B. Extinction

Of course, the channel between the transmit and receive antenna is **not** generally free-space, but instead is filled with all sorts of annoying **stuff**.

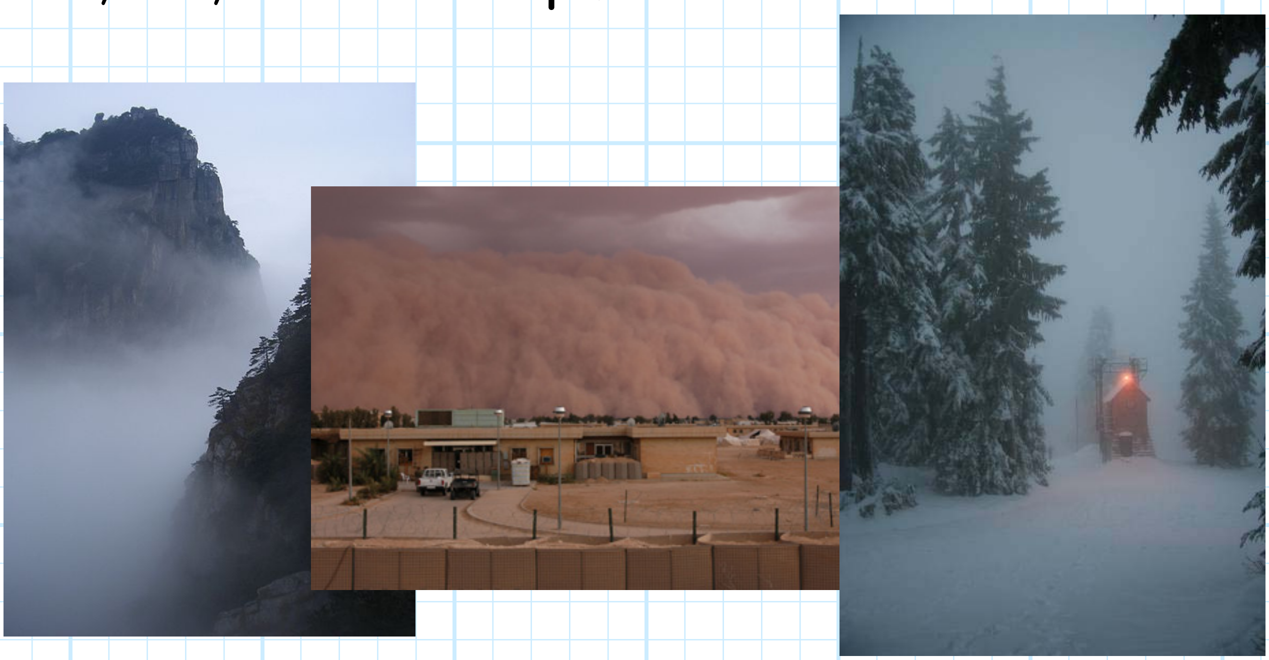
Among this stuff are atmospheric **gases** such as oxygen and water vapor. The gases can **surprisingly** affect radio wave propagation, particularly at specific frequencies!

HO: The Wave Attenuation

HO: The Extinction Coefficient

HO: Attenuation by Atmospheric Gases

In addition to gases, the atmosphere can also be filled with **particles**—particles such as **water droplets** (i.e., clouds/fog), **snow**, **dust**, and/or **rain drops**.



Each of these particles can **extract energy** from a propagating electromagnetic wave.

HO: The Extinction Cross-Section

If the atmosphere is **filled** with particles (otherwise know as hydrometeors), we find that a propagating electromagnetic wave likewise exhibits **exponential extinction** as it propagates!

HO: Extinction Due to Hydrometeors

Wave Attenuation

Recall that **a** time-harmonic **plane-wave** solution to the vector wave equation is:

$$\mathbf{E}(\bar{r},t) = \left(E_x\,\hat{\mathbf{x}} + E_y\,\hat{\mathbf{y}}\right) e^{j(k_0 z - \omega t)}$$

where:

$$k_0 = \frac{\omega}{c} = \omega\sqrt{\mu_0\,\varepsilon_0}$$

This of course describes a wave propagation in **free-space**, but in reality, e.m. waves on Earth propagate in an **atmosphere**.

Q: *Pfft! Aren't you being a bit picky? The Earth's atmosphere is **practically** free-space!*

A: Nope! From many problems, approximating the atmosphere as free space **is** accurate. But, for many **other** problems, it is **not at all** accurate!

The accuracy of the **free-space approximation** generally depends on **three** things:

1. The **frequency** of the propagating wave.

2. The **distance** over which we are attempting to propagate.

3. What's occurring in the atmosphere (i.e., the **weather** conditions!)

The **permeability** and **permittivity** of free space is, by definition:

$$\varepsilon = \varepsilon_0 \qquad \Rightarrow \qquad \varepsilon_r = \frac{\varepsilon}{\varepsilon_0} = 1$$

Likewise:

$$\mu = \mu_0 \qquad \Rightarrow \qquad \mu_r = \frac{\mu}{\mu_0} = 1$$

For the **Earth's atmosphere**, we find that $\mu_r = 1$, but the permittivity of the Earth's atmosphere is **complex**:

$$\varepsilon = \varepsilon' + j\varepsilon''$$

where:

$$\varepsilon' = Re\{\varepsilon\} \qquad \qquad \varepsilon'' = Im\{\varepsilon\}$$

Q: *Yikes! The dielectric constant of the Earth's atmosphere is **complex**?? What the heck does that **mean**?*

