EECS 861

Homework #9

1. A zero mean WSS random process, X(t) has the following autocorrelation function
$R_{XX}(\tau) = 2\, e^{-150|\tau|}$

a. Find E[X(t)]

b. Find the variance of X(t).

c. Find the effective bandwidth, $B_{\text{eff}}$ of X(t).

d. Find the correlation time $\tau_c$ in ms

Let

$$Y = \frac{1}{T}\int_0^T X(\eta)\, d\eta$$

e. For T = 5 ms find the variance of Y and the ratio of Var[X]/Var[Y].

f. For T = 2000 ms find the variance of Y and the ratio of Var[X]/Var[Y]; then compare the Var[Y] to the large $B_{\text{eff}}$T approximation of the of Var[Y].

g. For T = 2000 ms find P(Y> 0.1)

2. Let X(t) = A cos(2 $\pi f_c$t) where A is a random variable with E[A]=m and Var[$A$] = $\sigma^2$.

a. Find E[X(t)]

b. Find $\langle X(t)\rangle_T = \frac{1}{T}\int_{-T/2}^{T/2} X(t)\, dt$

c. Find $\lim_{T\to\infty} \langle X(t)\rangle_T$

d. Compare the results from part a and part b.

e. Repeat a. and d. with E[A]=0 and Var[$A$] = $\sigma^2$

3. Explain the concepts of strict sense stationarity and ergodicity.

4. Chapter 3: Problem 3.26

X(t) and Y(t) are two independent WSS random processes with the power spectral density functions shown below-Figure 3.34. Let Z(t) = X(t)Y(t). Sketch the Power Spectral Density of Z(t), and find $S_Z(0)$.

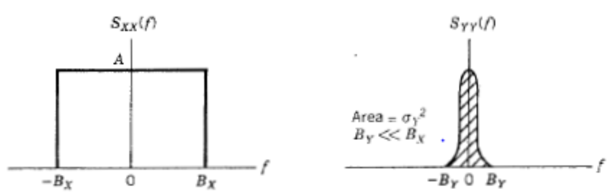


**Figure 3.34** Psd functions for Problem 3.26.

5. A zero mean Gaussian WSS random process, X(t) has the following PSD

$S_x(f) = \frac{2}{1000}\Lambda\left(\frac{f}{1000}\right)$

a. Plot $R_{XX}(\tau)$.

b Find E[X(t)]

c. Find Var[X(t)]

d. Are the random variables X(t) and X(t-2ms) uncorrelated (Yes or No); justify?

e. Are the random variables X(t) and X(t-2ms) statistically independent (Yes or No); justify?

f. What is P(X(t)>2| X(t-2ms)=3.29]?

g. What is the bivariate pdf for the random variables X(t) and X(t-0.5ms), Specify the pdf, mean vector and covariance matrix.

6. Given the random process given in problem 5, i.e., X(t) has the following PSD

$S_x(f) = \frac{2}{1000}\Lambda(\frac{f}{1000})$. The average of 20 samples are collected at a sample rate of 500 samples/sec the average is $Y = \frac{1}{20}\sum_{k=1}^{20} X(t - \mathrm{k}\Delta \mathrm{t})$ where $\Delta \mathrm{t} = 2$ ms

a. Find the variance of Y.

b. How long to you need to sample X(t) (that observe the signal X(t)) at a sample rate of 1000 samples/sec such that the varaince of the time average Var[Y]=0.01

7. Let $S_x(f) = \frac{2}{1000}\Lambda(\frac{f}{1000})$ and $S_Y(f) = \frac{2}{10}\mathrm{rect}(\frac{f}{10})$. Define Z(t)=X(t)Y(t).

a. Approximate $S_Z(0)$

b. Approximate the effective bandwidth of Z(t)