EECS 861
Homework #1

1. Plot
   a. $x_1(t) = 5\cos(2\pi 2000t)$,
   b. $x_2(t) = 5\cos(2\pi 2000(t\text{-}62.5\mu s))$
   c. $x_3(t) = 5\cos(2\pi 2000t - \frac{\pi}{4})$
   d. Compare these three signals and explain their similarities and differences.
   e. What is the energy and power in $x_1(t)$.

2. Plot
   a. $x(t) = \text{rect}(\frac{t-4}{4}) + \text{rect}(\frac{t+4}{4})$
   b. What is the energy and power in $x(t)$.

3. For

   $x_1(t) = \text{sinc}(1000t)$,
   $x_2(t) = \text{sinc}(2000\,t)$
   $x_3(t) = \text{sinc}(3000\,t)$
   (here $\text{sinc}(x)=\frac{\sin(\pi x)}{\pi x}$)

   a. Plot x_1(t), x_2(t), and x_3(t)
   b. What is the pulse width of each signal, defining pulse width as the time between the first zero crossings.
   b. Rank order the signals from lowest bandwidth to highest bandwidth.

4. A bit is transmitted as
$x(t) = A\cos(2\pi f_c t)$ if bit = "1" for T_b
or
$x(t) = -A\cos(2\pi f_c t) = -A\cos(2\pi f_c t - \pi)$ if bit = "0" for T_b
In another form $x(t) = \pm A\cos(2\pi f_c\, t)\,\text{rect}\left(\frac{t-\frac{T_b}{2}}{T_b}\right)$
For A=0.02 and T_b=10ms, f_c=100kHz
   a. Find the energy in x(t), that is the energy/bit = E_b.
   b. What is the bit rate in b/s?

5. For f_1=4000 and f_2=4150
   a. Find $\int_0^{1/50} \cos(2\pi f_1\, t)\cos(2\pi f_2\, t)\,dt$
   b. What property describes the relationship between $\cos(2\pi f_1 t)$ and $\cos(2\pi f_2 t)$.

6. For $x(t) = 2+6\cos(100\pi t) + 4\sin(300\pi t)$
   a. What is the fundamental frequency in Hz?
   b. Find the complex Fourier series for x(t).
   [Hint: no integration is required for this problem, convert to complex exponential form and note $\sin(\alpha)=\cos(\alpha-\pi/2)$ and $-\cos(\alpha)=\cos(\alpha-\pi)$]
   c. Plot the double-sided phase and amplitude spectrum for x(t).
   d. What is the bandwidth of x(t)?

7. A linear time-invariant system with input signal x(t) produces an output signal y(t) =2x(t-1ms), find the system transfer function and impulse response.

8. The transfer function for the voltage across the capacitor in an RC filter is

$$H(f) = \frac{1}{1+j2\pi RCf}$$

Set R=100Ω and C= 10pf

a. Plot 20 Log_{10} | $H(f_1)$ $|^2$)= 20 Log_{10}(|H(f)|)

b. Plot Phase of H(f) (or Angle H(f))

c. Given an input signal $x(t) = \cos(2\pi f_1 t) + 2\cos(2\pi 2 f_1 t)$ with f_1=160MHz, find the system output y(t).

9. A filter has an impulse response of h(t) = 1000sinc(1000t); here sinc(x)=$\frac{\sin(\pi x)}{\pi x}$.

a. Is this an ILPF? Yes or NO

b. What the filter bandwidth?

c. With an input signal x(t)=2δ(t- τ) + δ(t- 2τ) where τ=1ms input to this filter, plot the output signal y(t) in the time domain.

10. Consider a linear time invariant system with a impulse response of h(t), and input signal x(t) given below. The input signal x(t) given below produces and output of y(t). Here x(t)= rect(t+.5)+u(t)e^{-t} and h(t)=rect(t).

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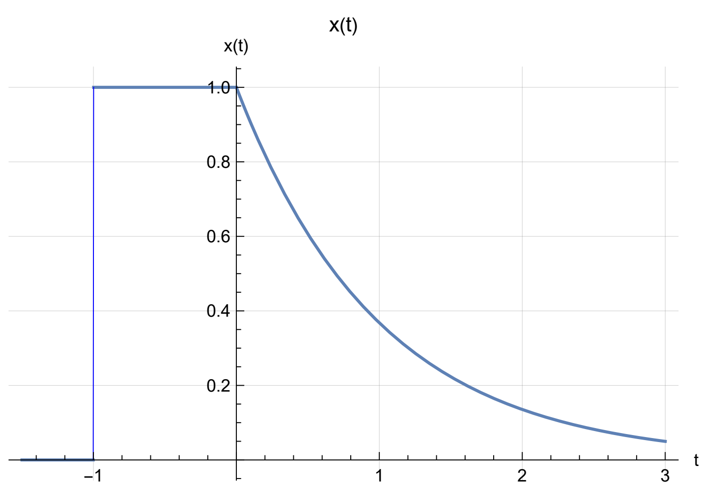