A: Since the permittivity is complex, so too is our **propagation constant** k:

$$k = \omega\sqrt{\mu_0 \varepsilon} = \omega\sqrt{\mu_0\left(\varepsilon' + j\varepsilon''\right)} = k_r + j\, k_i$$

where:

$$k_r = Re\{k\} \qquad \qquad k_i = Im\{k\}$$

And so a plane wave propagating in the **Earth's atmosphere** has the form:

$$
\begin{aligned}
\mathbf{E}(\bar{r},t) &= \left(E_x\ \hat{\mathbf{x}} + E_y\ \hat{\mathbf{y}}\right) e^{j(kz-\omega t)} \\
&= \left(E_x\ \hat{\mathbf{x}} + E_y\ \hat{\mathbf{y}}\right) e^{j(k_r + j k_i)z} e^{-j\omega t} \\
&= \left(E_x\ \hat{\mathbf{x}} + E_y\ \hat{\mathbf{y}}\right) e^{-k_i z}\ e^{j k_r z} e^{-j\omega t} \\
&= e^{-k_i z} \left(E_x\ \hat{\mathbf{x}} + E_y\ \hat{\mathbf{y}}\right) e^{j(k_r z-\omega t)}
\end{aligned}
$$

Compare and contrast this result with that of **free-space**:

$$\mathbf{E}(\bar{r},t) = \left(E_x\ \hat{\mathbf{x}} + E_y\ \hat{\mathbf{y}}\right) e^{j(k_0 z-\omega t)}$$

First, note that the propagating wave in the atmosphere is described using a **new** term:

$$e^{-k_i z}$$

Note this is a **real-valued** function of z!

If we plot the function, we see that it **decreases exponentially** with respect to z:

Q: What does this result **physically** mean?

A: Let's look at the **magnitude** of this propagating wave:

$$|\mathbf{E}(\bar{r},t)| = \sqrt{\mathbf{E}(\bar{r},t)\cdot\mathbf{E}^*(\bar{r},t)} = e^{-k_i z}\sqrt{|E_x|^2+|E_y|^2}$$

Note that **unlike** a plane-wave propagating in free space, the magnitude of a plane-wave in the atmosphere is **not a constant** throughout all space.

Instead, the magnitude of the wave **diminishes exponentially** as it propagates!

Q: *But this implies that that something is **extracting energy** from the propagating wave?*

A: Exactly! The constituents of the atmosphere (e.g., gases) can **absorb** electromagnetic energy. This is reflected in a permittivity that is complex—the **imaginary** part ε'' is a result of this fact.

In other words, **if** $\varepsilon''=0$, then $k_i=0$ and $e^{-k_i z}=1.0$. No attenuation occurs!

The **second** thing to note when comparing the two plane-wave expressions is the **phase** terms:

$e^{j(k_0 z-\omega t)}$ for a plane wave in **free-space**

$e^{j(k_r z-\omega t)}$ for a plane wave in the **atmosphere**

Thus, the value k_r is the spatial frequency (radians/meter) of a plane wave propagating in the **atmosphere**. From this value we can find the propagation **wavelength**:

$$k_r = \frac{2\pi}{\lambda} \qquad \Rightarrow \qquad \lambda = \frac{2\pi}{k_r}$$

and the propagation **velocity**:

$$k_r = \frac{\omega}{v_p} \qquad \Rightarrow \qquad v_p = \frac{\omega}{k_r}$$

We find that the velocity of a plane-wave in our atmosphere will be **less** than that of a plane-wave in free space ($v_p \le c$).

The Extinction Coefficient

Q: *So what's the **power density** of a plane-wave propagating in the Earth's atmosphere?*

A: *Recall the magnitude of the power density of a propagating plane wave was found from the **Poynting vector**:*

$$|\mathbf{W}| = \frac{|\mathbf{E}(\bar{r})|^2}{2\eta}$$

And so the power density of a plane-wave propagating in the **atmosphere** is:

$$\begin{aligned} |\mathbf{W}| &= \frac{\left(e^{-k_i z}\sqrt{|E_x|^2 + |E_y|^2}\right)^2}{2\eta} \\ &= \frac{|E_x|^2 + |E_y|^2}{2\eta} e^{-2k_i z} \end{aligned}$$

And so (not surprisingly), we find that the power density **diminishes exponentially** as:

$$|\mathbf{W}| \propto e^{-2k_i z}$$

We now define the **extinction coefficient** κ_e:

$$\kappa_e \doteq 2\,k_i$$

So that now:

$$|\mathbf{W}| \propto e^{-\kappa_e z}$$

It is apparent that the extinction coefficient describes the "rate" at which the power density **exponential decays**.

$$\kappa_e = 0 \qquad \Rightarrow \qquad \text{no attenuation of wave}$$

$$\kappa_e > 0 \qquad \Rightarrow \qquad \text{exponential reduction of wave}$$

Note that the extinction coefficient is a bit of a misnomer. The term $\kappa_e z$ **is** unitless, and so the extinction "coefficient" κ_e must have **units** of **inverse length** (e.g., m^{-1} or km^{-1}).

However, we typically express the extinction coefficient in terms of **"dB/km"**, as in (for example):

$$\kappa_e = 2.3 \quad \frac{dB}{km}$$

Q: *Yikes! Why would we do **that**? What does it mean?*

A: Consider the case where a plane wave propagates a distance ℓ, beginning at location z and ending at location $z+\ell$.

The **attenuation** (i.e., loss) of the plane wave power density over this distance can be described as:

$$\frac{\left|\mathbf{W}(z+\ell)\right|}{\left|\mathbf{W}(z)\right|} = \frac{e^{-\kappa_e(z+\ell)}}{e^{-\kappa_e z}} = \frac{e^{-\kappa_e z}\, e^{-\kappa_e \ell}}{e^{-\kappa_e z}} = e^{-\kappa_e \ell}$$

Expressed in **decibels**:

$$\begin{aligned} \text{Attenuation in dB} &= -10\,log_{10}\left(\frac{\left|\mathbf{W}(z+\ell)\right|}{\left|\mathbf{W}(z)\right|}\right) \\ &= -10\,log_{10}\left(e^{-\kappa_e \ell}\right) \\ &= \kappa_e \ell\; 10\,log_{10}\left(e^{1}\right) \\ &= \kappa_e \ell\; 4.343 \end{aligned}$$

