EECS 861

Homework #13

1. We want to estimate a received signal A from K observations of Y where Y is modeled as Y=A+N. Here the sample mean is

$\overline{y} = \frac{1}{K}\sum_{i=1}^{K} y_i = 15.$

N is Gaussian with E[N]=0 and Var[N] = σ_N^2

X is Gaussian with E[A]=5 and Var[A] = σ_A^2

N and X are statistically independent. For the following 3 cases:

Case 1: $\sigma_N^2 = 10$ $\sigma_A^2 = 2$ $K = 5$

Case 2: $\sigma_N^2 = 200$ $\sigma_A^2 = 2$ $K = 5$

Case 3: $\sigma_N^2 = 0.1$ $\sigma_A^2 = 2$ $K = 5$

Case 4: $\sigma_N^2 = 10$ $\sigma_A^2 = 2$ $K = 450$

Find

a. The MAP estimator for A.

b. The Mean Square (MS) estimator for A.

c. The Maximum Likelihood (ML) estimator for A.

Hint: check your results with **MAP Estimator with Gaussian Prior and Gaussian Noise**

2. An unconstrained Weiner filter is used to estimate S(t) from Y(t) =S(t)+N(t), where N(t) and S(t) are statistically independent and

$S_S(f) = \frac{40}{1+(20\,\pi f)^2}$

and

$S_N(f) = 10$

a. Find the unconstrained Weiner filter H[f]

b. Find the impulse response of the unconstrained Weiner filter H[f]. Is the corresponding linear system causal?

3. Let the desired signal, S[k], be characterized by a moving average process given by

S[k]= X[k] + 0.9X[k-1] +0.5 X[k-2]

where X[k] is a white Gaussian random process with zero mean and variance = 0.195

The observed signal is Y[k]=S[k] + N[k] where N[k] is Gaussian with R_{NN}[n]=0.4 for n=0 and R_{NN}[n]=0 elsewhere.

a. Find R_{SS}[n].

b. Find the optimum realizable Wiener filter h[k] where

$\hat{S}[k] = \sum_{k=0}^{\infty} h[n] \times Y[k-n]$

c. Apply the optimum realizable Wiener filter h[k] to this received signal

http://www.ittc.ku.edu/~frost/EECS_861/EECS_861_HW_Fall_2024/Received_signal-2024.xls

d. Calculate the resulting mean square error given the desired signal

http://www.ittc.ku.edu/~frost/EECS_861/EECS_861_HW_Fall_2024/Desired_signal-2024.xls

e. Repeat parts c and d using h[k] = 1/3, 1/3, 1/3, that is a three sample moving average and compare the resulting mean square errors.

(Note: MSE= 1/N $\sum_{k=0}^{N}\left(S[k]-\hat{S}[k]\right)^2$. In the provided files the length of the received and desired signals is 512. The desired signal, S[k], and the filtered signal, $\hat{S}[k]$, needs to be aligned to calculate the mean square error, i.e., there is a delay going through h[k]. Note the first 3 points in Y[k] are used to calculate the first point in $\hat{S}[k]$, also $\hat{S}[k]$ is longer than S[k], in this case the length of $\hat{S}[k]$ is 514)