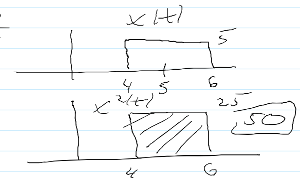
Thursday, January 29, 2026 7:37 AM

- Special function $\delta_{T_0}(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT_0)$

$x_p(t) = \sum_{\forall k} x(t - kT_0)$

- $\Pi(t)$
- $\delta(t)$
- $E_x = \int_{-\infty}^{\infty} |x(t)|^2 dt$

$x(t) = 5\,\text{rect}\left(\frac{t-5}{2}\right)$



- $P_x = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt$

if periodic $P_x = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} |x(t)|^2 dt$ T_0 = period

$x(t) = 5\cos(\omega_0 t)$ $P_x = 25/2 = 12.5$ $\frac{A^2}{2}$

$x_p(t) = \sum_{n=1}^{\infty} c_n \cos(n\omega_0 t + \theta_n)$

$P_{x_p} = \sum_{n=1}^{\infty} \frac{c_n^2}{2}$

- Energy signal E_x finite $P = 0$ Power Signal P finite $E = \infty$

- Linear System $x(t) \to \boxed{} \to y(t)$

$\alpha x_1(t) \to y_1(t)$ $\beta x_2(t) \to y_2(t)$

$\alpha x_1 + \beta x_2 \to y_3(t)$

If $y_1(t) + y_2(t) = y_3(t)$ Then Linear

Time invariance $x(t) \to \boxed{} \to y(t)$

$x(t - \tau) \to \boxed{} \to y(t - \tau)$

LTI

Let System be LTI $\delta(t) \to \boxed{} \to h(t)$

$h(t)$ = impulse response

$u(t) \to \boxed{} \to y_{step}(t)$ Step response

$y_{step}(t) = \int_{-\infty}^{t} h(\lambda)\, d\lambda$

$y(t) = \int_{-\infty}^{\infty} x(\lambda) h(t - \lambda) d\lambda$

$y(0) = \int_{-\infty}^{\infty} x(\lambda) h(-\lambda)\, d\lambda$

$h(\lambda) \to h(-\lambda)$ flip

multiply $x(\lambda) h(-\lambda)$

integrate $\to y(0)$

$y(1) = \int_{-\infty}^{\infty} x(\lambda) h(1 - \lambda)\, d\lambda$

$h(\lambda)$ flip $h(-\lambda)$

$h(-\lambda)$ shift $\to h(1 - \lambda)$

multiply

$x(\lambda) h(1 - \lambda)$

integrate $\to y(1)$

$x(t)$ is causal then $x(t) = 0$ $t < 0$

or $x(t) = x(t)u(t)$

$y(t) = \int_{-\infty}^{\infty} h(\lambda) x(t - \lambda) d\lambda$

or $x(t) = x(t)u(t)$

$y(t) = \int_{-\infty}^{\infty} h(\lambda)x(t-\lambda)d\lambda$

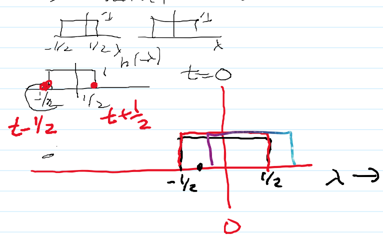
$\int_{-\infty}^{\infty} h(\lambda)\, u(t-\lambda)\, x(t-\lambda)\,d\lambda$

$u(t-\lambda) = 1 \quad t > \lambda$

$\int_{-\infty}^{t} h(\lambda)\, x(t-\lambda)d\lambda$

$h(t) = u(t)h(t)$

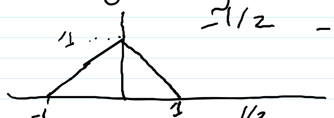
$x(t) = rect(t) \quad h(t) = rect(t)$



Case 1 $y(t) = 0 \quad -\infty < t < -1$

$t + 1/2 = -1/2 \quad t = -1$

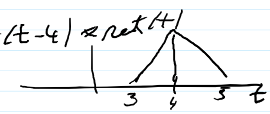
Case 2 $y(t) = \int_{-1/2}^{t+1/2} (1)(1)d\lambda \quad d\lambda$ $\quad -1 < t < 0$



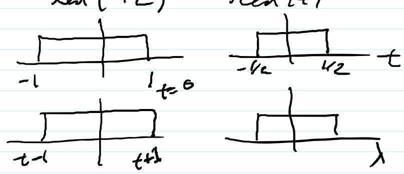
Case 3 $y(t) = \int_{t-1/2}^{1/2} (1)(1)d\lambda \quad 0 < t < 1$

Case 4 $y(t) = 0 \quad t > 1$

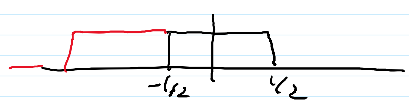
$y(t) = rect(t-4) * rect(t)$



$rect(t/2) \qquad rect(t)$

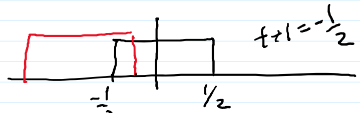


Case 1 $y(t) = 0 \quad -\infty < t < -3/2$

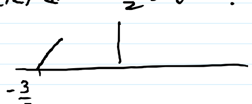


$t+1 = -1/2 \quad t = -3/2$

Case 2 $\quad t+1 = -\frac{1}{2}$



$y(t) = \int_{-\frac{1}{2}}^{t+1} (1)(1)\,d\lambda \quad -\frac{3}{2} < t < ?$

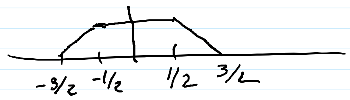


Case 3

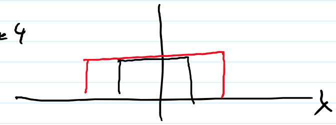
$y(t) = \int_{-1/2}^{1/2} (1)(1)\,d\lambda = 1 \quad -\frac{3}{2} < t < -1/2$

$t+1 = \frac{1}{2} \quad t = -\frac{1}{2}$

$y(t) = \int_{-1/2} (1)(1)\, d\lambda = \ldots \quad t+1 = \frac{1}{2} \quad t = -\frac{1}{2}$

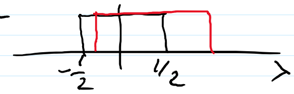


Case 4



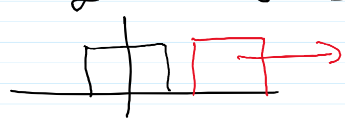
$y(t) = 1 \quad -\frac{1}{2} < t < \frac{1}{2}$

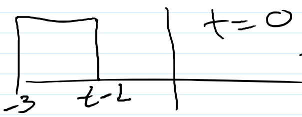
Case 5



$\int_{t-1}^{\frac{1}{2}} (1)(1)\, d\lambda$

Case 6 $\quad y(t) = 0 \quad t > 5/2$





$A^2/2 \qquad \frac{(\sqrt{2})^2}{2} + \frac{2^2}{2} = 3$

$A\cos(\omega_1 t) + B\cos(\omega_2 t)$ powers add

$\sqrt{2}\cos(\omega_1 t + 0.7)$ phase change

1

HW 3 #4 $\quad 100\times10^{-3}\cos(2\pi f_c t)$ "1" $\quad T_b = \frac{1}{10\times10^9}$

BPSK $\quad -100\times10^{-3}\cos(2\pi f_c t)$ "0" $\quad r_b = 10\times10^9$

$$\int_0^{T_b} (100\times10^{-3})^2 \cos^2(2\pi f_c t)\, dt$$

$\approx 5\times10^{-13} \qquad \cos^2\theta = \frac{1}{2} + \frac{1}{2}\cos(2\theta)$

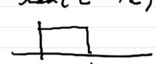
5. $\quad 10\cos(2\pi f_m t)\cos(2\pi f_c t) \quad$ m = message, c = carrier

$$\left.\begin{aligned} &5\cos(2\pi(f_c+f_m)t) \\ &+5\cos(2\pi(f_c-f_m)t) \end{aligned}\right\} \quad \frac{5^2}{2} + \frac{5^2}{2}$$

$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t) = \int_{-\infty}^{\infty} x(\lambda)\, h(t-\lambda)\, d\lambda$

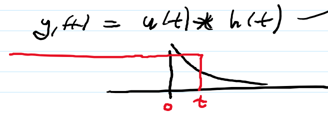
Example $\quad x(t) = \text{rect}(t - 1/2) \qquad h(t) = u(t)\, e^{-t}$

$y(t) = x * h$



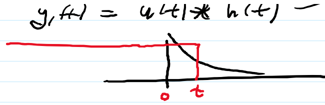
$x(t) = u(t) - u(t-1) \qquad y(t) = y_1(t) - y_1(t-1)$

$y_1(t) = u(t) * h(t)$



$y(t) = \int_0^t e^{-t}\, dt = u(t)(1 - e^{-t})$

$y_1(t) = u(t) * h(t)$ —

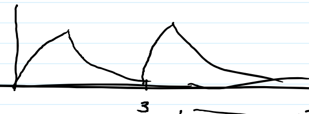


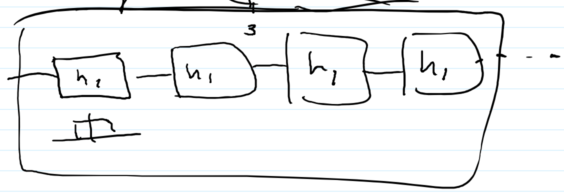
$y(t) = \int_0^t e^{-t}\,dt$
$= u(t)(1-e^{-t})$

$y(t) = u(t)(1-e^{-t}) + u(t-1)(1-e^{-(t-1)})$

$(1-e^{-1})$

$x_1(t) = rect(t-1/2) + rect(t-3)$

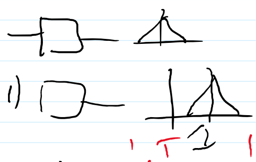




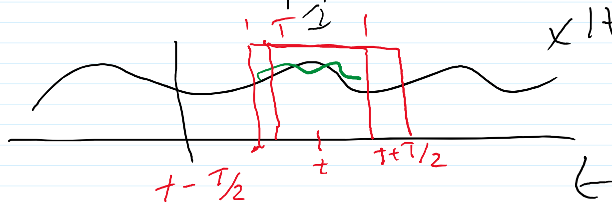
$h(t) = h_1 * h_1 * h_1 \dots$

$x(t)$

$x(t-1)$



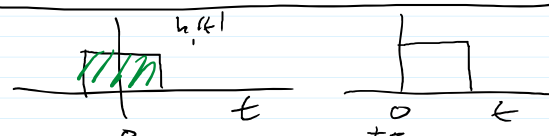
$x(t)$



$t - T/2$ t $t+T/2$

If $h(t) = 0$ $t < 0$ then causal

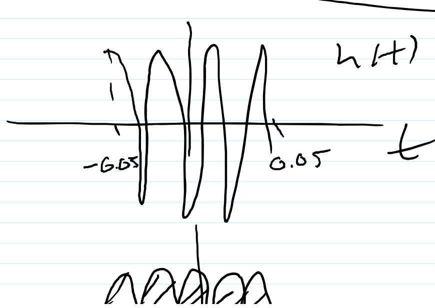
$h(t) = u(t)h(t)$



$y(t) = u(t)h(t) * x(t) = \int_{-\infty}^{t} x(\lambda)h(t-\lambda)$

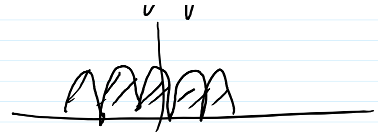
$\int_{-\infty} |h|\,d\lambda$

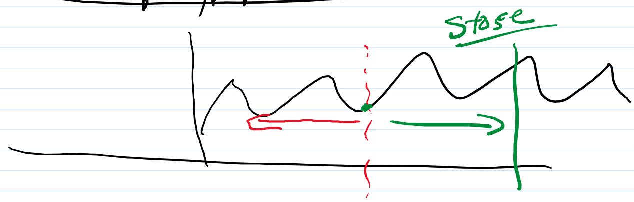
If $\int_{-\infty}^{\infty} |h(\lambda)|\,d\lambda$ is finite then BIBO stable