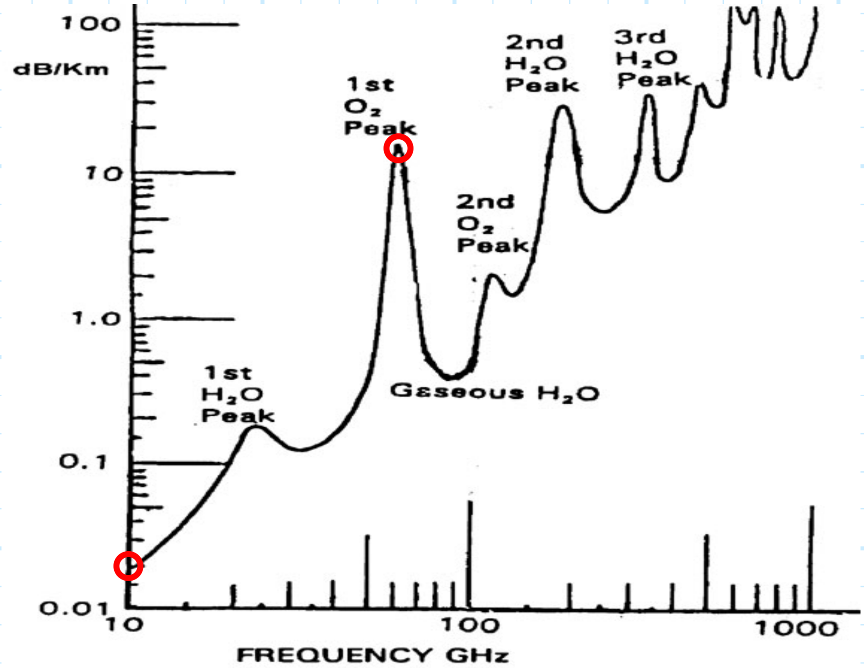
And so, we can define an "attenuation in dB/unit length" as:

$$\boxed{\frac{\text{Attenuation in dB}}{\ell} = \kappa_e\; 4.343 \qquad \frac{dB}{km}}$$

Attenuation by Atmospheric Gasses

Q: *So, what exactly is the value of the extinction coefficient κ_e of the Earth's* ***atmosphere****?*

A: This value depends on many things, including altitude, temperature, pressure, and weather conditions. But mostly, it depends on the **frequency** of the propagating signal!



For a "clear" atmosphere (i.e., **clear weather**), we find that the extinction coefficient **increases dramatically** with frequency.

For example, a propagating wave in the Earth's atmosphere with a frequency of **10 GHz** is attenuated an additional 0.03 dB for every km it propagates (i.e., $\kappa_e = 0.3\ dB/km$). Thus, if this wave propagates over a distance of **50 km**, it is attenuated by an additional **1.5 dB**.

If, however, the signal of this propagating wave is increased to **60 GHz**, we find that atmospheric attenuation has increased dramatically, to approximately **20.0 dB** for every km it propagates (i.e., $\kappa_e = 20.0\ dB/km$). Thus, if this wave propagates over a distance **50 km**, it is attenuated by an additional **1000 dB !!!!!**

Thus, we see that **below 10 GHz** the attenuation due to atmospheric gasses is largely regarded as **insignificant**.

Conversely, **above 50 GHz** the attenuation is so large that it make radio wave propagation over long distances effectively **impossible**!

→ Remember, atmospheric attenuation limits the "useable" region of the electromagnetic spectrum!

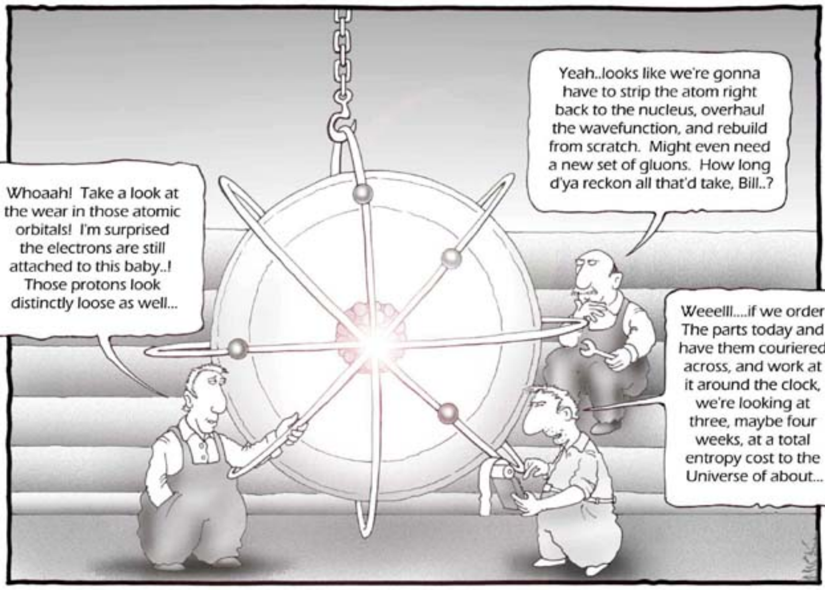
Q: *Why does the atmospheric attenuation* ***"peak"*** *at several frequencies?*

A: **Quantum mechanics**! Molecules and atoms absorb energy by discrete energy values (i.e., quanta). For each energy level E_n $(n = 1,2,3,\cdots)$, there is a corresponding wave frequency, a frequency that can be determined from Plank's Constant $\hbar$:

$$E_n = \hbar f_n \qquad \Rightarrow \qquad f_n = \frac{E_n}{\hbar}$$

It turns out that both a water molecule (H_2O) and oxygen molecule (O_2) exhibit an number of these absorption frequencies in the microwave region.

For example, we find that water vapor absorbs e.m. energy at about **22 GHz,** whereas oxygen absorbs e.m. energy at about **60 GHz**.



Quantum mechanics.

Q: *Now, in your example you said that the propagating wave is:*

"attenuated by an **additional** 1000 dB!"

