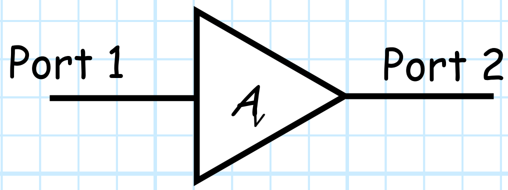
Amplifier Gain

Note that an amplifier is a **two-port** device.



As a result, we can describe an amplifier with a 2 x 2 **scattering matrix**:

$$\bar{\bar{\mathbf{S}}}(\omega) = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$

Q: What is the scattering matrix of an **ideal** amplifier??

A: Let's start with S_{11} and S_{22}.

To insure maximum power transfer, the input and output ports would ideally be matched:

$$S_{11} = S_{22} = 0$$

Now, let's look at scattering parameter S_{21}. We know that:

$$P_2^- = |S_{21}|^2 P_1^+$$

or, stated **another** way:

$$P_{out} = |S_{21}|^2 \, P_{in}$$

Therefore, we can **define** the amplifier **power gain** as:

$$G \doteq \frac{P_{out}}{P_{in}} = |S_{21}|^2$$

As the purpose of an amplifier is to boost the signal power, we can conclude that **ideally**:

$$|S_{21}| \gg 1$$

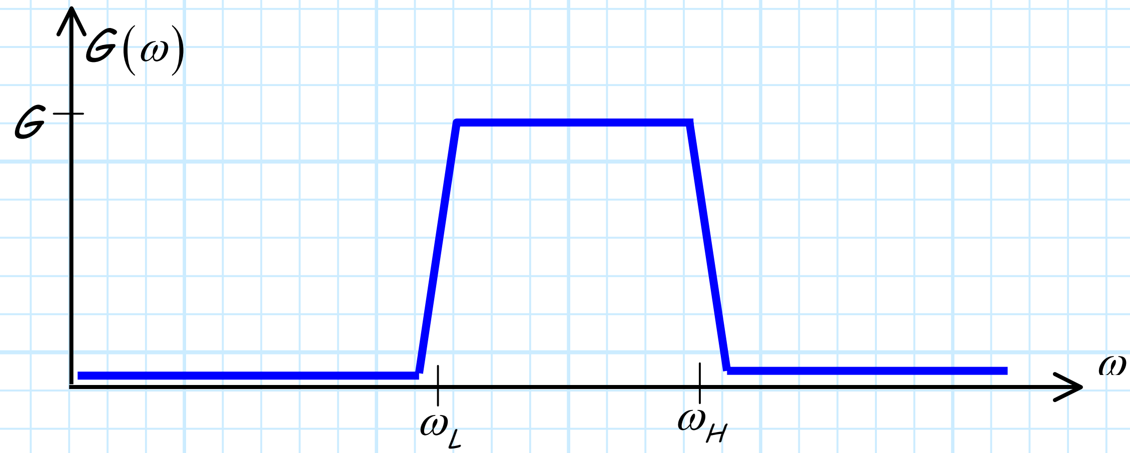
Clearly, an amplifier must be an **active** device!

As discussed earlier, the gain of an amplifier will change with signal frequency:

$$G(\omega) = |S_{21}(\omega)|^2$$

When radio engineers speak of amplifier **gain**, they almost always are speaking of this **power gain G**. However, they do not generally state it as a specific function of frequency!

Rather, amplifier gain is typically specified as a **numeric** value such as G =20 or G =13 dB. This value is a statement of the approximate amplifier gain **within** the amplifier **bandwidth**.

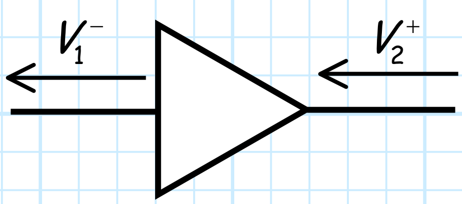


Thus, amplifier **gain** and **bandwidth** are the two most fundamental performance specifications of any microwave amplifier—together they (approximately) describe the amplifier transfer function!

Additionally, radio engineers almost always speak of amplifier gain in **decibels** (dB):

$$G(dB) = 10 \ log_{10} G$$

Finally, let's consider S_{12}. This scattering parameter relates the wave into port 2 (the output) to the wave out of port 1 (the input).



Q: *Are amplifiers **reciprocal** devices? In other words, is* $S_{12} = S_{21}$ *??*

A: No! An amplifier is strictly a **directional** device; there is a specific input, and a specific output—it does **not** work in reverse!

Ideally, $S_{12} = 0$. Any other value can just cause problems!

Typically though, S_{12} is small, but **not** zero. Generally speaking, radio engineers express S_{12} as a value called **reverse isolation**:

$$\text{reverse isolation} \doteq -10 \log_{10} \left|S_{12}\right|^2$$

Note when $S_{12} = 0$, reverse isolation will be **infinite**. Thus, the **larger** the reverse isolation, the **better**!

Summarizing, we find that the scattering matrix of the **ideal amplifier** is:

$$\bar{\bar{\mathbf{S}}}_{ideal} = \begin{bmatrix} 0 & 0 \\ S_{21} & 0 \end{bmatrix} \qquad \text{where } \left|S_{21}\right| \gg 1$$

Sort of like an **isolator** with gain!

The **non-ideal** reality is that the zero valued terms will be **small**, but not **precisely** zero. Moreover, each scattering parameter will change with signal **frequency**—although they remain **approximately** constant within the amplifier **bandwidth**.