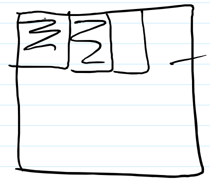


Not causal

Is stable







$\boxed{x(t) = e^{j\omega t}}$ $\boxed{y(t) = H(\omega)\, e^{j\omega t}}$

no func. of t

$\frac{dy}{dt} = j\omega H(\omega) e^{j\omega t}$

$j\omega H(\omega) \cancel{e^{j\omega t}} + \frac{1}{RC} H(\omega) \cancel{e^{j\omega t}} = \frac{1}{RC} \cancel{e^{j\omega t}}$

Solve for $H(\omega)$

$H(\omega)\left(j\omega + \frac{1}{RC}\right) = \frac{1}{RC}$

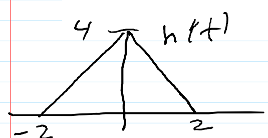
$H(\omega) = \dfrac{\frac{1}{RC}}{j\omega + \frac{1}{RC}} = \dfrac{1}{j\omega RC + 1}$

$x(t) = \sum_{n=1}^{\infty} c_n \cos(n\omega_0 t + \theta_n)$

$H(\omega)$

$y(t) = \sum_{n=1}^{\infty} c_n |H(n\omega_0)| \cos(n\omega_0 t + \theta_n + \phi_n)$

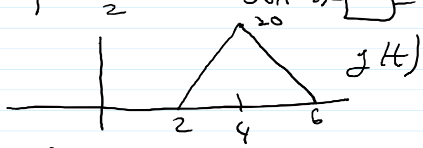
1. Let the system input be x(t) = 5δ(t-4) and the system impulse response of an LTI system be h(t) = 4tri($\frac{t}{2}$). Find the system output y(t) = x(t)∗h(t).



$\delta(t) \to \boxed{h} \to h(t)$

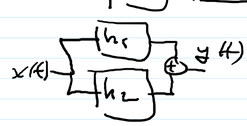
$5\delta(t) \to \boxed{} \to 5h(t)$

$5\delta(t-4) \to \boxed{} \to 5h(t-4)$



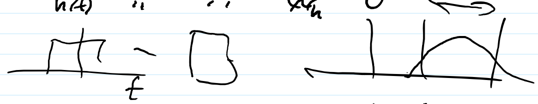
Properties of conv

- $x(t) \to \boxed{h_1} \to \boxed{h_2} \to y(t)$ $\quad x(t) \to \boxed{h_1 * h_2} \to y(t)$



$x(t) \to \boxed{h_1(t) + h_2(t)}$

x(t) has width W_x $\quad y \to W_y = W_x + W_h$
h(t) " " W_h



- causality If $h(t) = u(t)h(t)$
LTI then causal

- BIBO stability If $\int_{-\infty}^{\infty} |h(t)|$ is finite then stable

- BIBO Stability If $\int_{-\infty}^{\infty} |h(t)|$ is finite then stable

- $x(t) = e^{j\omega t}$ —[ ]— $H(\omega) e^{j\omega t} = y(t)$

- Find $H(\omega)$ for LCCDE

- $x(t) = A\cos(\omega_1 t + \theta)$

$y(t) = A\,|H(\omega_1)| \cos(\omega_1 t + \theta + \angle H(\omega_1))$

$x(t) = \sum_{n=1}^{\infty} c_n \cos(n\omega_0 t + \theta_n)$

$y(t) = \sum_{n=1}^{\infty} c_n |H(n\omega_0)| \cos(n\omega_0 t + \theta_n + \angle H(n\omega_0))$

Example: $\frac{C\,dy(t)}{dt} + \frac{y(t)}{R} = C\frac{dx(t)}{dt}$

$Cj\omega H(\omega) \cancel{e^{j\omega t}} + \frac{H(\omega)\cancel{e^{j\omega t}}}{R} = Cj\omega \cancel{e^{j\omega t}}$

Solve for $H(\omega)$ Not function of t

$H(\omega)$ is complex $H(-\omega) = H^*(\omega)$

$|H(\omega)|$ even $\angle H(\omega)$ is odd

$H(\omega) = \frac{j\omega}{j\omega + \frac{1}{RC}}$ $RC = \frac{1}{2\pi}$

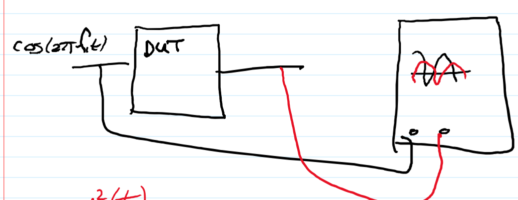
$x(t) = \cos(2\pi t)$ find $y(t)$ $\omega = 1$

$|H(1)| = \frac{1}{\sqrt{2}}$ $\angle H(1) = \pi/4$

$y(t) = \frac{1}{\sqrt{2}} \cos(2\pi t + \pi/4)$

$P_x = \frac{1}{2}$ $P_y = \frac{(1/\sqrt{2})^2}{2} = 1/4$

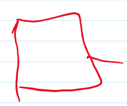
$P_y/P_x = \frac{1}{2}$ $10\log(P_y/P_x) = -3\text{dB}$



$x^2(t)$

$\cos^2(t) = \frac{1}{2} + \frac{1}{2}\cos(2\omega t)$

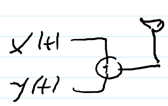
$\cos()$

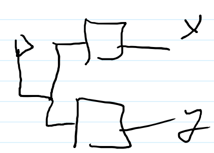


$x_p(t)$ T_0 $f_0 = \frac{1}{T_0}$ Hz

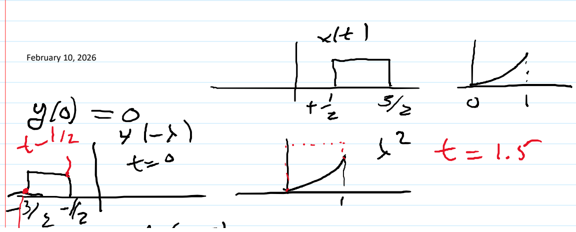
$\rightarrow \omega_0 = \frac{2\pi}{T_0}$

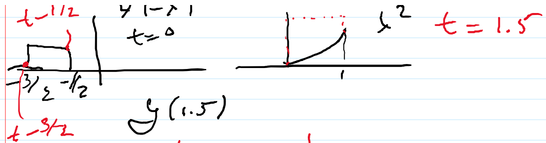
$n\omega_0$ $n > 1$ harmonics





February 10, 2026





$y(1.5)$

$y(1.5) = \int_0^1 \lambda^2 d\lambda = \frac{1}{3}$

- $H(\omega) \to$ given LCCDE

  $x(t) = \sum C_n \cos(n\omega_0 t + \theta_n)$ ω_0 = fundamental freq

  $y(t) = \sum |H(n\omega_0)| c_n \cos(n\omega_0 t + \theta_n + \angle H(n\omega_0))$

- Phasor $\bar{X} = |X| e^{j\phi}$ LCCDE

  solve for $\bar{Y}$ @ ω_1 $y(t) = \mathrm{Re}[\bar{Y} e^{j\omega t}]$

- Orthogonality x & y are real

  $\int_0^T x(t) y(t) dt = 0$

Example: $f_c = 100\,MHz$ $\Delta f = 20\,kHz$

$T = \frac{1}{\Delta f} = 50\,\mu s$

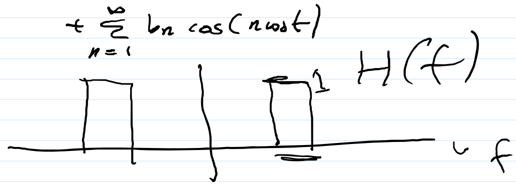
$\int_0^T \cos(2\pi f_c t) \cos(2\pi (f_c + \Delta f) t) = 0$

$\frac{f_c}{\Delta f}$ = integer

- $x_p(t)$ periodic T_0 $f_0\,(Hz) = \frac{1}{T_0}$

  $\omega_0 = \frac{2\pi}{T_0}$

$x_p(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) + \sum_{n=1}^{\infty} b_n \cos(n\omega_0 t)$



$y(t) = c_0 |H(0)| + \sum_{n=1}^{\infty} c_n |H(nf_0)| \cos(2\pi n f_0 + \theta_n + \angle H(nf_0))$

Feb 12, 2026

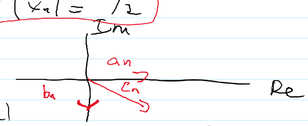
Fourier Series $x_p(t) = \sum_{k=-\infty}^{\infty} p(t - kT_0)$ $p(t)$ $-\tau/2$ $\tau/2$

$x_p(t) = c_0 + \sum_{n=1}^{\infty} c_n \cos(n\omega_0 t + \theta_n)$ $c_n \geq 0$ $n \neq 0$

$a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t)$

$\sum_{n=-\infty}^{\infty} x_n e^{jn\omega_0 t}$ $x_n = |x_n| e^{j\phi_n}$

$x_n = \frac{1}{2}(a_n + j b_n)$ $\boxed{|x_n| = c_n/2}$



- $\mathrm{sinc}(x) = \frac{\sin(x)}{x}$

- $p(t) = \mathrm{rect}(t/\tau)$

  $\frac{\tau}{T_0} + \sum_{n=-\infty}^{\infty} (\tau/T_0) \mathrm{sinc}\left(\frac{n\pi\tau}{T_0}\right)$

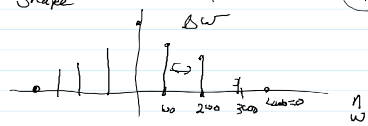
- $p(t) = \text{rect}(t/\tau)$

$$\underset{n=0}{\frac{\tau}{T_0}} + \underset{n\neq 0}{\sum_{n=-\infty}^{\infty}} \left(\tau/T_0\right) \text{sinc}\left(\frac{n\pi\tau}{T_0}\right)$$

Line spacing $= 1/T_0$ (Hz) $\quad \frac{2\pi}{T_0}$ rad/sec

Shape $\rightarrow \tau$ $\quad$ $\tau/T_0 = 1/4$

BW

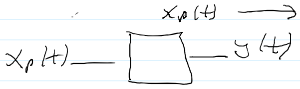


$\frac{n\pi}{4}$ $\quad$ $n = 4$

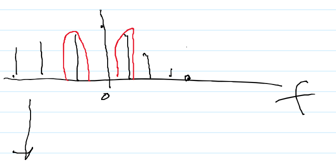
- If $p(t)$ is real $\quad c_n$ $|X_n|$ real θ_n odd