*What did you mean by "**additional**" attenuation?*

A: Remember, the power density of a spherical wave produced by an antenna diminishes as $1/r^2$:

$$\mathbf{W}(\bar{r}) = U(\theta,\phi)\,\frac{\hat{\mathbf{r}}}{r^2}$$

Thus, the attenuation by the atmosphere is in **addition** to this r^{-2} **"spreading loss".** A more **accurate** description of the power density of a spherical wave is:

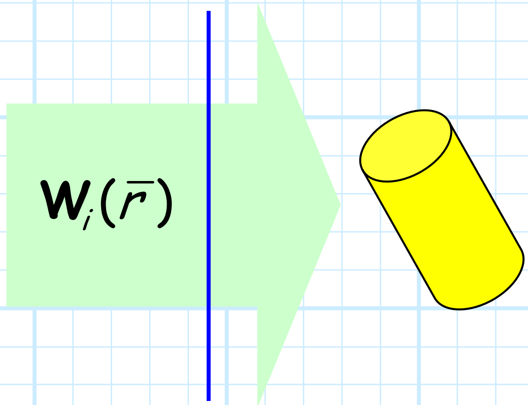
$$\mathbf{W}(\bar{r}) = U(\theta,\phi)\,\frac{e^{-\kappa_e r}}{r^2}\,\hat{\mathbf{r}}$$

And so, a more **accurate** form of the **Friis Transmission Equation** is likewise:

$$P_r = P_A\,\frac{G_0\,A_{em}}{4\pi r^2}\,e^{-\kappa_e r}$$

The Extinction Cross-Section

Say a lossy particle is illuminated by a plane wave with power density $\mathbf{W}_i(\bar{r})$:

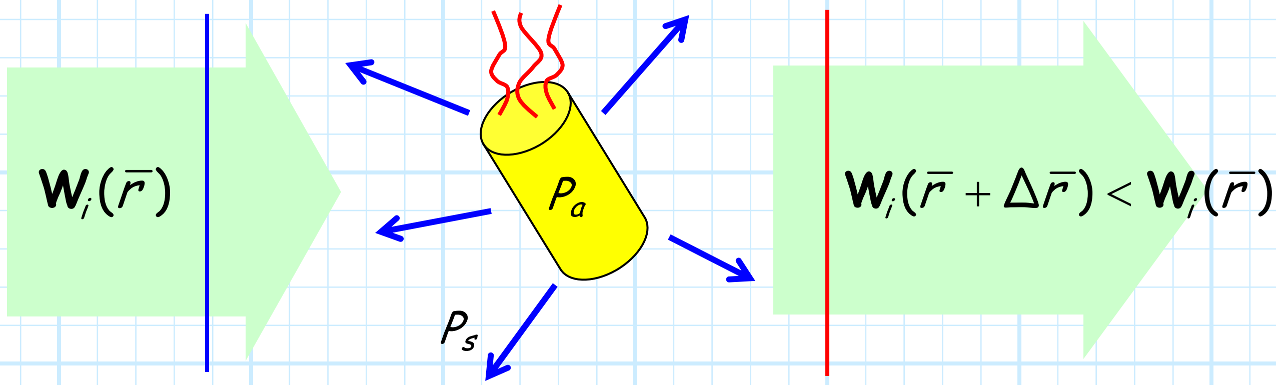


The particle will:

1. **Absorb** energy at a rate P_a Watts.

2. **Scatter** energy (in all directions) at a rate P_s Watts.

Because of conservation of energy, the power density of the incident wave must be diminished after interacting with the particle:



We can therefore define the **absorption cross-section** of this particle as:

$$\boxed{\sigma_a \doteq \frac{P_a}{\left|\mathbf{W}_i(\bar{r})\right|} \qquad \left[m^2\right]}$$

and likewise the **total scattering cross-section** as:

$$\boxed{\sigma_s \doteq \frac{P_s}{\left|\mathbf{W}_i(\bar{r})\right|} \qquad \left[m^2\right]}$$

Note that using these definitions we find that:

$$P_a = \sigma_a \left|\mathbf{W}_i(\bar{r})\right| \qquad \text{and} \qquad P_s = \sigma_s \left|\mathbf{W}_i(\bar{r})\right|$$

We can likewise define a particle **extinction cross-section** as:

$$\boxed{\sigma_e \doteq \frac{P_a + P_s}{\left|\mathbf{W}_i(\bar{r})\right|} = \frac{P_a}{\left|\mathbf{W}_i(\bar{r})\right|} + \frac{P_s}{\left|\mathbf{W}_i(\bar{r})\right|} = \sigma_a + \sigma_s \qquad \left[m^2\right]}$$

The extinction cross-section (as well as σ_a and σ_s), depend on the **physical** properties (size, shape, material, orientation) of the particle, as well as the **frequency** of the incident wave.

Q: *So what is the extinction cross-section of say, **a rain drop** or a **dust particle**, or a **snow flake**?*

A: A difficult question to answer, for **two** reasons:

1. Every rain drop/dust particle/snow flake is **different**, so we can only determine the extinction cross-section of a **typical** (i.e., average) particle.

2. Determining the extinction cross-section of even a typical particle is **extremely** difficult!

Evaluating the extinction cross-section of a particle requires an extremely complicated and difficult **electromagnetic analysis** (the solution must satisfy Maxwell's equations!).

In fact, solutions exist for only the **simplest** of objects; for example, a perfect **sphere**!

Even then, the solution for the extinction cross-section of a simple sphere is quite complex, and is expressed as a **infinite** series solution known as the **Mie Series**:

$$\sigma_e = \sum_{n=1}^{\infty} \sigma_{en} \left(k_0 a\right)^{2n} = \sigma_{e1} \left(k_0 a\right)^2 + \sigma_{e2} \left(k_0 a\right)^4 + \cdots$$

where the values σ_{en} are the **coefficients** of the Mie Series, and the value a is the **radius** of the sphere.

Thus, the quantity $k_0 a$ is simply the **electrical radius** of the sphere—the radius of the sphere expressed in radians:

$$k_0 a = \left(\frac{2\pi}{\lambda}\right) a = 2\pi \frac{a}{\lambda}$$

Note $k_0 a$ is **very** much like the value $\beta\ell$ we studied in transmission line theory (i.e., the electrical length of a transmission line, expressed in radians).

