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TM Mode Analysis of a Sievenpiper High-Impedance Reactive Surface

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1. Introduction

A high-impedance ground plane is an attractive boundary condition for electrically-thin antennas since the electric current on wire elements placed next to this boundary have co-directed images. This boundary condition is helpful for impedance matching bandwidth. Recently, Sievenpiper, *et al*, [1,2] have described a periodic structure, shown in Figure 1, which exhibits such properties over a finite bandwidth. It is characterized by an array of electrically-short, vertical, metal rods of length t and diameter d that are terminated on a horizontal PEC surface. Assume the rods form a square lattice of period a. On the upper end of each rod is attached a metal plate. The array of plates forms a capacitive FSS. This periodic structure also has an apparent surface wave bandgap for TM and TE modes that is believed to lie between the +/- 90^o reflection phase frequencies for plane waves at normal incidence.

The analysis presented here builds on the published work of King and Park [3] in which they analyze TM modes on a Fakir's grounded bed-of-nails structure. Sievenpiper's reactive surface is essentially a Fakir's structure with the addition of a capacitive FSS loading the ends of the rods. In this analysis, we show that an infinite number of TM modes can exist on this structure, and that the apparent TM mode cutoff, experimentally observed, is a manifestation of the two lowest order modes coalescing, and then being cut off. Furthermore, we show that this apparent TM mode cutoff can be adjusted independent of the $+90^o$ reflection phase frequency by controlling the density and diameter of the rods.

2. Transverse Resonance Analysis

According to the transverse resonance procedure the impedance looking to the left must be the negative of the impedance looking to the right at any transverse plane of the structure. Defining the x-axis as the direction of propagation parallel to the surface and the z-axis as the direction perpendicular to the surface, the observation boundary is located on the outside surface of the FSS (a distance t from the ground plane) as suggested in Figure 1. For guided TM surface waves $Z_{left}=Z_{TM}$:

$$Z_{TM} = \frac{\gamma_{z2}}{j\omega\varepsilon_0} = \frac{\alpha}{j\omega\varepsilon_0} \quad (1)$$

Since this must equal $-Z_{right,}$ it follows that,

$$\alpha = -k_0 \frac{jZ_{right}}{\eta_0} = k_0 X \quad (2)$$

where the quantity k_0/η_0 was substituted for ε_0 in order to define the normalized reactance of the surface according to $Z_{right}/\eta_0 = R+jX$. In the upper medium it must be true that $\gamma_0^2=\gamma_x^2+\gamma_z^2$, therefore,

$$k_x^2 = k_0^2\left(1 + X^2\right) \qquad (3)$$

Since the waves above and below z = t must propagate in phase, the x-direction propagation constant k_x is the same above and below z = t. Thus equation 3 can be used to obtain an equation relating the z-directed propagation constant in the lower medium (the bed-of-nails) to the normalized reactance.

A second such equation can be obtained by recognizing that the impedance looking to the right from the plane z = t is the short circuit rolled over a distance *t* by the same z-directed propagation constant in the lower medium, in parallel with the FSS sheet capacitance.

$$Z_{right} = 1/\left\{-\frac{j\varepsilon_x k_0}{\eta_0 k_{z1}}\cot\left(k_{z1}t\right) + j\omega C\right\} \qquad (4)$$

Here ε_x is the relative permittivity in the x-direction inside the slab, *t* is the thickness, k_0 is the free space phase constant, and k_{z1} is the phase constant inside the slab in the z direction (to be determined). The desired guided mode solutions for k_{z1} are found at the intersections of the two equations so derived from 3 and 4.

Given the construction, the bed-of-nails is equivalent to a uniaxial anisotropic material with $\varepsilon_x = \varepsilon_y$, and both of the order of $1+\delta$ (where δ is small), and ε_z is complex and given by the effective medium model of a rodded dielectric. The z-directed component of the propagation constant in a uniaxial anisotropic medium is given by:

$$k_{z1} = k_0\left[\mu_y\varepsilon_x\left(1 - \frac{(k_x/k_0)^2}{\mu_y\varepsilon_z}\right)\right]^{\frac{1}{2}} \qquad (5)$$