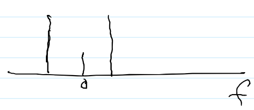
- Magnitude plots
- Given LCCDE $\longrightarrow H(\omega)$

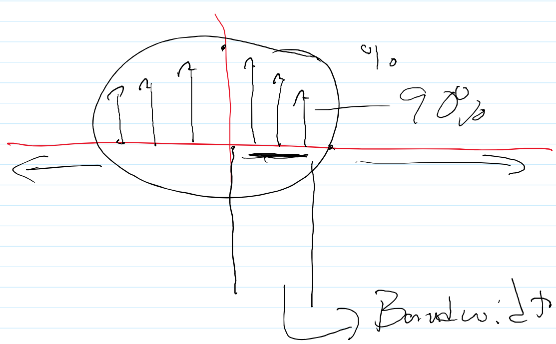
$x_p(t) \longrightarrow$ F.S



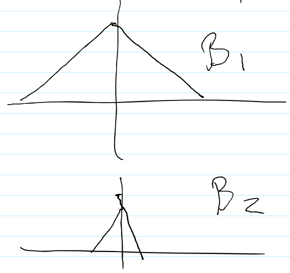
$$= c_0|H(0)| + \sum_{n=1}^{\infty} c_n |H(n\omega_0)| \cos(n\omega_0 t + \theta_n)$$







$$B \propto \frac{1}{\tau}$$



$B_2 > B_1$

Feb 17, 2026 $\quad$ HW 6 #9

$h(t) = u(t)e^{-t} - 16e^{-2t}u(t) + 13e^{-3t}u(t)$

$x(t) = \cos(2t) \quad \omega_1 = 2$

$\int_{-\infty}^{\infty} h(t)e^{-j\omega t}\,dt$

$h(t) = u(t)e^{-t} - 16e^{-2t}u(t) + 13e^{-3t}u(t)$

$x(t) = \cos(2t) \quad \omega_1 = 2$

Section 3.7 $\quad H(\omega) = \int_{-\infty}^{\infty} h(\tau)e^{-j\omega\tau}\,d\tau$

Ex 3.7.1 $\quad \int_0^{\infty} e^{-a\tau}e^{-j\omega\tau}\,d\tau = \frac{1}{a+j\omega}$

$$H(\omega) = \frac{5}{1+j\omega} - \frac{16}{2+j\omega} + \frac{13}{3+j\omega}$$

$$\boxed{y(t) = |H(\omega_1)|\cos(2t + \angle H(\omega_1))}$$

$$H(\omega_1) = \underbrace{\frac{5}{1+2j}}_{(1-2j)} - \underbrace{\frac{16}{2+2j}}_{-4+4j} + \underbrace{\frac{13}{3+2j}}_{3-2j} = 0$$

$\omega_1 = 2$

HW 6 #7 $\quad \frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 7y = 5\frac{dx}{dt}$

$H(\omega) \quad x(t) = e^{j\omega t} \quad y(t) = H(\omega)e^{j\omega t}$

$$H(\omega) = \frac{5j\omega}{(j\omega)^2 + 2j\omega + 7} = \frac{5j\omega}{(7-\omega^2) + 2j\omega}$$

b. Find ω_1 such that $\angle H(\omega_1) = 0$

$7 - \omega_1^2 = 0 \quad H(\omega_1) = 5/2$

$\omega_1 = \sqrt{7}$

– F.S. $\quad x_p(t) = \sum X_n e^{jn\omega_0 t}$

$H(\omega) \quad y(t) = |X_n||H(n\omega_0)|\cos(n\omega_0 t + \angle X_n + \angle H(n\omega_0))$

$H(\omega)$ weighting on each ω

– Parseval's Theorem $\quad \sum_{n=-\infty}^{\infty} |X_n|^2 = \frac{1}{T_0}\int_0^{T_0} |x(t)|^2\,dt$

– Example! $\quad x(t) = 10\,\mathrm{rect}(t/0.1) \quad x_p(t) = \sum_{k=-\infty}^{\infty} x(t-kT_0)$

$T_0 = .5$ Find power between 1Hz – 5Hz

$$x_p(t) = \frac{A\tau}{T_0} + \sum_{n=1}^{\infty} \frac{A\tau}{T_0}\,\mathrm{sinc}\left(\frac{n\pi\tau}{T_0}\right)\cos(n\omega_0 t)$$

$A = 10 \quad \tau = .1 \quad T_0 = .5 \quad \frac{\tau}{T_0} = .2 \quad f_0 = 2\text{Hz}$

$\mathrm{sinc}\left(\frac{5\pi}{.2}\right)$

$\mathrm{sinc}(\pi) = 0$

$$50\,\mathrm{sinc}\left(\frac{\pi}{5}\right)\cos(2\pi 2t) + 50\,\mathrm{sinc}\left(\frac{2\pi}{5}\right)\cos(2\pi 4t)$$

$P = 18.03$

End for Test 1

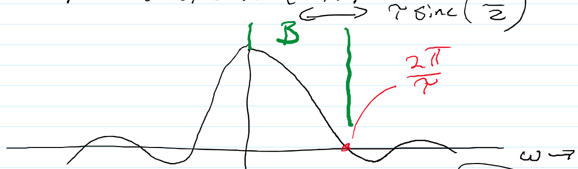
Fourier Transform $\quad X(\omega) = \int_{-\infty}^{\infty} x(\tau)e^{-j\omega\tau}\,d\tau$

$$H(\omega) = \int_{-\infty}^{\infty} h(\tau)e^{-j\omega\tau}\,d\tau$$

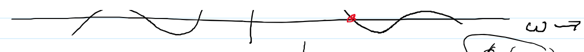
$H(\omega) = \mathcal{F}\{h(t)\}$

$$x(t) = \mathcal{F}^{-1}(X(\omega)) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}\,d\omega$$

Example $\quad x(t) = \mathrm{rect}(t/\tau) \longleftrightarrow \tau\,\mathrm{sinc}\left(\frac{\omega\tau}{2}\right)$



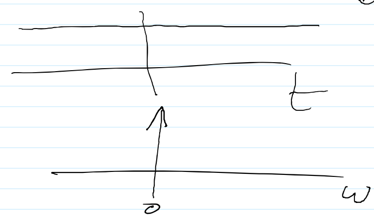
$B \propto \frac{1}{\tau} \quad$ Hz $\quad \boxed{\$/\text{Hz}}$



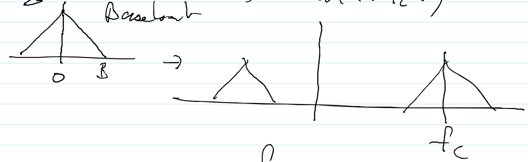
$B \propto \frac{1}{\tau}$ Hz $\boxed{\$/Hz}$

bits/sec $\neq$ Hz

D.C.

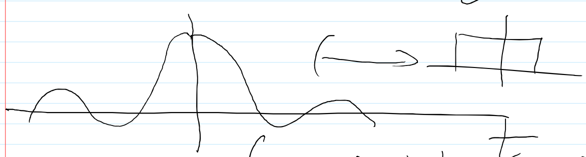


$X(f)$ Baseband $x(t)\cos(2\pi f_c t)$

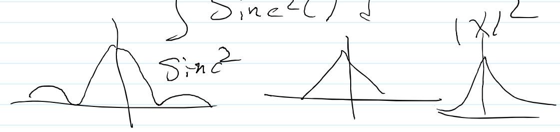


C = carrier freq. 91.5 MHz

$B = 3$ kHz AM $f_c = 610$ kHz



$\int \text{sinc}^2(\,)\, d$ $|X|^2$



February 19, 2026

$\frac{dy}{dt} + y(t) = x(t)$ $x(t) = e^{j\omega t}$

$y(t) = H(\omega) e^{j\omega t}$

$j\omega H(\omega) \cancel{e^{j\omega t}} + H(\omega) \cancel{e^{j\omega t}} = \cancel{e^{j\omega t}}$

$H(\omega) = \frac{1}{1+j\omega}$ $x(t) = 1 \rightarrow ?$ $\omega = 0$

$\cos(t) \rightarrow \omega = 1$ $H(1) = \frac{1}{1+j\omega} = \frac{1}{\sqrt{2}} e^{-j\pi/4}$

$\sqrt{2}\cos(t - \pi/4)$

Fourier Transform — properties

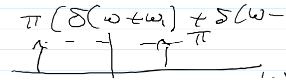
- Additive — Time scaling $B \propto \frac{1}{\tau}$ — time shift $x(t-t_0)$ linear phase
- freq shift — modulation — duality $x(t) \leftrightarrow X(\omega)$, $X(t) \leftrightarrow x(-\omega)$
- conv in time is multiply freq
  $x(t) * h(t) \longleftrightarrow X(\omega) H(\omega)$
- multiply time ?
  $x(t) \cdot y(t) \longleftrightarrow \frac{1}{2\pi} X(\omega) * Y(\omega)$
- If $x(t)$ is real then $|X(\omega)|$ is even, $\angle X(\omega)$ is odd

FT pairs

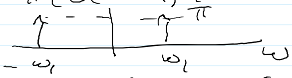
$\delta(t) \leftrightarrow 1$ $1 \leftrightarrow 2\pi\delta(\omega)$

$A\,\text{rect}(t/\tau) \longleftrightarrow A\tau\,\text{sinc}\left(\frac{\omega\tau}{2}\right)$

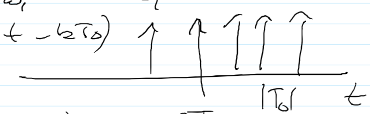
$\cos(\omega_0 t) \longleftrightarrow \pi\left(\delta(\omega+\omega_0) + \delta(\omega-\omega_0)\right)$



$\cos(\omega_1 t) \longleftrightarrow \pi\left(\delta(\omega+\omega_1) + \delta(\omega-\omega_1)\right)$



$\delta_{T_0}(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_0)$



$\updownarrow$

$\frac{1}{T_0} \sum_{n=-\infty}^{\infty} \delta(\omega - n\omega_0) \quad \omega_0 = \frac{2\pi}{T_0}$

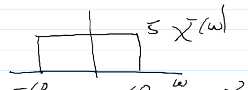
Example $x(t) = \frac{50}{\pi}\,\text{sinc}(10t)$

a) E_x

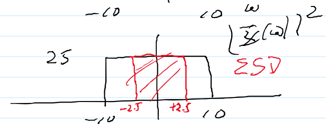
b) % energy $B = 2.5$ rad/sec

$\int_{-\infty}^{\infty} \left(\frac{50}{\pi}\,\text{sinc}(10t)\right)^2 dt$

$X(\omega) = 5\,\text{rect}\left(\frac{\omega}{20}\right)$

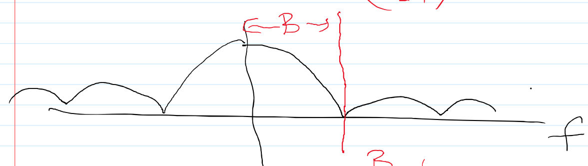


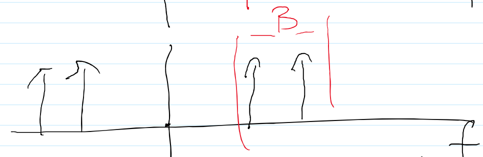
$\frac{1}{2\pi}\int |X(\omega)|^2 d\omega$