Q: *Yikes! What good is an **infinite** series solution? Wouldn't this require an **infinite** number of calculations?*

A: To get an **exact** answer yes. But we can **truncate** the infinite series (making it finite) and still get a very accurate (albeit imperfect) **approximation**. For **example**, if we retained the first three terms:

$$\sigma_e \approx \sigma_{e1}\left(k_0 a\right)^2 + \sigma_{e2}\left(k_0 a\right)^4 + \sigma_{e3}\left(k_0 a\right)^6$$

Q: *But how many terms **should** we retain? How do we know **when** to truncate the series?*

A: Note that as the sphere gets electrically **small** (i.e., $k_0 a \ll 1$) the higher order terms (e.g., $\left(k_0 a\right)^4$ or $\left(k_0 a\right)^{12}$) get **really** small, and thus can be ignored.

Conversely then, as the electrical size of the sphere increases, more and more terms of the Mie Series will become **significant**.

Q: *So how does all this help us determine the extinction cross-section of a* ***non****-spherical particle such as raindrop or dust particle?*

A: Note that your average raindrop has a diameter of a **few millimeters**, whereas the wavelength of a propagating wave at, say, 3.0 GHz is 100 mm.

Thus, your average hydrometer is **electrically small** (i.e., $k_0 a \ll 1$) in the RF/Microwave region of the electromagnetic spectrum.

From the standpoint of the Mie Series, this means that only a **few** terms will be significant. We find for this special case that the Mie Series provides us with a **relatively simple** answer:

$$\begin{aligned}\sigma_e &\approx (k_0 a)^4 A_p \frac{8}{3}\left(\frac{\varepsilon_r - 1}{\varepsilon_r + 2}\right) \\ &= \left(\frac{\omega}{c} a\right)^4 A_p \frac{8}{3}\left(\frac{\varepsilon_r - 1}{\varepsilon_r + 2}\right) \qquad \textit{for } k_0 a \ll 1\end{aligned}$$

where ε_r describes the material of the sphere, and A_p is its **physical cross-section**:

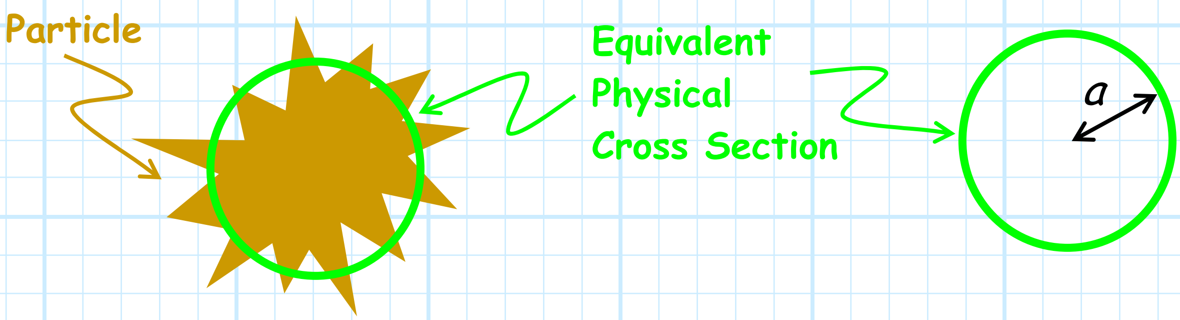
$$A_p = \pi a^2$$

This result is a quite famous one, known as the **Rayleigh** solution.

Q: *Terrific. But again, how does the solution for a **perfect sphere** help us determine the extinction cross-section of **non-spherical** particles such as raindrops or snowflakes!?!*

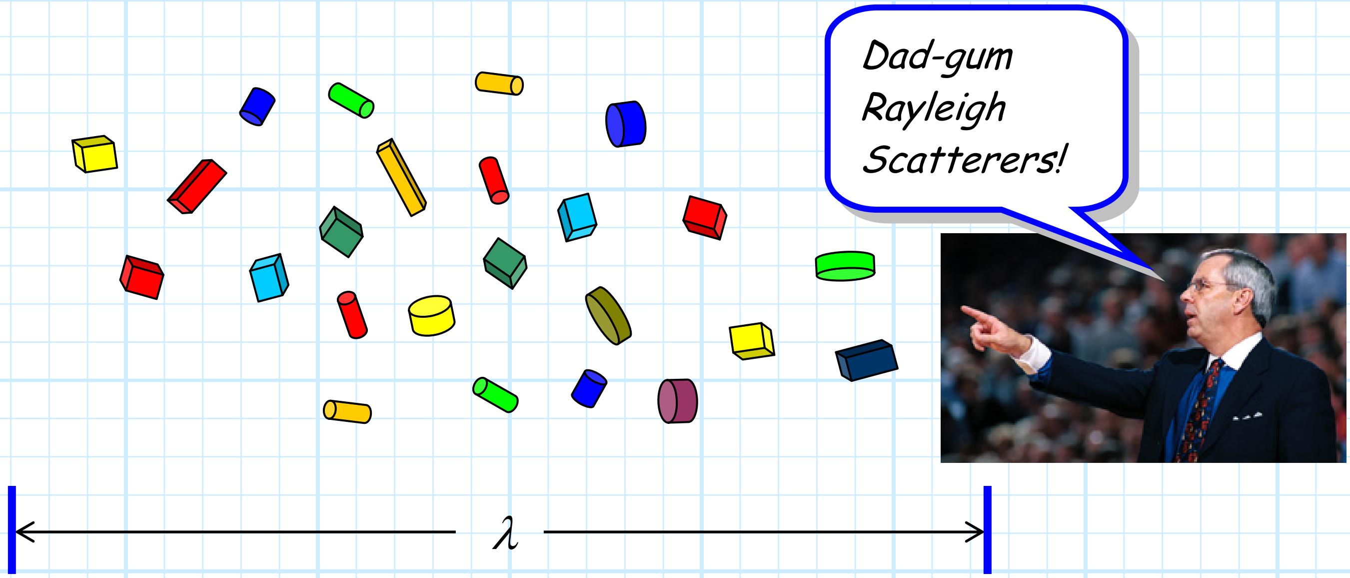
A: It turns out the Rayleigh solution for a **sphere** is a good **approximation** for electrically small **non-spherical** particles as well!

