

EECS 861
Homework #6

1. $Z(t) = Xt + Y$

Where X and Y are jointly Gaussian random variables with $E[X] = \mu_X = 0$, $E[Y] = \mu_Y = 0$, $\sigma_X = 4$, $\sigma_Y = 4$, and $\rho_{XY} = 0.75$.

- a. Find $E[Z(t)]$
- b. Find $\text{Var}[Z(t)]$
- c. What is the pdf of $Z(t)$, name of pdf and its parameters?
- d. Find $P(Z(1) > 17.4)$
- e. Find autocorrelation function, $R_{ZZ}(t_1, t_2)$
- f. Find autocovariance function, $C_{ZZ}(t_1, t_2)$
- g. Find the joint pdf of $Z(0)$ and $Z(1)$.
- h. Find $P(Z(1) > 17.4 | Z(0) = 15)$

2. Given $Z(t)$ in problem 1.

- a. Is $Z(t)$ strict sense stationary?
- b. Is $Z(t)$ wide sense stationary?

3. $X(t)$ and $Y(t)$ are SI WSS zero mean random process with $R_{XX}(\tau)$ and $R_{YY}(\tau)$. Find the autocorrelation function of $Z(t)$:

- a. $Z(t) = 1 + 2X(t) + 3Y(t)$
- b. $Z(t) = X(t)Y(t)$

c. Let $X(t)$ and $Y(t)$ be Gaussian SI WSS random processes with zero mean and unit variance, for $Z(t) = 2X(t) + 3Y(t)$ find $P(Z(1) < 4.6)$.

4. $Z(t) = X(t) + 0.5X(t-2)$

Where $E[X(t)] = \mu_X = 0$ and the autocorrelation function of $X(t)$ is $R_{XX}(t_1, t_2)$.

- a. Find $E[Z(t)]$
- b. Find $R_{ZZ}(t_1, t_2)$.
- c. Repeat b. assuming that $X(t)$ is a wide sense stationary random process.

5. For this problem use the data in this file.

http://www.ittc.ku.edu/~frost/EECS_861/EECS_861_HW_Fall_2023/HW-5-Problem-2.xls

Each Sheet contains data from one discrete time random process,

- Case 1 $X[n]$,
- Case 2 $Y[n]$,
- Case 3 $Z[n]$.

Each row is a sample function of that discrete time random process.

a. Consider column i and $i+1$ as a pair of random samples (e.g., $X[i]$, $X[i+1]$); calculate and plot the correlation coefficient between these samples for all i . That is, column 1 & 2 is random sample one (e.g., $X[1]$, $X[2]$), column 2 & 3 is random sample 2 (e.g., $X[2]$, $X[3]$), column 3 & 4 (e.g., $X[3]$, $X[4]$), is random sample 3.

- b. Repeat part a) for column i and $i+2$ (e.g., $X[3]$, $X[5]$).

- c. Repeat part a) for column i and $i+3$ (e.g., $X[3]$, $X[6]$).
 - d. Repeat a. - c. for $Y[n]$ and $Z[n]$
 - e. Comment on the stationarity of these random processes.
6. Find the autocorrelation function of the discrete time sequence $X[n]=A\cos(\omega n+\theta)$, where A is normally distributed with mean 0 and variance σ^2 , and the phase θ is uniformly distributed between $-\pi$ and π .