EECS 861

Homework #3

1. Show that Var[X]=E[$X^2$]-(E[X])$^2$

2. 1. X is a discrete random variable with
   P(X=-2) = a, P(X= -1) = 0.3, P(X=1) = 0.3, P(X= 2) = 0.2
   a. Find "a"
   Given "a" find:
   b. Find P(X> 3)
   c. Find P(X=0)
   d. Find E[X]
   e. Find E[$X^2$]
   f. Find Var[X]

3. Using 200 samples {$x_1$.....$x_{200}$} of a discrete random variable X is given in http://www.ittc.ku.edu/~frost/EECS_861/EECS_861_HW_Fall_2024/data_HW3_Prob_2.csv
   a. Given this data what is an estimate for $p_k$ = P(X=k) for k=-2, -1, 1, 2?
   b. Find the sample mean of X using $\overline{X} = \frac{1}{200}\sum_{i=1}^{200} x_i$
   c. Estimate the mean of X using $\widehat{X} = \sum_{k=-2}^{2} kp_k$
   d. Find the sample mean square of X using $\overline{X}^2 = \frac{1}{200}\sum_{i=1}^{200} x_i^2$
   e. Estimate the mean square of X using $\widehat{X}^2 = \sum_{k=-2}^{2} k^2 p_k$
   f. Is the pmf given in problem 1 a "good" probabilistic model for this data?

4. X is a random variable with $f_X(x) = 0.1\delta(x) + 0.9u(x)e^{-x}$ where u(x)= unit step function
   a. Sketch $f_x(x)$.
   b. Verify that the total probability is 1.
   c. What is P(X= 0)?
   d. What is P(X= 2)?
   e. What is P(-2<X< 1)?
   f. Find E[X]
   g. Var[X]

5. X is a Gaussian random variable X with $\mu_X = 0$ and $\sigma_X$=0.577
   a. What is P(-0.5<X<.5)?
   b. Plot $P(X < x_i)$ for $x_i$= -4.0,-1.0, -0.4, -0.3, -0.2, -0.1, 0.0, 0.1, 0.2, 0.3, 0.4, 1.0, 4.0
   c. Confirm your answers using
   https://www.mathportal.org/calculators/statistics-calculator/normal-distribution-calculator.php

d. Assuming X is a Uniform random variable [-1, 1] repeat part b.

e. Is a Uniform random variable [-1, 1] “good” probabilistic model for the data given in Homework 2-Problem 2?

6. X and Y have the following joint distribution function

| □ | X = -2 | X = 0 | X = 2 |
|---|---|---|---|
| Y = -2 | 1/8 | 1/8 | 0 |
| Y = 0 | 0 | 0 | 1/4 |
| Y = 2 | 1/8 | 1/8 | 1/4 |

a. Find P(X=0).

b. Find P(Y=2).

c. Find P(X=0|Y=2).

d. Find $\rho_{XY}$.

e. Are X and Y SI random variables?

7. Show (from Chapter 2: Problem 2.18)

a. $E\{a + bX\} = a + bE\{X\}$

b. $E\{aX + bY\} = aE\{X\} + bE\{Y\}$

c. Variance of $aX + bY = a^2 \operatorname{Var}[X] + b^2 \operatorname{Var}[Y] + 2ab \operatorname{Covar}[X, Y]$