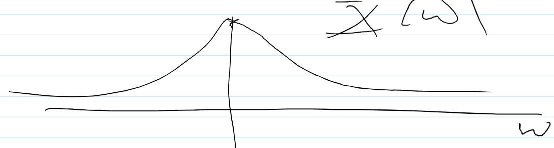


$\frac{20 \cdot 25}{2\pi} = 79.6$

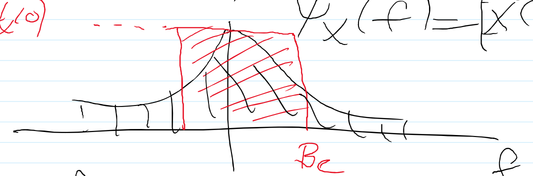
$\% = 100\left(\frac{\frac{5 \cdot 25}{2\pi}}{\frac{20 \cdot 25}{2\pi}}\right) = 25\%$







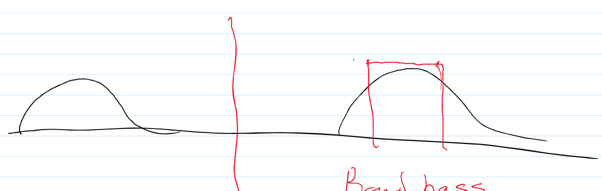
$\Psi_x(f) = |X(f)|^2$



$\int_{-\infty}^{\infty} \Psi_x(f)\,df = \Psi_x(0)\,2B_e$

$B_e = \frac{\int_{-\infty}^{\infty} \Psi_x(f)\,df}{2\Psi_x(0)}$

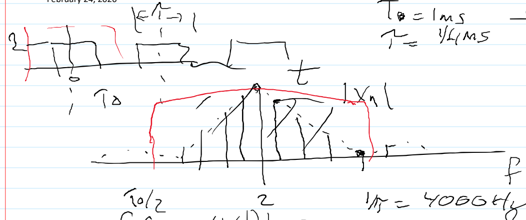
$= \frac{\int_0^{\infty} \Psi_x(f)\,df}{\Psi_x(0)}$



$B \propto \frac{1}{\tau}$

$B \propto \frac{1}{\tau}$

February 24, 2026



$T_0 = 1ms$ $\quad f_0 = 1000$

$\tau = 1/4 ms$

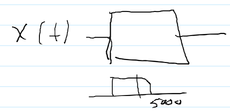
$1/\tau = 4000 Hz$

$P_x = \int_{-T_0/2}^{T_0/2} (2\, rect(t/\tau))\, dt$

$= \int_{-\tau/2}^{\tau/2} 4\, d\tau = 1$

$y(t) = \quad n = -5 \dots 5$

$P_y = \sum_{n=-5}^{5} |x_n|^2$



$x_0 = 1/2$

$x_n = A \frac{\tau}{T_0} \operatorname{sinc}\left(\frac{n\pi\tau}{T_0}\right)$

$x_1 = 0.45 \quad x_4 = 0$

$x_2 = .35 \quad x_5 = .09$

$x_3 = .15$

$P_y = 0.91 \qquad 91\%$

$x_n = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} x(t) e^{jn\omega_0 t} dt$

$\frac{1}{T_0} \int_{-\tau/2}^{\tau/2} 2 e^{jn\omega_0 t} dt$

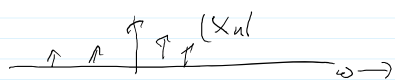
$x(t) = \cos(2\pi 500 t) + \frac{1}{2} + \frac{1}{2}\cos(2\pi 1000 t)$

$a_0 = \frac{1}{2} \quad a_1 = 1 \quad a_2 = 1/2$

- $E_x = \int_{-\infty}^{\infty} |x(t)|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(\omega)|^2 d\omega$

$x(t) \longleftrightarrow X(\omega)$

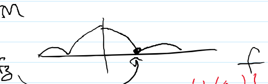
- $x_p(t) = \sum X_n e^{jn\omega_0 t} \longleftrightarrow \sum 2\pi X_n \delta(\omega - n\omega_0)$



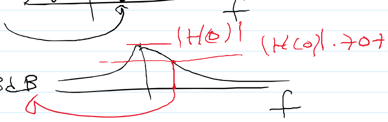
- $x_p(t) = \sum x(t - kT_0)$ $\qquad \omega_0 = \frac{2\pi}{T_0}$

$x_n = \frac{1}{T_0} X(n\omega_0)$

- Bandwidth
  - B_{f_3}



  - B_{3dB}



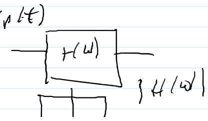
  - B_e

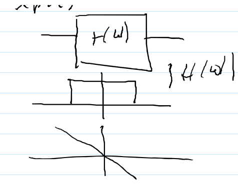
  - B_{rms}

- $B\tau \to$ constant

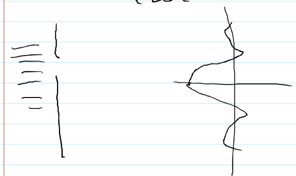
$B \propto \frac{1}{\tau}$

$x_p(t)$





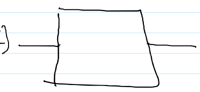
$y(t) = A|H(\omega)| \cos(\omega t + \theta \angle H(\omega))$

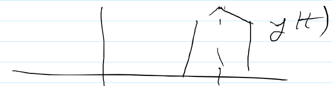


Ideal filter

$K \neq 0$

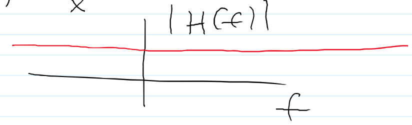
$\delta(t)$ $\quad K\delta(t - t_0)$

$x(t)$  $Kx(t - t_0)$



$X(f)$ $\quad Y(f) = K e^{-j2\pi f t_0} X(f)$

$H(f) = \frac{Y}{X} = K e^{j2\pi f t_0}$



Constant Magnitude

Linear phase

$h(t) = K\delta(t - t_0)$

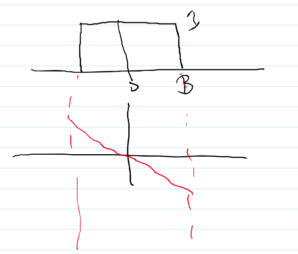
dB $\quad$ dB_m $\quad$ Power wrt 1 mS

1 mW	$0\,dB_m$
10 mW	$= 10\,dB_m$
dB_W	1 watt

dB_{stuff}

ILPF $\quad K e^{-j2\pi f t_0}$ $\quad |f| < B$

$= 0$ $\quad |f| > B$



February 26, 2026

HW 7 $\quad$ 15 $\quad 5\cos(2\pi 300 t) - 2\sin(2\pi 600 t)$

$5\cos(2\pi 300 t) - 2\cos(2\pi 600 t - \pi/2)$

$5\cos(2\pi 300 t) + 2\cos(2\pi 600 t - \pi/2 - \pi)$ $\quad -\frac{3\pi}{2}$

$\underbrace{\frac{2}{2} e^{-j3\pi/2}}_{X_{-2}} e^{-j2\pi 600 t} + \underbrace{\frac{5}{2}}_{X_{-1}} e^{-j2\pi 300 t} + \underbrace{\frac{5}{2}}_{X_1} e^{+j2\pi 300 t} + \underbrace{\frac{2}{2} e^{+j\frac{3\pi}{2}}}_{X_2} e^{+j2\pi 600 t}$

$P = \sum |X_i|^2 = 1^2 + \left(\frac{5}{2}\right)^2 + \left(\frac{5}{2}\right)^2 + 1^2$

$= 5^2/2 + 2^2/2$

- $H(\omega) = |H(\omega)| e^{j\phi(\omega)} \longrightarrow \phi(\omega)$ is important
- LPF, BPF, BRF, HPF $\rightarrow$ passband $\rightarrow$ BW

$\approx h(t)$ $\quad$ If $B_{system} \gg B_{signal}$

- LPF, BPF, BRF, HPF → passband → BW
- $x(t)$ B_{signal} — B_{system} — $y(t)$

If $B_{system} \gg B_{signal}$ then $y(t) \approx x(t)$

- RLC bandpass & bandreject filter, tune by change $\underline{C}$

$\omega_0 = \frac{1}{\sqrt{LC}}$

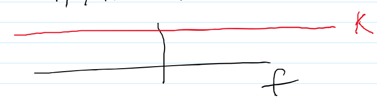
$\boxed{Q\text{-factor}}$

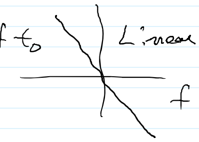
- Distortionless transmission — Ideal

$x(t)$ — ☐ — $y(t) = K x(t - t_0)$ $\forall f$

$H(f) = K e^{-j2\pi f t_0}$ $\phi(f) = -2\pi f t_0$

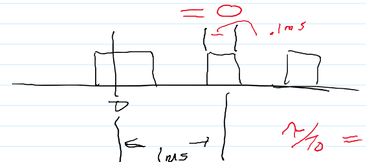
phase — Linear



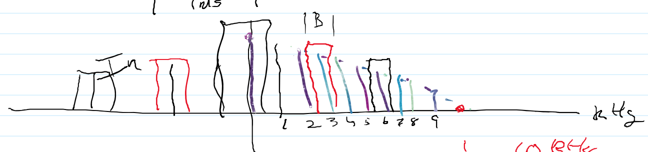


ILPF $H(f) = K e^{-j2\pi f t_0}$ $|f| < B$

$= 0$



$\tau/T_0 = .1$ 10%



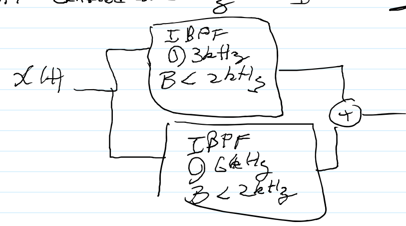
$\frac{1}{\tau} = 10\,kHz$

$f_0 = 1\,kHz$

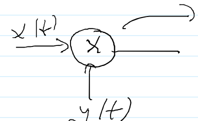
Out

a) IBPF centered at 3 kHz $B < 2\,Hz$

b)