We can define an **equivalent** physical cross-section A_p and thus an equivalent radius a for any non-spherical particle.



Inserting these **equivalent** values into the Rayleigh solution for spherical particles provides an "acceptably" **accurate** value of the particle extinction cross-section—provided of course that the non-spherical particle is **electrically small**!

In fact, all electrically small particles are known as **Rayleigh Scatterers**.



Of course, what makes a particle electrically small is not just its physical size, but instead its physical size with respect to a wavelength. Thus, the **frequency** of the propagating wave makes a **BIG** difference!

We can see this in the **Rayleigh solution** of extinction cross-section:

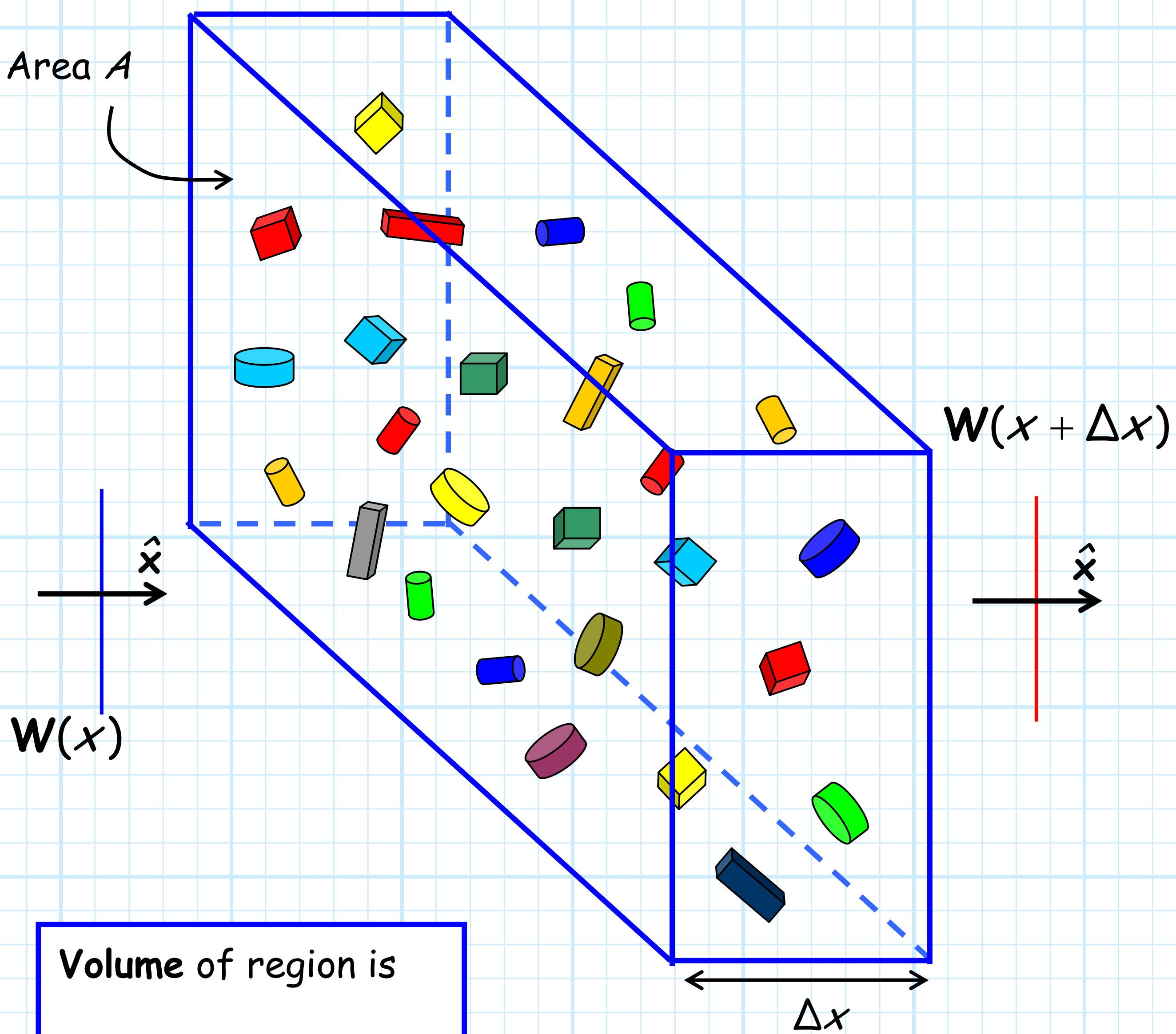
$$\sigma_e \approx \left(\frac{\omega}{c}a\right)^4 A_p \, \frac{8}{3}\left(\frac{\varepsilon_r - 1}{\varepsilon_r + 2}\right) \qquad \text{for } k_0 a \ll 1$$

Note that the extinction cross-section is proportional to ω^4. Thus, if we **double** the frequency of the propagating wave, the extinction cross-section will increase **8 times** (providing the particle is still electrically small)!

We will find that this make propagating through rain or fog exceeding **difficult** at **high frequencies**!

Extinction Due to Hydrometeors

Consider a plane wave that propagates a distance Δx through a thin volume with cross-sectional area A.



Volume of region is

$$V = A\,\Delta x$$

Say that this volume is filled with N particles. The **particle density** n_o is therefore defined as:

$$n_o \doteq \frac{N}{V} = \frac{N}{A\,\Delta x} \qquad \left[\frac{particles}{m^3}\right]$$

Now, we know that energy is flowing into the **front** of this volume at a rate of:

$$P_{in} = \left|\mathbf{W}(x)\right| A \qquad [W]$$

while the power of the plane wave **exiting** the back of the volume is:

$$P_{in} = \left|\mathbf{W}(x+\Delta x)\right| A \qquad [W]$$

Of course, the particles within the volume will **extract power** from this plane wave (due to absorption and scattering). Thus, the power lost due to **extinction** can be written as:

$$\begin{aligned}\Delta P &= P_{out} - P_{in} \\ &= \left|\mathbf{W}(x+\Delta x)\right| A - \left|\mathbf{W}(x)\right| A \\ &= \Delta\left|\mathbf{W}\right| A\end{aligned}$$

Where $\Delta\left|\mathbf{W}\right| \doteq \left|\mathbf{W}(x+\Delta x)\right| - \left|\mathbf{W}(x)\right|$.

