EECS 861

Homework #8

1. X(t) is a wide sense stationary zero mean, Gaussian random processes with a power spectral density of $S_x(f) = \frac{2}{1+(0.2*\pi*f)^2}$. [Hint: see Homework 7-problem 4]

a. Find E[X(t)], Var[X(t)], E[X(t+0.1)], and Var[X(t+0.1)]

b. What is the distribution of X(t), i.e., name of pdf and its parameters?

c. Find P(X(t)>4)

d. What is the covariance matrix for X(t) and X(t+0.1)?

e. What is the joint distribution of X(t) and X(t+0.1), i.e., name of pdf and its parameters?

f. What is the correlation coefficient between X(t) and X(t+0.1)?

g. Find P(X(t+0.1)>4|X(t)=3.5)

h. Approximate P(X(t+20)>4|X(t)=3.5)

Hint: Check the result for part g. with **Study of the Conditional probability P(X(t+Tau)>L|X(t)=y) for Gaussian Random Process given Different Autocorrelation Functions**

2. For a random process with a PSD of $S_x(f) = \frac{1}{1000} * \text{sinc}^2\left(\frac{\pi*f}{1000}\right)$

a. Find the B_{eff}

b.Find the $B_{3\,\text{dB}}$

c. Find $B_{\text{first zero}}$ defined as the first frequency where S_X(f) =0

d. Compare the above definitions of bandwidth.

3. Given the random process from problem 2,

a. Find the correlation time τ_c

b. Compare the correlation time to $\frac{1}{2B_{3\,\text{dB}}}$ and $\frac{1}{2B_{\text{first zero}}}$

c. What is the correlation coefficient between X(t) and X(t+2ms)?

4. Determine whether the following functions can be the power spectral density for a WSS real valued random process (YES or NO).

a. $-\text{rect}\left(\frac{f}{10}\right)$

b. $e^{+\pi f^2}$

c. $10\,\Lambda(10f)$

d. $10e^{-(f+0.10)}$

e. $e^{-\pi f^2}$

f. $5\,\delta(f) + \sin(200\,\pi f)$

g. $\delta(f) + 4\,\delta(f+20\ 0) + 4\,\delta(f-200)$

5. The random process X(t) is WSS. For each of the autocorrelation functions below find and plot the corresponding power spectral density, S_x(f).

a) $R_{XX}(\tau) = 4\cos(2\pi 1000\tau)$

b) $R_{XX}(\tau) = \frac{2}{1+4\pi^2\tau^2}$

c) $R_{XX}(\tau) = 4\Lambda(\frac{\tau}{4})$

d) $R_{XX}(\tau) = 4e^{-\left|\frac{\tau}{4}\right|}$

e) $R_{XX}(\tau) = 4e^{-\pi(\frac{\tau}{4})^2}$

6. A power spectral density for a WSS random process X(t) is
$S_X(f) = 0.001\Lambda(\frac{f}{10\,\text{kHz}})$

a. Find $E[X^2(\text{t})]$.

b. Find the Average power.

c. Find the % power the band [0, 5 kHz]

7. Given $S_X(f) = \frac{2L}{1+4f^2L^2\pi^2}$

a. Find $R_{XX}(\tau)$.

b. Given $R_{XX}(\tau)$ from part a. and $L = 5.0$ and τ_1=1. and τ_2=2.0 and τ_3=3 find the covariance matrix for X(t), X(t-τ_1), X(t-τ_2), X(t-τ_3).

c. Given X(t)=0 and X(t-τ_1)=0, find the correlation coefficient between X(t-τ_2), X(t-τ_3).

To confirm your result use: Interactive Gaussian Process Visualization

Hint: see Conditional Bivariate distributions from a 4-Dimensional Mulitvariate Gaussian