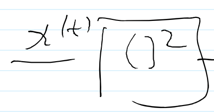
c) ILPF $B < 1\,kHz$



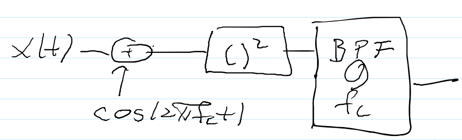
→ mixer product multiplier

typically $y(t) = \cos(2\pi f_c t)$

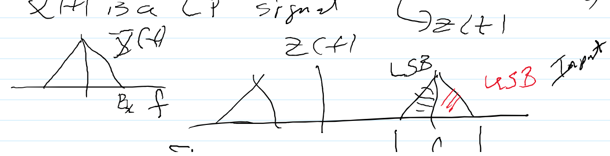
C = Carrier

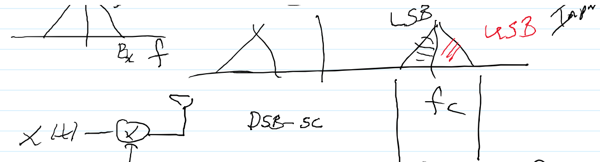


$y(t) = x^2(t)$

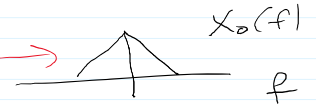


$x(t)$ is a LP signal — $K x(t)\cos(2\pi f_c t)$ → $z(t)$



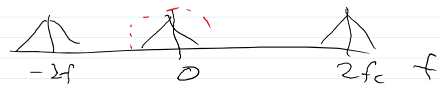


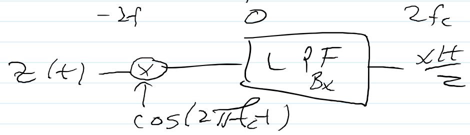
$B_{RF} = 2B_x$



$z(t)\cos(2\pi f_c t) = [x(t)\cos(2\pi f_c t)]\cos(2\pi f_c t)$

$x(t)\cos^2(2\pi f_c t) = \frac{1}{2}x(t) + \frac{1}{2}x(t)\cos(2\pi 2 f_c t)$





March 3, 2026

$x_p(t)$ $P = \frac{1}{T_0}\int_{T_0} |x(t)|^2 dt$

$E_x = \int |x(t)|^2 dt$

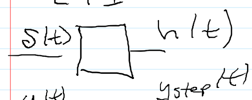
$x(t)$

$x\left(\frac{t-t_0}{\tau}\right)$

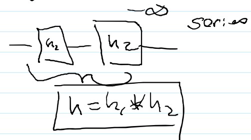
$A^2/2$

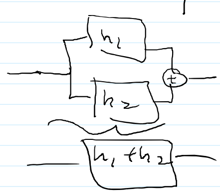
$A_n \cos(n\omega_0 t + \theta_n)$

LTI



$y_{step}(t) = \int_{-\infty}^{t} h(\lambda)\, d\lambda$





BIBO

If $\int |h(t)|dt$ finite then BIBO

If $h(t) = 0$ $t < 0$ then causal

If $h(t) = h(t)u(t)$

$x(t) = e^{j\omega t}$ LTI

$H(\omega)e^{j\omega t}$

If $h(t) = h(t)u(t)$

$x(t) = e^{j\omega t}$ LTI

$H(\omega) e^{j\omega t}$

$H(\omega)$

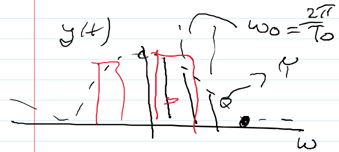
If $x(t) = A\cos(\omega_1 t + \theta)$

$y(t) = A\,|H(\omega_1)|\cos(\omega_1 t + \theta + \angle H(\omega_1))$

Find $H(\omega)$

Find c_n & ϕ_n

$y(t)$ $\omega_0 = \frac{2\pi}{T_0}$

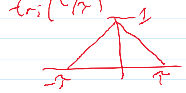


$\sum A_n \cos(n\omega_0 t + \phi_n)$

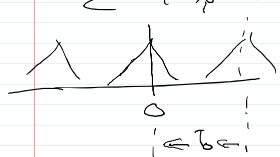
$P = \sum_{n=0}^{\infty} A_n^2/2$

$\sum_{n=-\infty}^{\infty} |X_n|^2$

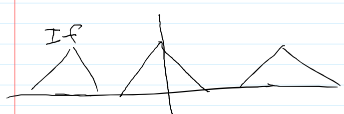
% power in Band of frequencies



$\sum tri\left(\frac{t - nT_0}{\tau}\right)$

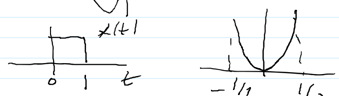


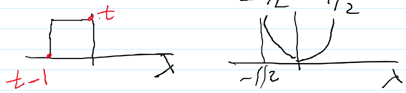
If



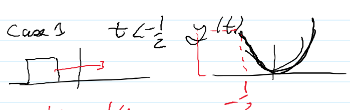
$b_n = 0$

$a_n = 0$

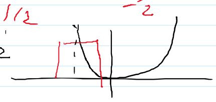




Case 1 $t < -\frac{1}{2}$ $y(t)$

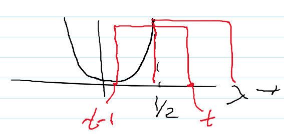


case 2



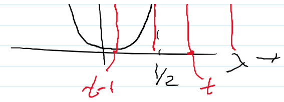
$\int_{-1/2}^{t} \lambda^2 \, d\lambda$ $-\frac{1}{2} < t < \frac{1}{2}$

Case 3



$\int^{1/2}$

Case 3



$\int_{t-1}^{3/2} \lambda^2 \, d\lambda$ $\quad \frac{1}{2} < t < \frac{3}{2}$

$t-1 = 1/2$

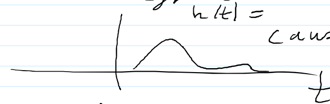
Dynamic $y(t) = x(t) - 5x(t+3)$

$y(5) = x(5) - 5x(8)$

Causal $\rightarrow$ not causal

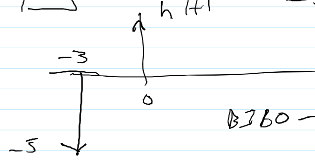
LTI for $h(t) \neq K$ dynamic

$h(t) =$ causal



Find $h(t)$

$\underline{\delta(t)} \to \boxed{} \to h(t)$ $\quad h(t) = \delta(t) - 5\delta(t+3)$



BIBO - stable

$a x_1(t) \longrightarrow a x_1(t) - 5a x_1(t+3) = y_1$

$b x_2(t) \rightarrow b x_2(t) - 5b x_2(t+3) = y_2$

$a x_1 + b x_2 \rightarrow y_1 + y_2$ so linear

$x(t-\tau) = x_1(t-\tau) - 5x_1(t+3-\tau)$

$= y_1(t-\tau)$ TI

$y(t) = x(t) - 5x(t+3) + 3$

NL

$\mathrm{sinc}(x) = \frac{\sin(x)}{x}$

$c_1 \frac{dy}{dt} + c_2 y(t) = x(t)$

$c_1 j\omega H(\omega) \cancel{e^{j\omega t}} + c_2 H(\omega) \cancel{e^{j\omega t}} = \cancel{e^{j\omega t}}$

$H(\omega) = \frac{1}{c_1 j\omega + c_2}$

$= |H(\omega)| e^{j\phi(\omega)}$

$x_1(t) = 4\cos(300t - \pi/6)$

$y_1(t) = 4|H(300)| \cos(300t - \frac{\pi}{6} + \phi(300))$

$x_2(t) = -8\cos(600t - \pi/4)$

$y_2(t) = -8|H(600)| \cos(600t - \frac{\pi}{4} + \phi(600))$

$y(t) = y_1(t) + y_2(t)$

HW 7 #7 $\quad n=1 \quad n=2$

$x(t) = 8\cos(300t) - 3\sin(600t)$

$\omega_0 = 300$

$a_0 = 0 \quad a_1 = 8 \quad b_1 = 0$

$a_2 = 0 \quad b_2 = -3$

$c_n \quad c_0 = 0$

$8\cos(300t) - 3\cos(600t - \frac{\pi}{2})$

$8\cos(300t) + 3\cos(600t - \frac{\pi}{2} - \pi)$

$c_1 = 8 \quad \phi_1 = 0 \quad c_2 = 3 \quad \phi_2 = -\frac{3\pi}{2}$

$$8\cos(300t) + 3\cos(600t - \tfrac{\pi}{2} - \pi)$$

$$c_1 = 8 \quad \phi_1 = 0 \qquad c_2 = 3 \quad \phi_2 = -\tfrac{3\pi}{2}$$

$$\tfrac{3}{2} e^{+j\frac{3\pi}{2}} e^{-j600t} \quad n = -2$$

$$+ \tfrac{8}{2} e^{-j300t} \quad n = -1$$

$$+ \tfrac{8}{2} e^{+j300t} \quad n = +1$$

$$\tfrac{3}{2} e^{-j\frac{3\pi}{2}} e^{+j600t} \quad n = +2$$

$$x_{-2} = \tfrac{3}{2} e^{+j3\pi/2} \qquad X_{-1} = \tfrac{8}{2}$$

$$x_1 = \tfrac{8}{2} \qquad x_2 = \tfrac{3}{2} e^{-j\frac{3\pi}{2}}$$

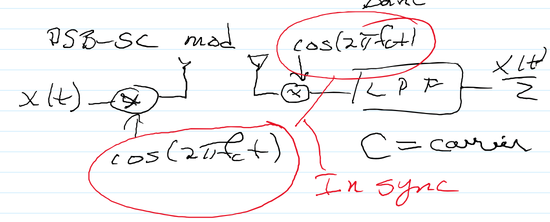
$$x_0 = 0$$

March 10, 2026

- Ideal filters $\quad H(\omega) = K e^{-j\omega t_0}$ over some band

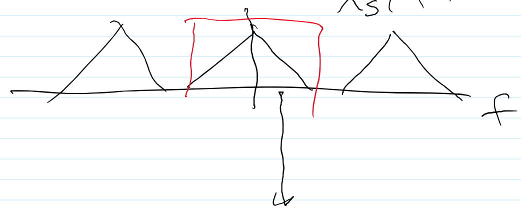
ILPF IHPF
IBPF IBRF

- DSB-SC mod

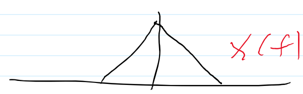


C = carrier

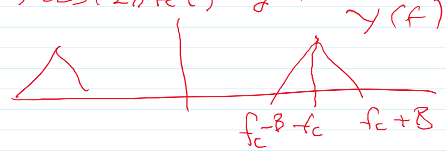
- DSB-LC AM $\quad X_s(f)$



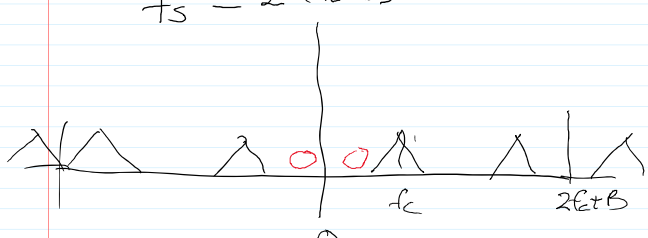
want Recover $X(f)$



$x(t)\cos(2\pi f_c t) = y(t)$ $\quad Y(f)$



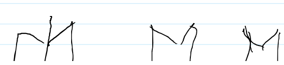
$$f_s = 2(f_c + B)$$



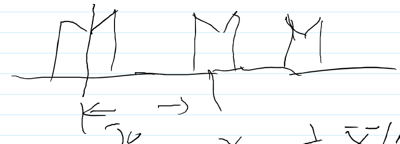
March 12, 2026

$$x(t) = \text{rect}(t/2) - \tfrac{1}{2}\,\text{tri}(t) \qquad \tau = 1$$

$$X(\omega) = 2\,\text{sinc}(\omega) - \tfrac{1}{2}\,\text{sinc}^2\!\left(\tfrac{\omega}{2}\right)$$



$\bar{X}(\omega) = 2\,\mathrm{sinc}(\omega) - \frac{1}{2}\,\mathrm{sinc}(\frac{\omega}{2})$



