EECS 861

Homework #10

1. A discrete time R.P. X[n] has an autocorrelation function of

$$R_{xx}[k] = \begin{array}{l} 1 \text{ for } k=0 \\ 0 \text{ for } k \neq 0 \end{array}$$

For Y[n]=.333(X[n]+ X[n-1]+ X[n-2]) find

a. E[Y[n]] and Var[Y[n]]

b. Find R_{YY}(k)

2. N(t) is bandpass white Gaussian noise with $\eta/2 = 10^{-10}$, centered at 10 Ghz with a bandwidth $B_N = 500$ Mhz.

a. Find the noise power.

b. Find P(|N(t)|>0.6).

3. Let Y(t)= $\frac{1}{T}\int_{t-T}^{t} X(t)\,d t$, this system is an integrator.

a. Find the transfer function, H(f), for this system, i.e., Y(f)=X(f)H(f), hint find the impulse response first.

b. Let X(t) be white Gaussian noise with $S_X(f) = \eta/2$, find E[Y(t)].

c. Let X(t) be white Gaussian noise with $S_X(f) = \eta/2$, find Var[Y(t)].

4. The transfer function of a linear time invariant filter is

$$H(f)=\frac{200}{1+\frac{j2\pi f}{f_0}} \text{ with } f_0 = 7500$$

The input to this filter is zero-mean white Gaussian Noise with S_x(f)= $5 * 10^{-10}$

a. Find the PSD of the filter output Y(t), S_Y(f).

b. Find E[Y(t)]

c. Find Var[Y(t)]

d. Find the noise power at the filter output Y(t).

e. What is P(Y(t)>0.5)

f. P(Y(t)>0.6|Y(t-100μs)=0). Hint: first find $R_{yy}(\tau)$.

5. Using Y(t) from problem 4, let

$$Z = \frac{1}{T}\int_0^T Y(\tau)\,d\tau \quad \text{with T = 53.35 ms}$$

Find P(Z>0.02), apply appropriate approximations.

6. A signal X(t) is corrupted by statistically independent additive white Gaussian noise, N(t) with a bandwidth B_N and is input to a linear time-invariant filter H(f).

$$S_N(f) = \begin{cases} \frac{10^{-11}}{2} & \text{for } |f| < 100 \text{ kHz} \\ 0 & \end{cases}$$

$X(t) = A\cos(2\pi f_i t)$ where $A = 0.005$ and f_c is a constant

$$H(f) = \frac{1}{1+\frac{j2\pi f}{f_0}} \text{ with } f_0 = 7500$$

a. Find the input signal-to-noise ratio (in dB) for f_i=5kHz

b. Find the output signal-to-noise ratio (in dB)for for f_i=5kHz

c. Find the output signal-to-noise ratio (in dB)for for f_i=10kHz

d. Find the output signal-to-noise ratio (in dB) for for f_i=20kHz

e. Why does the output signal-to-noise ratio change as f_c changes?

7. A discrete time process is defined as $Y[n] = a_1 Y[n-1] + N[n]$. Where N[n] is zero-mean white Gaussian Noise with a variance of σ^2=0.5. Let a_1=0.8.

a. Is this an autoregressive or moving average process?

b. Find E[Y[n]], $E\left[Y^2[n]\right]$, Var[Y[n]], and $R_{YY}[k]$.

8. One way to define a second order moving average process is defined by
$X[n] = e[n] + b_1\, e[n-1] + b_2\, e[n-2]$
where e[n] is zero-mean white Gaussian Noise with a variance of σ^2.

a. Find the covariance matrix for X[n], X[n-1], and X[n-2]. For $b_1 = 0.9$ & b_2=0.9 and σ^2=0.15.

b. Plot three member functions given the data in sheet 2 in the file specified below. Find the autocorrelation function for the data given in this file. Would the second order moving average process model given in the problem be a good representation for this data?

http://www.ittc.ku.edu/~frost/EECS_861/EECS_861_HW_Fall_2024/HW-5-Problem-2.xls

9. Use the data in given file.

a. Find and plot the autocorrelation function of this data.

http://www.ittc.ku.edu/~frost/EECS_861/EECS_861_HW_Fall_2024/Homework-10-Problem-9.xls

b. Could this data be a sample function from an MA(2) process, justify your answer.

c. Assuming this data is a sample function from an AR(1) process suggest a value for α_1. Hint experiment with the AR(1) example in **http://www.ittc.ku.edu/~frost/EECS_861/Mathematica_files/ARMA_study-V4.cdf**