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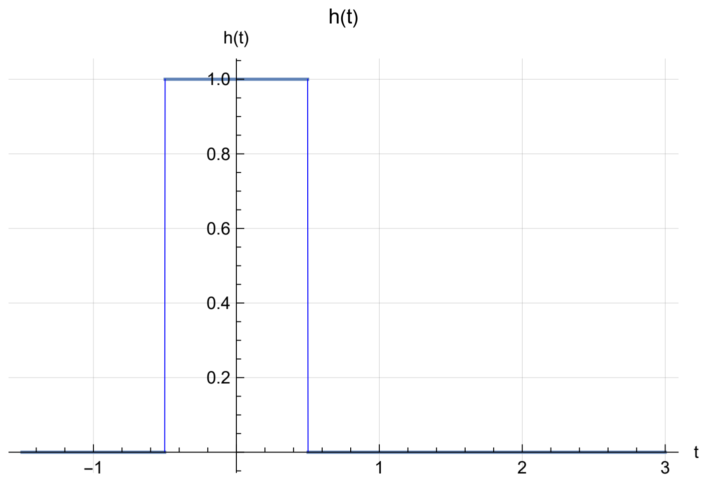
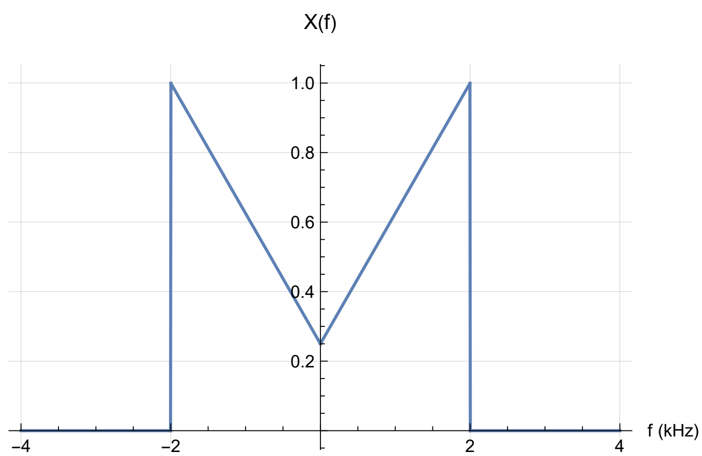


a. What is the range of time such that y(t) =0

b. What is y(-0.5).

11. A signal x(t) has a Fourier Transform given as



a. The signal x(t) is sampled at a rate of f_s =5000 samples/sec to create $x_s(t)$. Sketch the spectrum of the sampled signal $x_s(t)$.

b. The signal x(t) is sampled at a rate of 5000 samples/sec. Describe, e.g., draw a block diagram, a system to recover x(t) from $x_s(t)$.

c. What is the minium value of f_s where x(t) can be recovered from $x_s(t)$?

12. Let $s(t) = x(t)\cos(2\pi f_0 t)+z(t)\sin(2\pi f_0 t)$ with f_0=100khz and $X(f) = \text{rect}\left(\frac{f}{20000}\right)$ and $Z(f)= \Lambda\left(\frac{f}{10000}\right)$ as shown below.

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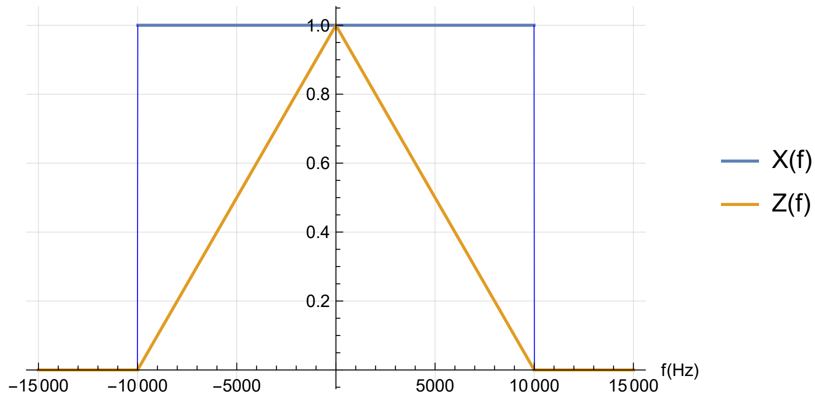

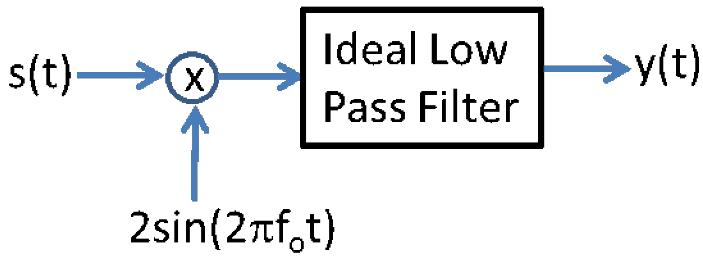

Find the output y(t) in terms of x(t) and z(t) of the system above. The bandwidth of the ILPF is 11 kHz. [Hint: use the trigonometry identities for $\sin^2(\theta)$ and $\cos^2(\theta)$]