$x_n = \frac{1}{T_0}\bar{X}(n\omega_0)$

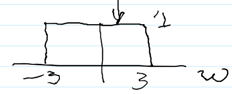
$\omega_0 = 2\pi/T_0$

HW 9 #7

$x(t) = u(t)e^{-.5t}$ → [ ] → $y(t)$

$|X(\omega)|^2 = \frac{16}{\frac{3}{4}+\omega^2}$

$\frac{3}{\pi}\,\mathrm{sinc}(3f)$



$Y(\omega) = \bar{X}(\omega)\,\mathrm{rect}(\frac{\omega}{6})$

$E_y = \int_{-3}^{3} |\bar{X}(\omega)|^2\, d\omega = 14.31$

— FDM

— Frequency conversion

$x(t)\cos(2\pi f_c t)$ → ⊗ → BPF $B = B_x$ → $x(t)$

$\cos(2\pi f_{LO} t)$

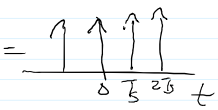
$x(t)\cos(2\pi(f_c+f_{LO})t) + x(t)\cos(2\pi(f_c-f_{LO})t)$

$f_c - f_{LO}$

— Superhet

— Sampling theorem

$\delta_{T_s}(t) =$



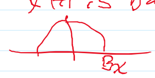
$x_s(t) = x(t)\,\delta_{T_s}(t)$

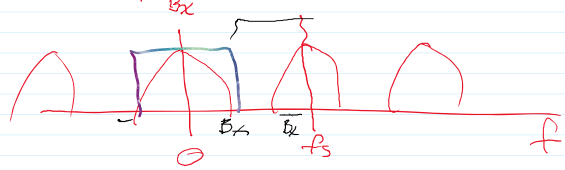
$X(f)$

$\mathcal{F}[\delta_{T_s}(t)] = f_s \sum_{n=-\infty}^{\infty} \delta(\omega - nf_s)$

Assume $x(t)$ is band limited

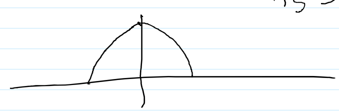
$f_s = \frac{1}{T_s}$



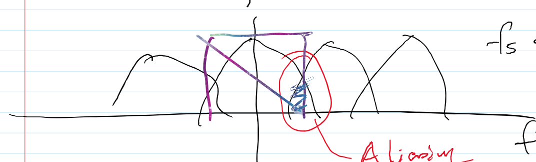


$f_s > 2B_x$

$2B_x$ = Nyquist rate



$f_s < 2B_x$



$x(t)$ not band limited → Aliasing

① Combat aliasing

→ [Anti Aliasing filter ILPF $B = f_s/2$] → [Sample]

② f_s ↑

$x_s(kT_s) \to x[k]$ discrete time

March 24, 2026

$x(t) = \cos(\omega_0 t)$

$= \cos(2\pi f_0 t)$

as $f_0 \uparrow$ $T_0 \downarrow$

$T_0 = 1/f_0$

$\text{rect}\left(\frac{t-0.05}{0.25}\right)$ $\quad \text{rect}(t/\tau) \Longleftrightarrow \tau\,\text{sinc}\left(\frac{\omega\tau}{2}\right)$

$x(t-t_0) \Longleftrightarrow X(\omega)e^{-j\omega t_0}$

$0.25\,\text{sinc}(0.125\omega)e^{-j0.05\omega}$

$x(t) = \underbrace{\text{rect}(t/0.25)}\,\underbrace{\cos(200t)}$

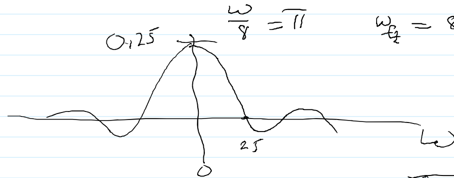
$\pi\delta(\omega+200) + \pi\delta(\omega-200)$

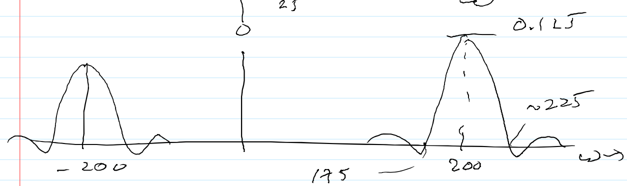
$0.125\,(\text{sinc}(0.125(\omega+200))$

$+\text{sinc}(0.125(\omega-200)))$

$0.25\,\text{sinc}(0.125\omega)$ $\quad 0.125 = 1/8$

$\frac{\omega}{8} = \pi$ $\quad \omega_{f} = 8\pi \approx 25$

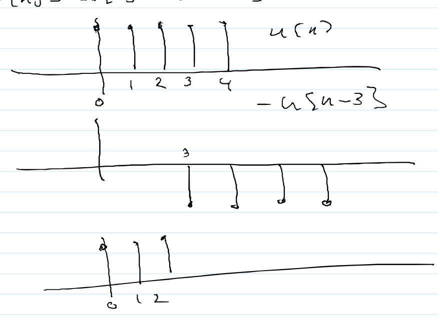




- Sampling $\quad f_s > 2B$
- $x(kT_s) = x[n] = \{3\ \underline{7}\ 9\ 0\ 1\}$

$x[-1] = 3 \quad x[0] = 7 \quad x[1] = 9$

- $x[-n] \quad x[n-k] \quad x\left[\frac{n}{k}\right]$
- $\delta[n] \quad u[n]$
- $x[n] = u[n] - u[n-3]$



- $u[n]\rho^n$
- $\cos(\omega_0 t + \theta)$ sample @ f_s $\quad T_s = \frac{1}{f_s}$

$x[n] = \cos(\omega_0 n T_s + \theta)$

$= \cos(\Omega_0 n + \theta)$

$\underline{\underline{\Omega_0}} = \omega_0 T_s = \frac{2\pi f_0}{f_s} \quad \left(\frac{f_0}{f_s}\right)$

Period $N_0 = \frac{kT_0}{T_s}$ $\quad T_0 = \frac{2\pi}{\omega}$

Find smallest k such that

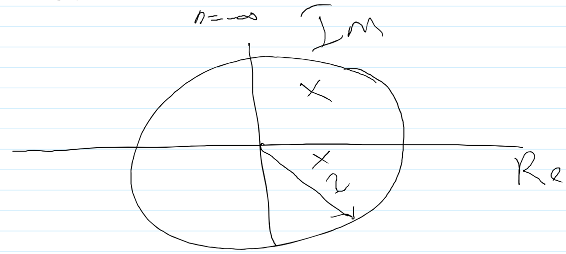
N_0 is integer

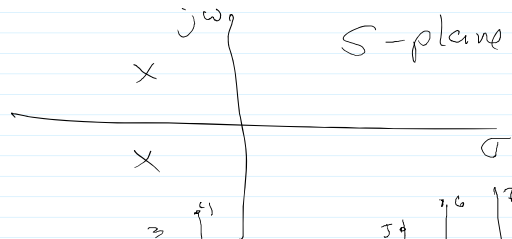
$$y[n] = \frac{1}{3}x[n] + \frac{1}{3}x[n-1] + \frac{1}{3}x[n-2]$$

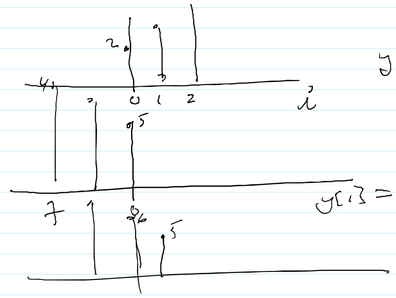
$$h[n] = \frac{1}{3}\delta[n] + \frac{1}{3}\delta[n-1] + \frac{1}{3}\delta[n-2]$$

Causal $\quad h[n] = u[n]h[n]$

BIBO Stable $\quad \sum_{n=-\infty}^{\infty} |h[n]| < \infty$







$y[0] = 10$

$y[1] = 6\cdot 2 + 5\cdot 3$

March 26, 2026

$y(t) = 1 + \cos(.5t) + \cos(t)$

$H(\omega) = \mathrm{rect}(\omega) + 4\,\mathrm{rect}\left(\frac{\omega}{4}\right)$

$h(t) = \frac{1}{2\pi}\mathrm{sinc}\left(\frac{t}{2}\right) + \frac{8}{\pi}\mathrm{sinc}(2t)$

HW 10 #9 $x(t) =$

$x(t) \rightarrow \boxed{\text{ILPF } B} \rightarrow y(t) \quad B > 5000$

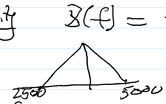
$H(\omega) = Ke^{j\omega t_0}$ over B_x

$5000\,\mathrm{sinc}^2(5000\pi f)$

$\mathrm{tri}(t/\tau) \longleftrightarrow \tau\,\mathrm{sinc}^2\left(\frac{\omega\tau}{2}\right)$

Duality $\quad X(f) = \mathrm{tri}\left(\frac{f}{B}\right)$

$B = 5000$



$$\int_{-2500}^{2500} X^2(f)\,df$$

$$= 2\int_0^{2500} \left(1 - \frac{f}{5000}\right)^2 df$$

$x(t) = \cos(\omega_0 t)$ Sample at f_s

$= \alpha \int_0 (\; \frac{\;}{5000})$

$x(t) = \cos(\omega_0 t)$ Sample at f_s
$T_s = 1/f_s$

$x[n] = \cos(\Omega_0 n)$ $\quad \Omega_0 = \frac{2\pi f_0}{f_s}$

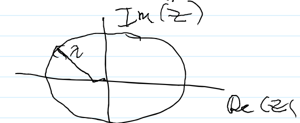
$2\pi f_0 = \omega_0$

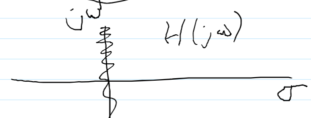
$\left(\frac{f_0}{f_s}\right)$

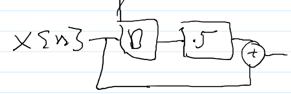
- DT LTI Systems
- Causal if $h[n] = h[n]u[n]$
- BIBO Stable $\sum_{n=-\infty}^{\infty} |h[n]| < \text{finite}$
- ARMA

$$y[n] = \underbrace{\sum_{i=0}^{M} b_i x[n-i]}_{MA} - \underbrace{\sum_{k=1}^{N} a_k y[n-k]}_{AR}$$

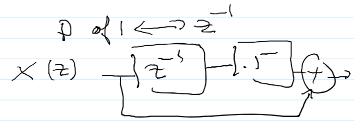
- Block diagram
- $\delta[n] \rightarrow \boxed{\;\;} \rightarrow h[n]$
- MA $h[n] = \{b_0, b_1, \dots b_M\}$
- Case $h[n] = \sum C_i p_i^n u[n]$
  BIBO stable if all $|p_i| < 1$
- D.T conv $y[n] = \sum_{i=-\infty}^{\infty} x[i]h[n-i]$
- N_x length of x $\quad N_h$ length of h
  $N_y = N_x + N_h - 1$