Note since $P_{out} < P_{in}$, both ΔP and $\Delta\left|\mathbf{W}\right|$ will be **negative** values (i.e., the power **decreases** as it passes through the volume).

Q: *What happened to the **missing energy** ?*

A: The particles within the volume **extract** this energy by absorbing or scattering the incident plane wave. If P_{en} is the rate at which energy is extracted by the ***n*-th** particle, then the **total** rate of energy extinction by the entire collection is:

$$\begin{aligned} P_e &= \sum_{n=1}^{N} P_{en} \\ &= \sum_{n=1}^{N} \sigma_{en} \left|\mathbf{W}(x)\right| \end{aligned}$$

By **conservation of energy**, we can conclude that:

$$P_e = P_{in} - P_{out} = -\Delta P$$

And therefore:

$$\begin{aligned} \Delta P &= -P_e \\ &= -\sum_{n=1}^{N} \sigma_{en} \left|\mathbf{W}(x)\right| \end{aligned}$$

Q: *Yikes! How are we supposed to know σ_e for **each** and **every** one of the N particles?*

A: Look closer at the equation! We don't need to all the values σ_{en}--we simply need to know the **sum** of all the values (i.e., $\sum_{n=1}^{N} \sigma_{en}$)!

To determine this sum, we just need to know the **average** value of the extinction cross-sections ($\bar{\sigma}_e$), defined as:

$$\bar{\sigma}_e \doteq \frac{1}{N}\sum_{n=1}^{N}\sigma_{en}$$

Therefore:

$$\begin{aligned}\Delta P &= -\sum_{n=1}^{N}\sigma_{en}\left|\mathbf{W}(x)\right| \\ &= -\left|\mathbf{W}(x)\right|\sum_{n=1}^{N}\sigma_{en} \\ &= -\left|\mathbf{W}(x)\right|\left(N\,\bar{\sigma}_e\right) \\ &= -N\left|\mathbf{W}(x)\right|\bar{\sigma}_e\end{aligned}$$

Recall, however, that:

$$N = n_o V = n_o A\,\Delta x$$

Therefore:

$$\begin{aligned}\Delta P &= -N\left|\mathbf{W}(x)\right|\bar{\sigma}_e \\ &= -n_o A\left|\mathbf{W}(x)\right|\bar{\sigma}_e\,\Delta x\end{aligned}$$

and thus:

$$\Delta\left|\mathbf{W}\right| = \frac{\Delta P}{A} = -n_o\left|\mathbf{W}(x)\right|\bar{\sigma}_e\,\Delta x$$

Finally (whew!) we can say:

$$\frac{\Delta\left|\mathbf{W}\right|}{\Delta x} = -n_o\left|\mathbf{W}(x)\right|\bar{\sigma}_e$$

And taking the limit as $\Delta x \to 0$, we have determined the following **differential equation**:

$$\boxed{\lim_{\Delta x \to 0} \frac{\Delta |\mathbf{W}|}{\Delta x} = \frac{d\,|\mathbf{W}(x)|}{dx} = -(n_o \bar{\sigma}_e)\,|\mathbf{W}(x)|}$$

This differential equation is **easily solved**:

$$|\mathbf{W}(x)| = |\mathbf{W}(x=0)|\, e^{-(n_o \bar{\sigma}_e)x}$$

And thus:

$$\mathbf{W}(x) = |\mathbf{W}(x=0)|\, e^{-(n_o \bar{\sigma}_e)x}\ \hat{\mathbf{x}}$$

Note the **similarity** of the function

$$e^{-(n_o \bar{\sigma}_e)x} \qquad \text{with} \qquad e^{-\kappa_e x}$$

Extinction due to **particles**

Extinction due to **gases.**

It is apparent that the extinction due to atmospheric particles has **precisely** the same form as the extinction due to atmospheric gases!

In fact, we find that the **extinction coefficient** for atmospheric particles (such as hydrometeors) is:

$$\boxed{\kappa_e \doteq n_o \bar{\sigma}_e}$$

As a result, we can describe the power density of a plane wave propagating in the atmosphere as:

$$\mathbf{W}(\bar{r}) = U(\theta,\phi)\frac{e^{-\kappa_e r}}{r^2}\,\hat{\mathbf{r}}$$

Where the extinction coefficient κ_e is due to gasses, particles, or **both**!

Now, consider the (likely) situation where the atmospheric particles are **Rayleigh scatterers**. Since the extinction coefficient is **proportional** the average particle extinction cross-section:

$$\kappa_e \propto \bar{\sigma}_e$$

And since the extinction cross-section of a Rayleigh scatterer is **proportional** to frequency as:

