EECS 861

Homework #12

1. A decision is based on 2 samples, Y_1 and Y_2. Y is a multivariate Gaussian random vector

$$Y = \begin{pmatrix} Y_1 \\ Y_2 \end{pmatrix} \quad E[Y \mid H_0] = \begin{pmatrix} -1 \\ -1 \end{pmatrix} \text{ and } E[Y \mid H_1] = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \text{ with } \Sigma = \begin{pmatrix} 0.984 & 0.48 \\ 00.48 & 0.984 \end{pmatrix}$$

a. Design the optimum detector. Assume $P(H_0)= P(H_1)=1/2$.

b. Find P_e.

c. Repeat parts a. and b. with

$$E[Y \mid H_0] = \begin{pmatrix} -1 \\ -1 \end{pmatrix} \text{ and } E[Y \mid H_1] = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \text{ with } \Sigma = \begin{pmatrix} 0.984 & -0.48 \\ -0.48 & 0.984 \end{pmatrix}$$

d. Explain why the results, i.e., P_e, from b. and c. are different.

e. The 500 bits in the file below (sheet labeled Volts maps 0->-1 and 1->+1) are sampled at two sample/bit, the resulting transmitted signal is corrupted by additive Gaussian correlated noise with $\Sigma = \begin{pmatrix} 0.984 & 0.48 \\ 0.48 & 0.984 \end{pmatrix}$, apply optimum detector found in part a. to the received signal given in the file below and estimate the P_e of your detector, then compare to P_e found in part b.

Here are the transmitted bits

http://www.ittc.ku.edu/~frost/EECS_861/EECS_861_HW_Fall_2024/Homework-12-Problem-1-bits.xls

The received signal is given in

http://www.ittc.ku.edu/~frost/EECS_861/EECS_861_HW_Fall_2024/Homework-12-Problem-1.xls

2. Let $s_1[k]$ = -2, -2, -2 for k =0, 1, 2 and $s_0[k]$ = - $s_1[k]$, i.e., $s_0[k]$ = 2, 2, 2 for k =0, 1, 2

Assume

$P(s_1) = 0.5 = P(s_0)$

Y[k] = S[k] + N[k] for k =0…2 where

S[k] & N[k] are statistically independent

N[k] is white Gaussian noise with a zero mean and variance= 2, i.e., $\sigma_N = \sqrt{2}$.

a. Find the MAP decision algorithm.

b. Find the probability of error.

c. Apply the MAP decision algorithm for the follow observations

y[k] = {-0.4, 0.1, -0.1}, is the receiver output is $s_1[k]$ or $s_2[k]$?

d. Repeat a)-b) for $s_1[k]$ = 0.5, −3.4, 0.5

e. Why is P_e from part b. and part d. the same.

3. A received signal X(t) contains pulses of amplitude +5.5 with width 10 μs plus bandlimited white Gaussian noise N(t). The noise PSD is

$$S_N(f) = \begin{cases} 10^{-4} & |f| < 1\text{MHz} \\ 0 & elsewhere \end{cases}$$

X(t) is sampled at a rate of 10 Msamples/sec. Samples are collected in time synchronization with the pulses. The received signal is N(t) if no pulse or X(t)+N(t) for 10 μs is a pulse is present.

a. Design a MAP pulse detector assuming Prob(pulse)=0.5

b. For your pulse detector and given the parameters above calculate the probability of detection and false alarm.

c. Design a pulse detector using a Neyman-Pearson (N-P) rule with a $P_{\mathrm{fa}} = 0.01$ and find probability of detection.

d. Apply the MAP detector with Prob(pulse)=0.5 to the data set given below. How many pulses are in this record. This file contains a record of collected samples.

http://www.ittc.ku.edu/~frost/EECS_861/EECS_861_HW_Fall_2024/HW-12_Problem-3_data.xls

4. Let $f_X(x;\theta) = \frac{1}{\sqrt{5\pi}} e^{\frac{-(x-\theta)^2}{5}}$. Given $x_1 \ldots.\ x_N$ be N statistically independent samples from $f_X(x)$

Is $\overline{x} = \frac{1}{N}\sum_{i=1}^{N} x_i$ an biased estimator for θ; yes or no and justify.