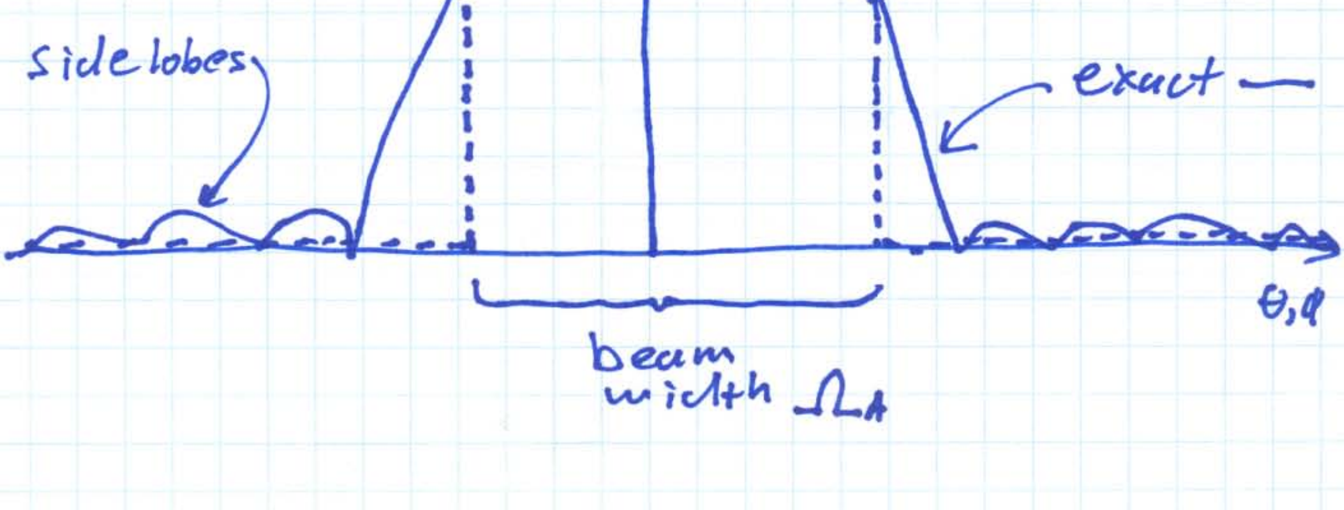
Beamwidth + Directivity

Consider an antenna with low sidelobes and a well-defined main beam. Its directivity pattern can be approximately written as:

$$\boxed{D(\theta,\phi) \approx \begin{cases} D_0 & \text{within the main beam} \\ 0 & \text{outside the main beam} \end{cases}}$$

e.g.,



Recall

$$\frac{1}{4\pi}\int_0^{2\pi}\int_0^{\pi} D(\theta,\phi)\sin\theta\, d\theta\, d\phi = 1$$

So using our approximation, we get

$$\frac{1}{4\pi}\iint_{\substack{\text{main}\\\text{beam}}} D_0 \sin\theta\, d\theta\, d\phi + \frac{1}{4\pi}\iint_{\substack{\text{side}\\\text{lobes}}} 0 \sin\theta\, d\theta\, d\phi = 1$$

So we find that

$$\iint_{\substack{\text{main}\\\text{beam}}} \sin\theta\, d\theta\, d\phi = \frac{4\pi}{D_0}$$

But the integral $\iint_{\substack{\text{main}\\\text{beam}}} \sin\theta\, d\theta\, d\phi$ has a physical meaning! It is equal to the beamwidth of the antenna in steradians!

$$\Omega_A = \iint_{\substack{\text{main}\\\text{beam}}} \sin\theta\, d\theta\, d\phi$$

Thus, we come to the conclusion that the beam width Ω_A of an antenna with directivity D_o is approximately:

$$\boxed{\Omega_A \approx \frac{4\pi}{D_o}}$$

Or rearranging, we find:

$$D_o = \frac{4\pi}{\Omega_A} \Rightarrow D_o \Omega_A = 4\pi$$

In other words, the directivity of an antenna is approximately the ratio of its beam width (in steradians) to the beam width of an <u>isotropic</u> radiator (i.e., 4π steradians).