$$\bar{\sigma}_e \propto f^4$$

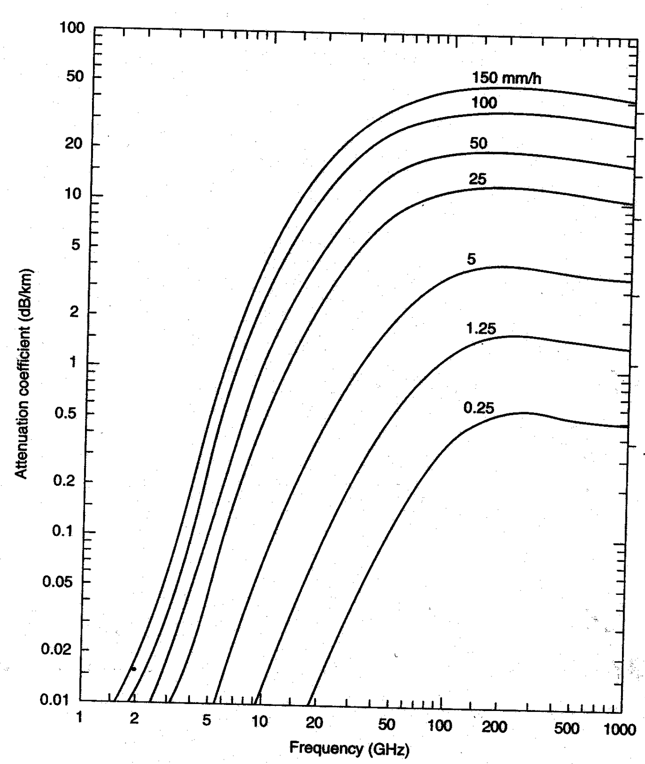
We can likewise conclude that the **extinction coefficient** is proportional to **frequency** as:

$$\kappa_e \propto f^4$$

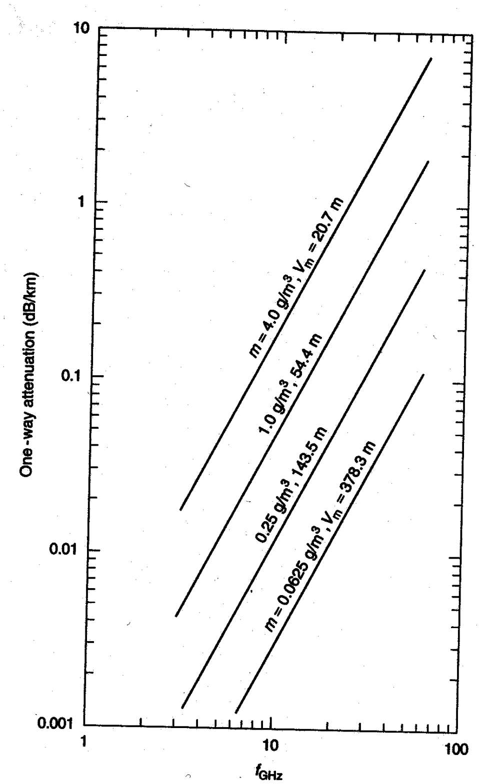
Thus, the extinction coefficient of a volume of Rayleigh Scatterers is proportional to the **forth power** of the wave frequency. If we **double** the frequency, we increase the extinction coefficient **8 times**!

This is unfathomably **significant**, as we know that the power density of a propagating wave is **exponentially** related to the extinction coefficient (i.e., $|\mathbf{W}| \propto e^{-\kappa_e r}$).

We find that as signal frequency increases, it rapidly becomes **nearly impossible** to propagate through an **average rainstorm**!



Extinction Coefficient as a function of frequency for various rainfall rates



Extinction Coefficient as a function of frequency for various densities of fog/clouds

For **example**, using the charts above, the extinction coefficient for an atmosphere containing **rain** falling at a rate of 1.0 inches/hour (i.e., 25 mm/hour), is approximately:

0.01 dB/km at 3.0 GHz

0.08 dB/km at 6.0 GHz

0.60 dB/km at 12.0 GHz

3.5 dB/km at 24.0 GHz

8.0 dB/km at 48.0 GHz

Thus, if this rainstorm extends over **10 km**, there will be an additional wave attenuation of:

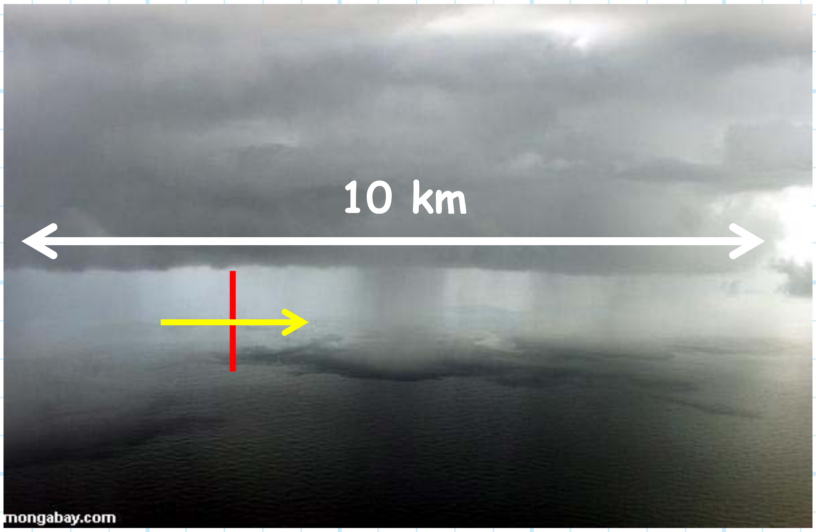
0.1 dB at 3.0 GHz

0.8 dB at 6.0 GHz

6.0 dB at 12.0 GHz

35 dB at 24.0 GHz

80 dB at 48.0 GHz



Note the **big** difference between 3.0 GHz and 48.0 GHz!

Q: *I notice the effect of **Rayleigh Scattering** when the frequency is **low**. When we **double** the frequency from 3.0 to 6.0 GHz, the extinction increases **8 times** from 0.01 dB/km to 0.08 dB/km.*

*Again, when we go from 6.0 GHz to 12.0 GHz, the extinction increases approximately **8 times** to 6 dB/km. **But**, when we increase the frequency to 24.0 GHz and then to 48.0 GHz, this "f^4" Rayliegh Scattering effect seems to **disappear**—the extinction does **not** increase 8 times?!*

A: Remember, as the wave frequency increases, the **electrical size** of the raindrops also increase. If the frequency becomes too high, then the particles will **no longer** be electrically **small** (i.e., $k_0 \ll 1$ is **not** true).

As a result, **other** terms in the Mie Series become significant, and the extinction **no longer** increases as f^4.

In fact, the extinction coefficient eventually **"plateaus"** as frequency is increased—the Mie Series has **converged**!