For a medium containing infinitely long perfectly conducting rods embedded in a uniform dielectric (of constitutive properties μ_d and ε_d) it can be shown that,

$$\mu_y\varepsilon_x = \mu_d\varepsilon_d\,. \qquad (6)$$

In a uniaxial medium a plane wave propagating in the x direction with field components E_z and H_y will see a normalized index of refraction given by:

$$n_{zy} = \sqrt{\frac{\varepsilon_z\mu_y}{\varepsilon_d\mu_d}}\ \text{, where }\ \cos(n_{zy}\theta) = \cos\theta + \frac{A}{\theta}\sin\theta\text{, with }\ A = \frac{\pi}{\ln\left(a/\pi d\right)} \qquad (7)$$

and where $\theta = k_d a$ is the electrical spacing between rods, and *d* is the diameter of the rods. For small electrical spacings the index of refraction in equation 7 is purely imaginary, corresponding to a negative effective permittivity. Defining the variable $p = -jn_{zy}$, and using equations 7, 6, and 3, equation 5 becomes:

$$k_{z1}^2 p^2 = k_d^2 p^2 + k_0^2 + X^2 k_0^2 \qquad (8)$$

Now, recognizing $k_d^2 = \varepsilon_d k_0^2$, solving for *X*, and multiplying both sides by $k_0 t$ yields

$$k_0 Xt = \sqrt{k_{z1}^2 p^2 t^2 - k_0^2 t^2\left(1 + \varepsilon_d p^2\right)} \qquad (9)$$

This is the first of the desired equations. Equation 4 can be likewise rearranged to give the second desired equation:

$$k_0 Xt = \frac{1}{\frac{\varepsilon_x}{k_{z1}t}\cot\left(k_{z1}t\right) - c\frac{C\eta_0}{t}} \qquad (10)$$

where it has been recognized that $k_0=\omega/c$, and c is the free space speed of light. For the sake of simplicity, the capacitance of the FSS layer is treated as if it were independent of the angle of incidence (given by $\cos\phi=k_{z1}/k_d$). It can be shown that for the TM modes in question including the dependence on the angle of incidence leaves the results virtually unchanged. Now, if k_0Xt is used as the dependent variable and $u=k_{z1}t$ as the independent variable, equation 10 is independent of frequency, whereas equation 9 gives a family of curves labeled by the frequency through the dependence on k_0. The surface wave modes exist at the intersections of the curves 9 and 10.

3. Numerical Results

Equations 9 and 10 are plotted in Figure 2, for a structure with the following parameters: a=2.4mm, d=0.36mm, t=1.6mm, ε_d=2.2. Two of the infinite number of "resonances" of equation 10 are shown (solid curve), the first around $k_{z1}t=\pi/4$ and the second around 3.3. In their neighborhood the Reactance can take on very large values. The dashed curves correspond to equation 9 for frequencies 5.5, 8.5, 9.5, and 10.5 GHz. The Figure shows that at any given frequency (dashed curves) there are an infinite number of modes (intersections with the solid curve) able to propagate. The first two modes occur near the first resonance and have relatively low reactance. Then comes a third intersection in the neighborhood of the second resonance, and so forth. King et al point out that an antenna suspended at a certain height above the surface z = t, will tend to excite the lowest modes most efficiently because, having lowest reactances, the modal fields decay weakly in z, and extend the farthest above the plane.

Note in the detail of Figure 2b that as frequency is increased, the reactance of the lower intersection increases whereas that of the second one decreases. In the neighborhood of 10.2 GHz the two modes coalesce. Then above 10.2 GHz the dashed curve of equation 9 cannot intersect the solid curve of equation 10 near the quarter wave resonance any more. The First TM mode abruptly moves off this curve and onto the second resonance region, at a much higher reactance. It thus becomes very difficult to excite. Thus a wire antenna held at a fixed height above the surface and scanned in frequency would suddenly appear to meet a "band edge" around 10.2 GHz, when the two lowest order surface waves it was exciting no longer can be excited. This is the lower TM band edge reported in [1] and [2]. This "band edge" can be moved by altering the complex index of refraction of the bed-of-nails, either by changing the diameter of the rods (Figure 2c for d=0.33mm, 0.24mm, 0.15mm, 0.06mm) or their lattice period (Figure 2d for a= 2.52mm, 2.88mm, 3.24mm, 3.6mm.)

3. Conclusions

We have extended the TM mode transverse resonance analysis of Fakir's bed-of-nails structure to model the Sievenpiper high impedance surface. Although an infinite number of TM modes are predicted to exist at any given frequency, an apparent TM mode cutoff is explained by the eigenvalues of the two lowest reactance modes converging and then disappearing. The remaining higher reactance modes are tightly bound to the surface and much more difficult to excite. Our predictions of the apparent TM mode band edge are consistent with full-wave predictions from Sievenpiper and Zhang [1,2].

4. References

[1] D. F. Sievenpiper, "High-Impedance Electromagnetic Surfaces," UCLA, Ph. D. dissertation, January 1999.

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[3] R. J. King, D. V. Thiel, and K. S. Park, "The Synthesis of Surface Reactances Using an Artificial Dielectric," IEEE Trans. Ant. Propaga., Vol AP-31, May, 1983, pp. 471-476.

