$$

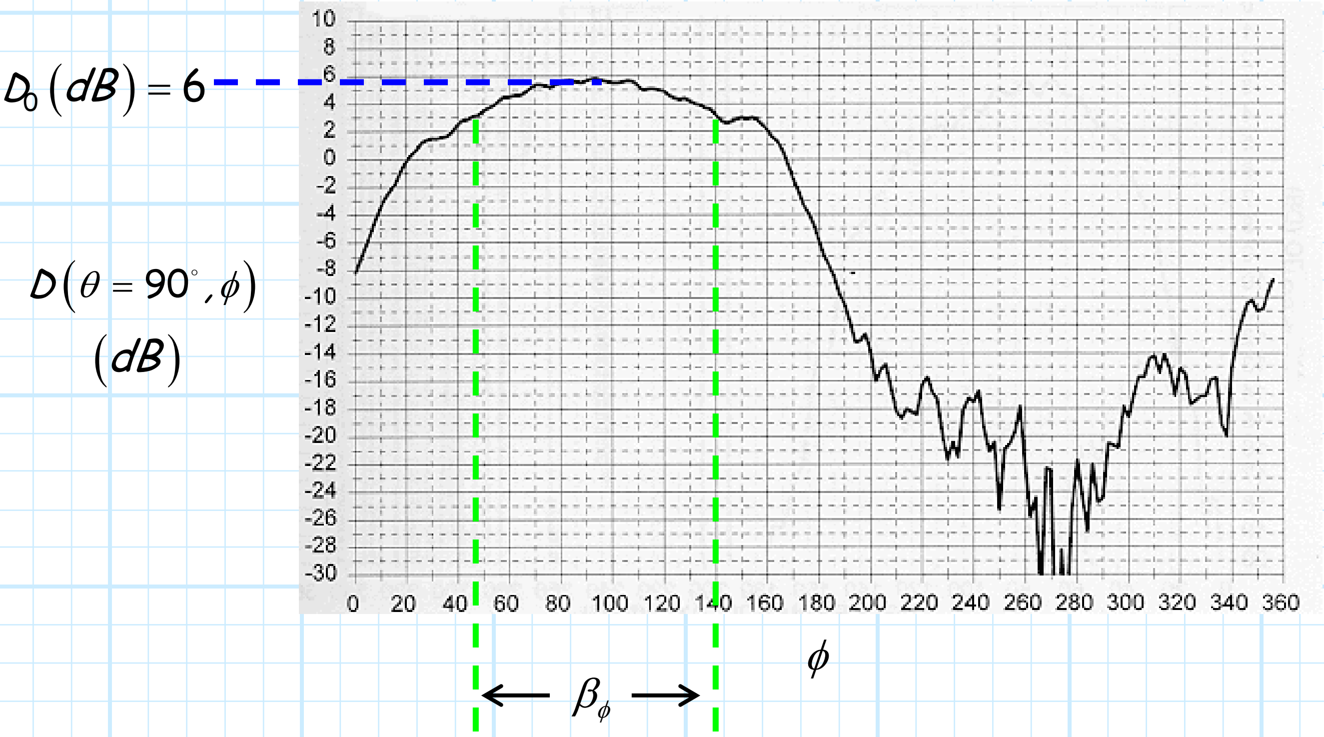
Thus, by simply determining the **maximum** value of function $D(\theta,\phi)$ (i.e., D_0), we can easily determine an **approximate** value of antenna **beamwidth** (in steradians) using the equation shown **above**!

Q: *Now, Ω_A tells us the **size** of the mainlobe solid angle (in steradians), but it does **not** tells its **shape**. Didn't you say that solid angles with **different** shapes can have the **same** size Ω_A?*

A: That's exactly correct!

Recall that our **3-D** beam pattern $D(\theta,\phi)$ is often plotted on two, orthogonal **2-D planes**. We can define the beamwidth on each of these two **planes** in terms of **radians** (or degrees).

For example, we might plot $D(\theta,\phi)$ on the ***x-y* plane** (i.e., $D(\theta = \pi/2, \phi)$) and find that its **(2-D)** 3dB beamwidth has a value (in radians) that we'll call β_ϕ.