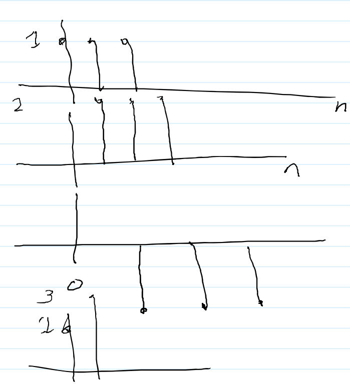






D of 1 $\longleftrightarrow z^{-1}$



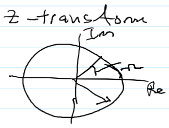


March 31, 2026

– DT conv

– Z-transform $X(z) = \sum x[n] z^{-n}$

z complex



FT of sampled signal

$X(z)\big|_{e^{j\Omega}}$

$\delta[n] \longleftrightarrow 1$ $\quad u[n] \longleftrightarrow \frac{z}{1-z}$ $\quad a^n u[n] \longleftrightarrow \frac{z}{z-a}$

Table

$\delta[n-k] \longleftrightarrow z^{-k}$

$-\boxed{D}-$

z^{-1}

– Properties

+ Add + Associative $x * h = h * x$

$x_1 + x_2 \longleftrightarrow X_1(z) + X_2(z)$

$x[n-k] \longleftrightarrow z^{-k} X(z)$

LTI $x[n] * h[n] \longleftrightarrow X(z)\, H(z)$

– Finite Impulse Response filter or MA (FIR)

$y[n] = \sum_{k=0}^{M} b_k\, x[n-k]$

$h[n] = \{b_0, b_1, b_2, \ldots b_M\}$

$H(z) = \sum_{k=0}^{M} b_k z^{-k} = b_0 + b_1 z^{-1} + b_2 z^{-2}$

$= \dfrac{b_0 z^M + b_1 z^{M-1} \cdots b_M}{z^M}$

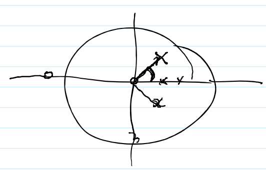
– Example $x[n] = \{\underline{0}, 1, 1\}$ $\quad h[n] = \{\underline{0}, 0, 2, 1\}$

$y[n]$ $\quad X(z) = z^{-1} + z^{-2}$ $\quad H(z) = 2z^{-2} + z^{-3}$

$Y(z) = 2z^{-3} + z^{-4} + 2z^{-4} + z^{-5}$

$2z^{-3} + 3z^{-4} + z^{-5}$

Inverse Z-T $Y(z)$ $\quad y[n] = \{\underline{0}, 0, 0, 2, 3, 1\}$



April 2, 2026

$\delta \rightarrow \boxed{} \rightarrow h[n]$

~~h[~~ $h[n] = \cancel{x}\,\delta(n+2)$

$+ 2\delta(n+1)$

$+ 3\delta[n]$

$+ 2\delta[n-1)$

$+ 1\,\delta[n-2]$

$\sum |h[n]| = 9 < \infty$

Stable

$h[n] \neq h[n]u[n]$ $\quad$ non causal

$x(n+2)$ $\quad n=0$ $\quad x[2]$

– Inverse Z-tranform

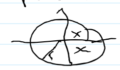
+ match terms + polynomial divide + from table + PFE

– $H(z) = \dfrac{Y(z)}{X(z)} = \dfrac{N(z)}{D(z)}$ $\quad N(z) \rightarrow$ zeros, $\quad D(z) \rightarrow$ poles

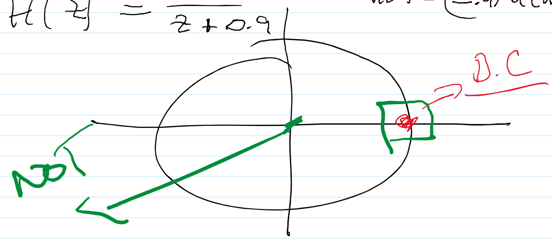
– BIBO Stable poles are inside unit circle

$X(z)$ $D(z) \longrightarrow$ poles

- BIBO Stable poles are inside unit circle



$$H(z) = \frac{z}{z+0.9}$$ $h[n] = (-.9)^n u[n]$



April 7, 2026

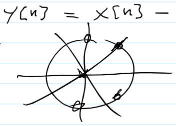
$$\frac{z^2}{z^2+0.81} = \frac{1}{1+0.81z^{-2}}$$ $\frac{Y}{X}$

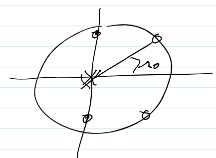
$X(z) = Y(z) + 0.81 z^{-2} Y(z)$

$Y(z) = X(z) - 0.81 z^{-2} Y(z)$

$y[n] = x[n] - 0.81\, y[n-2]$

BRF





$$\frac{z^2+0.81}{z^2}$$

HW13
11c

$y[n] = x[n] + 1.66\, y[n-1] - 0.81\, y[n-2]$

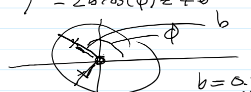
$Y(z) = X(z) + 1.66 z^{-1} Y(z) - 0.81 z^{-2} Y(z)$

$$H(z) = \frac{1}{1-1.66z^{-1}+0.81z^{-2}}$$

$$= \frac{z^2}{z^2-1.66z+0.81}$$

12. $y[n] = x[n] + 2b\cos(\phi)\, y[n-1] - b^2$

$$\frac{z^2}{1-2b\cos(\phi)z+b^2}$$



$b = 0.9$
$\phi = \pi/8$

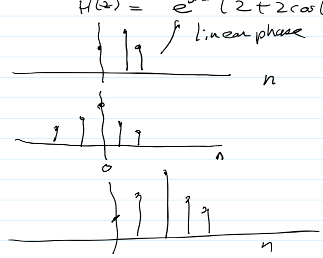
#13 $h[n] = \{1\ \underline{2}\ 1\} = \delta(n+1) + 2\delta[n] + \delta[n-1]$

$z^{1} + 2 + z^{-1}$

$z = e^{j\Omega}$ $H(z) = e^{j\Omega} + 2 + e^{-j\Omega}$

$2 + 2\cos(\Omega)$

$h[n] = \{\underline{1}\ 2\ 1\}$
$H(z) = e^{-j\Omega}(2 + 2\cos(\Omega))$ linear phase



- $H(z)$ Freq response $H(z)\big|_{e^{j\Omega}}$

- $x[n] = A\cos(n\Omega_0 + \theta) \to \boxed{LTI}$

$y[n] = A|H(e^{j\Omega_0})|\cos(n\Omega_0 + \theta + \angle H(e^{j\Omega_0}))$

$x(t) = \cos(2\pi f_0 t)$ sample @ f_s

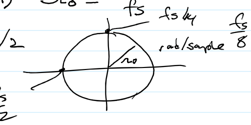
- $x[n] = A\cos(n\Omega_0 + \theta) \rightarrow \boxed{LTI}$

$y[n] = A\,|H(e^{j\Omega_0})|\cos(n\Omega_0 + \theta + \angle H(e^{j\Omega_0}))$

- $x(t) = \cos(2\pi f_0 t)$ sampled @ f_s

$x[n] = \cos\left(\frac{2\pi f_0}{f_s} n\right)$ $\Omega_0 = \frac{2\pi f_0}{f_s}$

Highest freq $= f_s/2$

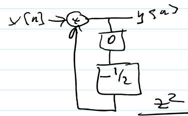


- $|H(e^{j\Omega})|$ even $\angle H(e^{j\Omega})$ odd

- Representations
  + D.Eq + Block diagram + $H(z)$ + $H(e^{j\Omega})$
  + $h[n]$

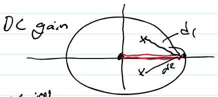
Ex: $H(e^{j\Omega}) = \frac{e^{j\Omega}}{e^{j\Omega} - 1/2}$ $H(z) = \frac{z}{z - 1/2}$

$h[n] = (\frac{1}{2})^n u[n]$ $y[n] = x[n] + \frac{1}{2} y[n-1]$



$H(z) = \frac{z^2}{(z + .5e^{j\pi/4})(z + .5e^{-j\pi/4})}$

DC gain



$|H(e^{j0})| = \frac{(1)(1)}{d_1 d_2}$

April 9, 2026

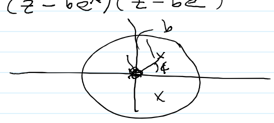
April 14, 2026

HW14 #4 $y[n] = x[n] + 2b\cos(\phi) y[n-1] - b^2 y[n-2]$

$\frac{z^2}{z^2 - 2b\cos(\phi) z + b^2}$ 2 zeros @ 0

$= \frac{z^2}{(z - be^{j\phi})(z - be^{-j\phi})}$ poles are at $be^{\pm j\phi}$

BIBO $b < 1$



HW 14 #6 $y[n] = x[n] + 2x[n-1] + x[n-2]$

$H(z) = \frac{z^2 + 2z + 1}{z^2}$

$x(t) = 2\cos(2\pi 750 t)$ $f_s = 4000 \rightarrow x[n]$

$y[n]$ $\Omega_1 = \frac{2\pi 750}{4000} = 0.375\pi = 1.17$

$|H(e^{j\Omega_1})| = \left| \frac{e^{j2(1.17)} + 2e^{j1.17} + 1}{e^{j(2\cdot 1.17)}} \right| = 2.76$

$\angle H(e^{j\Omega_1}) = -1.17$

$y[n] = (2 \cdot 2.76)\cos(n\,1.17 - 1.17)$

- $H(z)$ $H(e^{j\Omega})$

- LP, HP, BP, BR, notch, comb

April 16, 2026

$y[n] = x[n] + 2x[n-1] + x[n-2]$

$Y(z) = X(z) + 2z^{-1}X(z) + z^{-2}$

$\frac{Y}{X} = 1 + 2z^{-1} + z^{-2} = \frac{z^2 + 2z + 1}{z^2}$

$f_1 = 1000$ $f_s = 2000$

$\Omega_1 = \frac{2\pi f_1}{f_s} = \pi$

$H(e^{j\pi}) = e^{j2\pi} + 2e^{j\pi} + 1$

$f_1 = 1000 \quad f_s = 2000$

$\Omega_1 = \frac{2\pi f_1}{f_s} = \pi$

$z = e^{j\pi} \qquad H(e^{j\pi}) = \frac{e^{j2\pi} + 3e^{j\pi} + 1}{e^{j2\pi}}$

$e^{j\pi} = -1 \quad e^{j2\pi} = 1 \qquad = \frac{1 + (-3) + 1}{1} = -1$

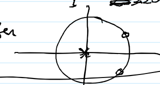
$H(e^{j0}) = \frac{e^{j0} + 3e^{0} + 1}{e^{j0}} = 5$

$y[n] = \frac{1}{5}x[n] + \frac{3}{5}x[n-1] + \frac{1}{5}x[n-2]$

HW 14 #13 $\quad y(t) = x(t) + \cos(2\pi 2000 t)$ Interfere

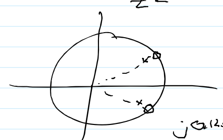
$f_s = 32000 \qquad \Omega_I = \frac{2\pi\, 2000}{32000} = 0.125\pi$

notch filter



$H(z) = \frac{1 - 1.8z + z^2}{z^2}$

$\frac{(z - e^{j(0.125\pi)})(z - e^{-j(0.125\pi)})}{z^2}$



$H(z) = \frac{(z - e^{j0.125\pi})(z - e^{-j0.125\pi})}{(z - 0.92e^{j0.125\pi})(z - 0.92e^{-j0.125\pi})}$

DFT (FFT) $\quad$ FS of a period DT Signal

$x[0] \ldots x[N_0-1]$ — DFT — $X[0] \ldots X[N_0-1]$ complex

Parameters $\quad \Delta f = \frac{1}{L} \quad L$ (sec)