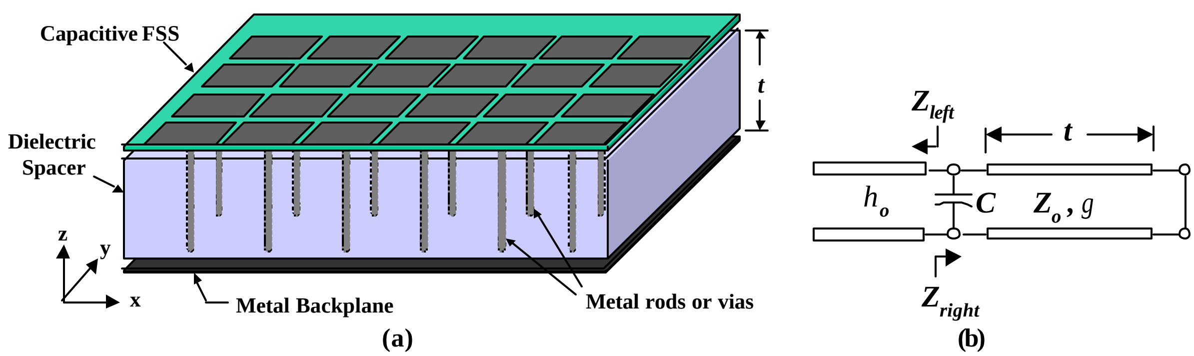

Figure 1. The Sievenpiper high impedance ground plane (a) is an electrically-thin periodic structure whose TM mode equivalent circuit is shown in (b).

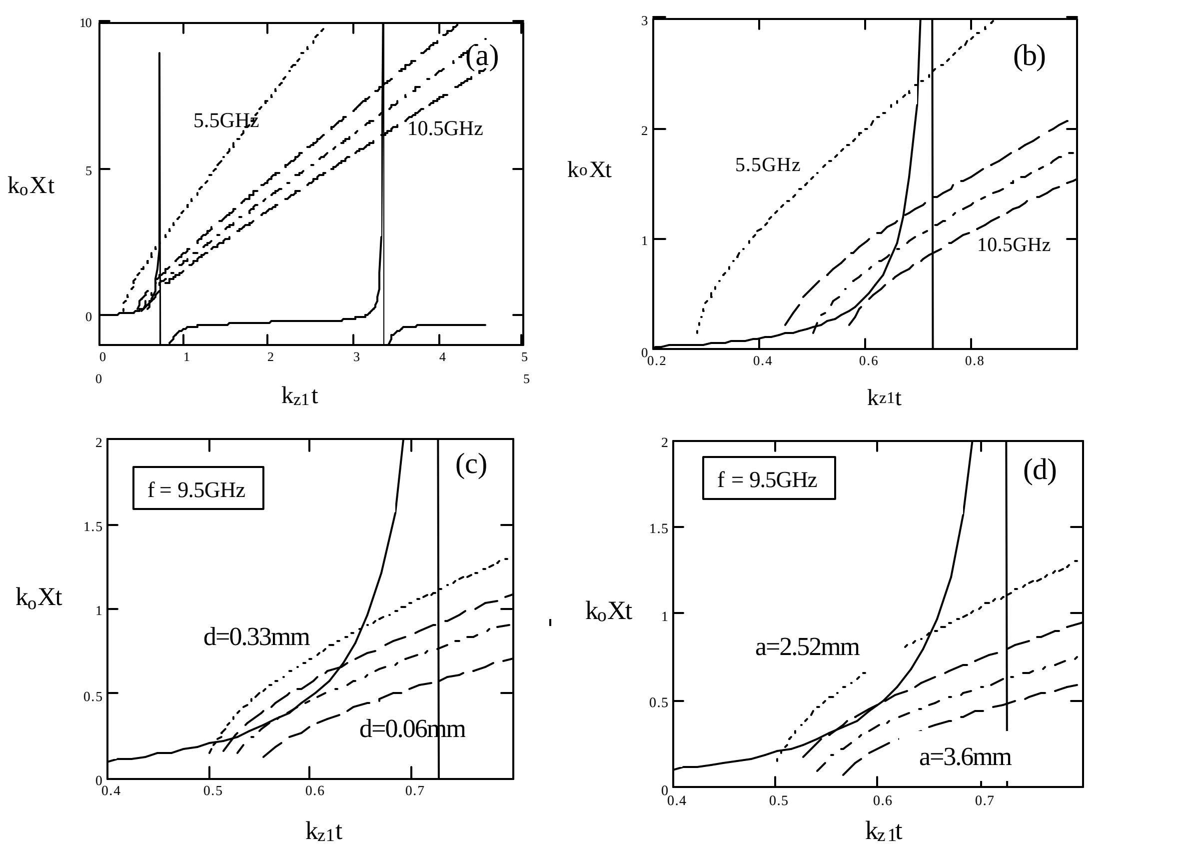

Figure 2. Normalized reactance versus electrical thickness shows allowed TM surface wave modes at the points of intersection as a function of frequency (a) and (b) (detail), rod diameter (c), and period (d).