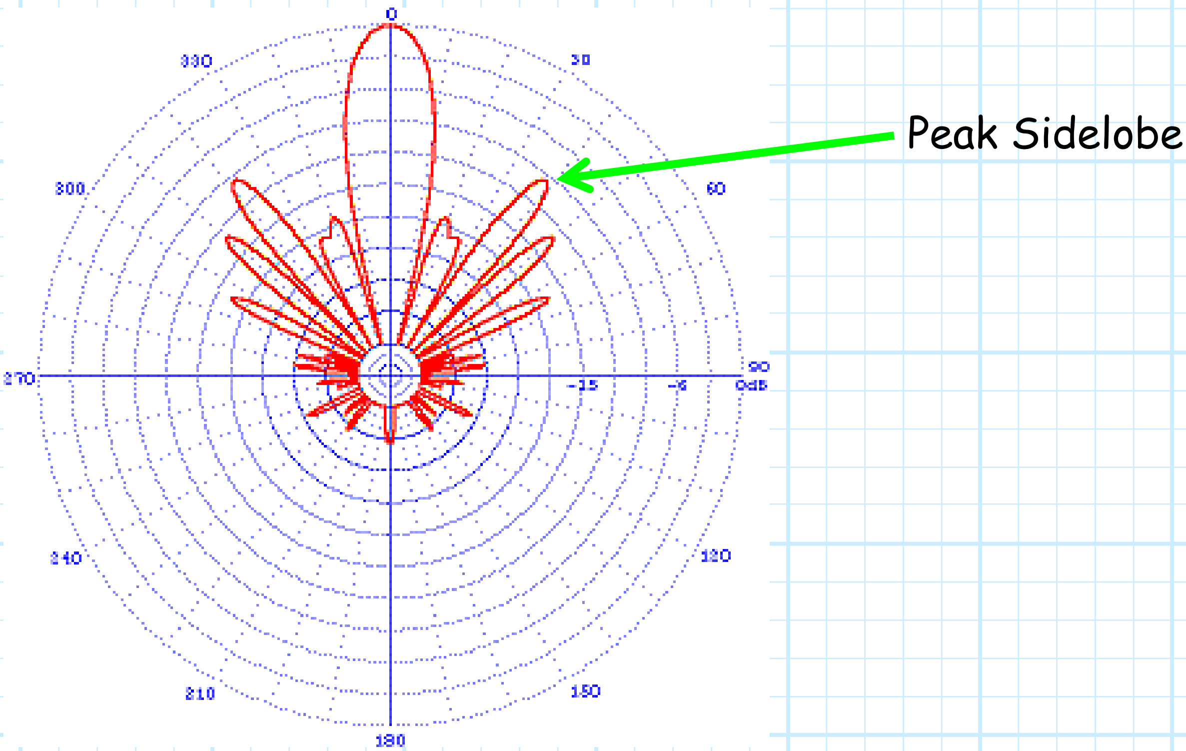
We could likewise plot $D(\theta,\phi)$ on the ***x-z* plane** (i.e., $D(\theta, \phi = 0)$) and find that its **(2-D)** 3dB beamwidth has a value (in radians) that we'll call β_θ.

We find that antenna beamwidth is **often** expressed in terms of these two angles (β_θ and β_ϕ), as opposed to the value of the solid angle Ω_A in steradians.

Finally, a third fundamental antenna parameter that we can extract from antenna pattern $D(\theta,\phi)$ is the **peak sidelobe level**.

This provides a measure of the magnitude of the **sidelobes**, as **compared** to the directivity of the mainlobe. Say we define the largest value of $D(\theta,\phi)$ found in the **sidelobes** (i.e., **outside** the mainlobe) as the **peak sidelobe directivity**:

$$D_{sl} \doteq \max_{\substack{side\\lobes}} \left\{ D(\theta,\phi) \right\}$$



Peak Sidelobe

We can then **normalize** this value to antenna directivity D_0. This value is known as the **peak sidelobe level**, and is typically expressed in **dB**:

$$\boxed{\text{peak sidelobe level} \doteq 10\, log_{10}\left[\frac{D_{sl}}{D_0}\right]}$$

Sidelobes are generally considered to be a **non-ideal** artifact in antenna patterns. Essentially, sidelobe levels represent a **waste of energy**—electromagnetic propagation in directions **other** than the desired direction of the mainlobe.

Thus, we generally desire a peak sidelobe level that is a **small** as possible (e.g., < -40 dB).