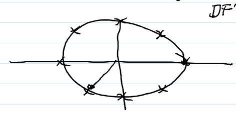
$f_s \quad \Delta t = 1/f_s$

$L\uparrow \quad \Delta f \downarrow$

Picket fence $\qquad$ Discrete frequency Δf

Spectral Leakage

– $x[n] \longleftrightarrow X(z)$



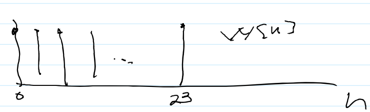
$DFT\{x[n]\} = X(z)\Big|_{e^{j\frac{2\pi k}{N_0}}}$

Properties — Linear — Time Shift
— freq shift — Energy

$y[n] = x[n] * h[n] \qquad N_x = $ length of $x[n]$
$N_h = $ " " $h[n]$

$N_y = N_x + N_h - 1$

$DFT^{-1}\left[[DFT\{x\} \cdot DFT\{h\}]\right] = y[n]$



$w[n]$

$W(z)$

April 21, 2026

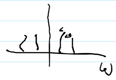
$e^{j\omega_0 t} \longleftrightarrow 2\pi\delta(\omega - n\omega_0)$

$x_p = \sum x_n e^{jn\omega_0 t}$

$\sum 2\pi x_n \delta(\omega - n\omega_0)$

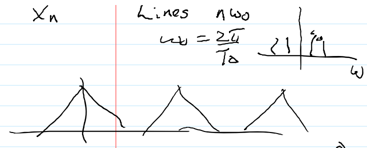
$x_n \qquad$ Lines $n\omega_0$

$\omega_0 = \frac{2\pi}{T_0}$



x_n Lines $n\omega_0$

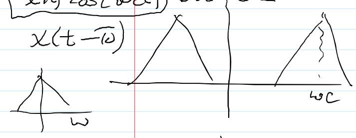
$\omega_0 = \frac{2\pi}{T_0}$



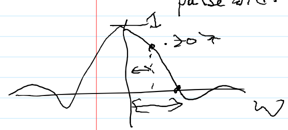
$$x_n = \frac{1}{T_0}\,\mathrm{sinc}^2\left(\frac{n\omega_0 \tau}{T_0}\right)$$

$\boxed{x(t)\cos(\omega_c t)}$ DSB-SC

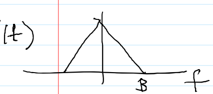
$x(t-t_0)$



$B \propto \frac{1}{\text{pulse width}}$



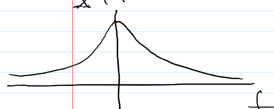
$X(t)$

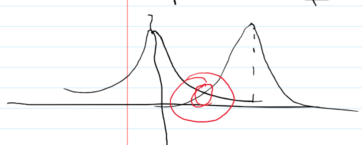


$f_s > 2B$

output $g(t)$ $G(f) \rightarrow x(t)$ $X(f)$

$X(f)$





Anti aliasing filter (LPF $B = f_s/2$)

$x(t)$ — $H(f)$ — Sample —

f_s

$x(t)$ f_s $T_s = 1/f_s$

$x[n] = x[nT_s]$

$\cos(2\pi f_1 t)$ $\cos(2\pi f_1 \frac{n}{f_s})$

$\Omega_1 = \frac{2\pi f_1}{f_s}$

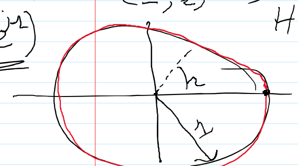
$$\frac{z^2 + \frac{1}{2}z - 1}{z^2}$$

$1 + \frac{1}{2}z^{-1} - z^{-2}$

$x[n] + \frac{1}{2}x[n-1] - x[n-2]$

$h[n] = \{\underline{1}, \frac{1}{2}, -1\}$

$H(e^{j\Omega})$ $H(z)$



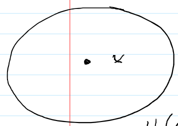
$y[n] = x[n] - a y[n-1]$

$H(e^{j0}) = 1$

$Y(z) = X(z) - a z^{-1} Y(z)$

$Y(z) + a z^{-1} Y(z) = X(z)$

$H = \frac{Y}{X} = \frac{1}{1 + a z^{-1}} = \frac{cz}{z+a}$



$H(e^{j0}) = 1$

$\frac{c e^{j0}}{e^{j0} + a} = \frac{c}{1+a} = 1$

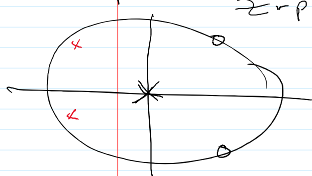
solve for given a
find c

$\frac{cz}{z+a}$

$y[n] = c x[n] - a y[n-1]$

$e^{j\pi/2} = 1$

z-plane



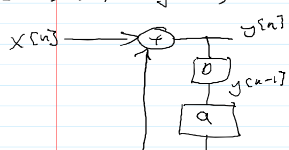
$y[n] = |A| \, |H(e^{j\omega_1})|$
$\cos(\omega_1 + \phi + \angle H(e^{j\omega_1}))$

$(z - \underline{b e^{j\omega_1}})(z - \underline{b e^{-j\omega_1}})$

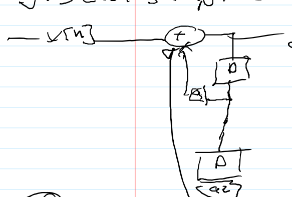
$\cdot(\quad)(\quad)$

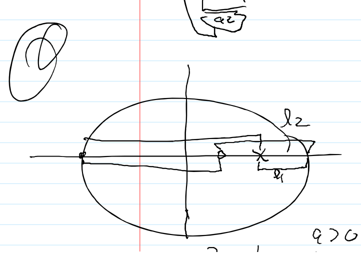
$\frac{z}{z-a}$

$y[n] = x[n] + a y[n-1]$

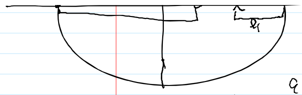


$y[n] = x[n] + a_1 y[n-1] + a_2 y[n-2]$





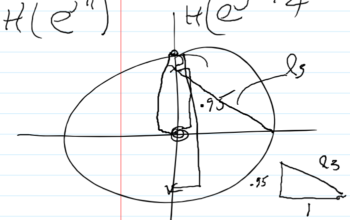
$a > 0$



$$\frac{z-b}{z-a} \qquad a>0,\ b>0$$

$$H(e^{j0}) = \ell_2/\ell_1$$

$H(e^{j\pi})$ $\quad$ $H(e^{j\pi/2})$



$$H(z) = \frac{z^2}{(z+\underset{.95}{1}e^{j\pi/2})(z+\underset{.95}{e}^{-j\pi/2})}$$

$$H(e^{j\pi/2}) = \frac{(1)(1)}{(.05)(1.95)}$$

$\cos(2\pi 1000 t)$

$$\frac{2\pi 1000}{f_s} = \frac{\pi}{2}$$

$$f_s = 4000$$

$$x(t) \longrightarrow \boxed{\underset{B}{\text{ILPF}}} \longrightarrow$$

$$u(t)\, e^{-\frac{t}{.25}} \longleftrightarrow \frac{1}{4+j\omega}$$

$a = \frac{1}{.25} = 4$

$$\frac{1}{2\pi}\int_{-\infty}^{\infty}\left|\frac{1}{4+j\omega}\right|^2 d\omega = \mathcal{E}_T$$

$$\mathcal{E}_B \quad \frac{1}{2\pi}\int_{-B}^{B}\left|\frac{1}{4+j\omega}\right|^2 d\omega = \mathcal{E}_B$$

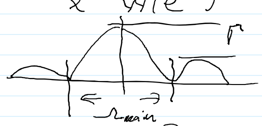
$$\Psi_y = \Psi_y \underline{|H(\omega)|^2}$$

April 28, 2026

Windowing

DFT$\{x[n]\}$ $\qquad$ $w[n]x[n]$

$x \quad W(e^{j\Omega})$

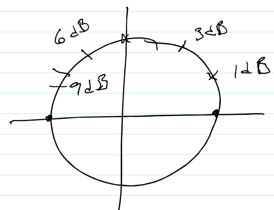


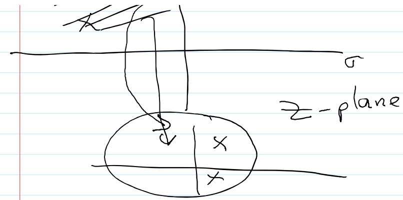
$$H(z) = \frac{z}{z-.5}$$

$$\underline{h[n]} = (0.5)^n u[n] \quad \forall n$$

$$y[n] = x[n] + 0.5y[n-1]$$

1 add
1 multiply





April 30, 2026

- Deconv $X = \frac{Y}{H}$ $N_y = N_x + N_h - 1$

$N_y \gg N_h$ $N_{FFT} \geq N_y$

Example: $N_y = 512$, $N_h = 20$

Pad h to 512

FIR

+ Linear phase FIR making $h[n]$ symmetric about its center

+ Solving set of Linear equations $H(e^{j\omega_i}) = V_i$, $i = 1 \cdots n$

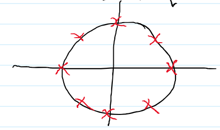
$h[n] = \{ b_0 \cdots b_n \}$

+ Target $\underline{H(z)}$ sample $H(e^{j\omega})\Big|_{\omega_k = \frac{2\pi k}{N_0}}$

$H[k]$

$DFT^{-1}[H[k]] = h[n]$

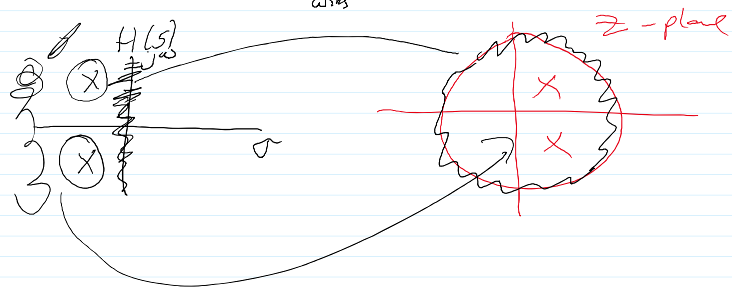
$0, \cdots N_0 - 1$



- $H(s)$ $\mathcal{L}^{-1}[H(s)] = h(t)$

$h_1[n] = T_s h(nT_s)$ = closed form

$\mathcal{Z}[h_1[n]] \rightarrow H(z) \rightarrow \frac{z - \cdots}{z - \cdots}$



May 5, 2026

- IIR Filter Design

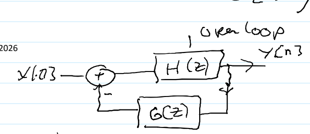
+ Impulse Invariance $H(s) \rightarrow h(t)$

$T_s h(nT_s) = h_D[n]$ $\mathcal{Z}[h_D[n]] = H(z)$

differ Eq

+ $H(s)$ $s = \frac{2}{T}\left(\frac{z-1}{z+1}\right) \rightarrow H(z)$

May 7, 2026



$Q(z) = \frac{Y(z)}{X(z)} = \frac{H(z)}{1 + H(z)G(z)}$ closed loop

$H(z)$ plant parameters not controll

$G(z)$