EECS 861

Homework #5

1. Define a random process X(t) based on the outcome k of tossing a fair 6 sided die as:

$$X(t) = \begin{cases} -2 & k = 1 \\ -1 & k = 2 \\ 1 & k = 3 \\ 2 & k = 4 \\ t & k = 5 \\ -t & k = 6 \end{cases}$$

a. Find the joint probability mass function of X(0) and X(2).

b. Find the marginal probability mass functions of X(0) and X(2).

c. Find E{X(0)}, E{X(2)}, and E{X(0)X(2)}.

2. For this problem use the data in this file.
http://www.ittc.ku.edu/~frost/EECS_861/EECS_861_HW_Fall_2024/HW-5-Problem-2.xls

Each Sheet contains data from one discrete time random process,

Case 1 X[n],

Case 2 Y[n],

Case 3 Z[n].

Each row is a sample function of that discrete time random process.

a. For Sheet 1 create 3 plots, one plot per row for the first 3 rows.

b. For Sheet 1 create 3 plots, one plot per column for the first 3 columns.

c. For Sheet 1 calculate the average and variance of all the values in each row, plot the row averages.

d. For Sheet 1 calculate the average and variance of all the values in each column, plot the column averages.

e. For Sheet 1 consider column 2 and 3 as a pair of random samples; estimate the correlation coefficient between these samples.

f. For Sheet 1 repeat part e. for column 2 and 4.

g. For Sheet 1 repeat part e. for column 2 and 5.

h. Repeat e.-g. for Sheet 2

i. Repeat e.-g. for Sheet 3

h. Discuss the differences in the estimate the correlation coefficient for the three discrete time random processes.

3. $X(t) = A\cos(2\pi t+\varphi)$

For $\varphi=0$ and $P(A=-1)= P(A=1)= P(A=-2)= P(A=2)=0.25$.

a. Sketch all possible sample functions of X(t)

b. What is $P(X(1)=0)$?

c. What is P(X(0.25=0)?

d. What is the PMF for the RV X(0.5)?

e. Find E[X(t)].

For A=1 and P(φ=+π/4)= P(φ=- π/4)=0.5

f. Sketch 2 sample functions of X(t)

g. Find E[X(t)].

4. X[n] is a discrete random sequence. Sample function X_1[n] = 2 for all n and sample function X_2[n] = -2 for all n. The $P(X_1[n])$ = 0.5 and $P(X_2[n])$ = 0.5.

a. How many member (sample) functions are in the random process?

b. Plot all the sample functions of X[n].

c. What is the pmf for X[n]?

d. Find E[X[n]].

e. What is the joint pmf for X[n] and X[n+1]?

f. Find the autocovariance function of X[n], $R_{XX}[k]$.

5. X[n] is a discrete random sequence.

$$X[n] = \sum_{i=1}^{n} J_i \text{ with } P(J_i = 1) = P(J_i = -1) = \frac{1}{2} \text{ and } J_i\text{'s are S.I and } X[0]=0$$

a. Sketch two sample functions of X[n] for n=1….10

b. Is X[n] and random walk? Yes or No

c. Find P(X[3]=1)

d. Find E[X[3]]

e. Find P(X[6]=0| X[3]=1)

6. A random process is described by X(t)= Yt+10. Y ~N(0,1).

a. Find E[X(t)]

b. Find Var[X(t)]

c. Find P(X(1)>11)

7. A random process X(t) has 4 member functions that occur with equal probability:

$X_1(t) = t^2$

$X_2(t) = \sin(2\pi t)$

$X_3(t) = -t^2$

$X_4(t) = -\sin(2\pi t)$

a. Plot the sample functions.

b. Find PMF for X(0)

c. Find P[X(1)=1]

d. Find E[X(t)]

e. Find Var[X(1)]

f. For t>1 find P(X(t)>1)