2. Amplifiers

We will find that the signal power collected by a receiver antenna is often **ridiculously small** (e.g., less than one **trillionth** of a Watt!)

To accurately recover the information impressed on this signal, we must **increase** the signal power a whole bunch—**without** modifying or distorting the signal in any way.

But first, a few comments about the **decibel**!

HO: dB, dBm, dBw

Q: *So what is an amplifier exactly? What is it suppose to accomplish?*

A: HO: Amplifiers

Q: *By how much will an amplifier increase signal power?*

A: HO: Amplifier Gain

Q: *Can we increase this signal power an unlimited amount?*

A:

HO: Amplifier Output Power

Q: *So, just how precisely does an amplifier reproduce a signal at its output?*

A: HO: Intermodulation Distortion

Q: *Is intermodulation distortion really that big of a problem?*

A:

HO: Two-Tone Intermodulation Distortion

Every good radio engineer knows and understands that parameters of the amplifier **spec sheet**!

HO: The Amplifier Spec Sheet

dB, dBm, dBw

Decibel (dB), is a specific function that operates on a **unitless** parameter:

$$dB \doteq 10 \log_{10}(x)$$

where x is <u>unitless</u>!

Q: *A unitless parameter! What good is that !?*

A: **Many** values are unitless, such as **ratios** and **coefficients**.

For example, amplifier **gain** is a unitless value!

E.G., amplifier gain is the ratio of the output power to the input power:

$$\frac{P_{out}}{P_{in}} = G$$

$$\therefore \text{ Gain in dB } = 10 \log_{10} G \doteq G(dB)$$

Q: *Wait a minute! I've seen statements such as:*

.... the output <u>power</u> is 5 dBw ....
or
.... the input <u>power</u> is 17 dBm ....

*Of course, Power is **not** a unitless parameter!?!*

A: True! But look at how power is expressed; not in dB, but in **dBm** or **dBw**.

Q: *What the heck does dB**<u>m</u>** or dB**<u>w</u>** refer to ??*

A: It's sort of a **trick** !

Say we have some power *P*. Now say we divide this value *P* by one 1 Watt. The result is a unitless value that expresses the value of *P* in relation to 1.0 Watt of power.

For example, if $P = 2500\,mW$, then $P/1W = 2.5$. This simply means that power *P* is 2.5 times larger than one Watt!

Since the value $P/1W$ is unitless, we can express this value in decibels!

Specifically, we define this operation as:

$$P(dBw) \doteq 10 \log_{10}\left(\frac{P}{1\,W}\right)$$

For example, $P = 100$ Watts can alternatively be expressed as $P(dBw) = +20\,dBw$. Likewise, $P = 1\,mW$ can be expressed as $P(dBw) = -30\,dBw$.

Q: *OK, so what does* ***dBm*** *mean?*

A: This notation simply means that we have normalized some power P to one **Milliwatt** (i.e., $P/1mW$)—as opposed to one Watt. Therefore:

$$P(dBm) \doteq 10 \log_{10}\left(\frac{P}{1\,mW}\right)$$

For example, $P = 100$ Watts can alternatively be expressed as $P(dBm) = +50\,dBm$. Likewise, $P = 1\,mW$ can be expressed as $P(dBm) = 0\,dBm$.

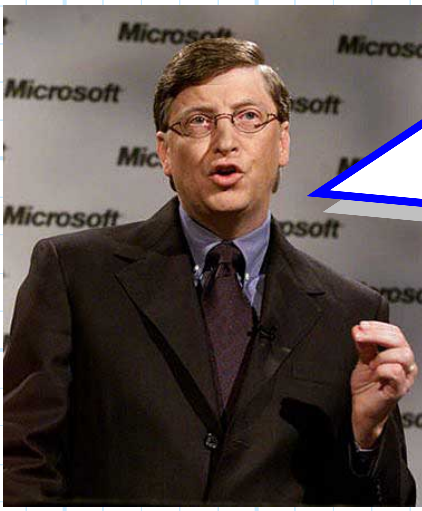
Make sure you are very **careful** when doing math with decibels!

Standard dB Values

Note that $10\,log_{10}(10) = 10dB$

Therefore an amplifier with a gain G = **10** is likewise said to have a gain of **10** dB.

Now consider an amplifier with a gain of **20** dB……



Q: *Yes, yes, I know. A **20 dB** amplifier has gain **G=20**, a **30 dB** amp has **G=30**, and so forth.*

*Please speed this lecture up and quit wasting my valuable time making such **obvious** statements!*

A: **NO!** Do **not** make this **mistake**!

Recall from **your** knowledge of logarithms that:

$$10\log_{10}\left[10^{n}\right] = n\,10\log_{10}\left[10\right] = 10n$$

Therefore, if we express gain as $G = 10^n$, we conclude:

$$G = 10^n \quad \leftrightarrow \quad G(dB) = 10n$$

In other words, G=100 = 10^2 (n=2) is expressed as 20 dB, while 30 dB (n=3) indicates G= 1000 = 10^3.

Likewise 100 mW is denoted as 20 dBm, and 1000 Watts is denoted as 30 *dBW*.

Note also that 0.001 mW = 10^{-3} mW is denoted as –30 *dBm*.

Another important relationship to keep in mind when using decibels is $10\log_{10}[2] \approx 3.0$. This means that:

$$10\log_{10}[2^n] = n\ 10\log_{10}[2] \simeq 3n$$

Therefore, if we express gain as $G = 2^n$, we conclude:

$$G = 2^n \quad \leftrightarrow \quad G(dB) \simeq 3n$$

As a result, a 15 dB (n=5) gain amplifier has G= 2^5 = 32. Similarly, 1/8 = 2^{-3} mW (n=-3) is denoted as –9 dBm.

Multiplicative Products and Decibels

Other logarithmic relationship that we will find useful are:

$$10\log_{10}[x\,y] = 10\log_{10}[x] + 10\log_{10}[y]$$

and its close cousin:

$$10\log_{10}\left[\frac{x}{y}\right] = 10\log_{10}[x] - 10\log_{10}[y]$$

Thus, the relationship $P_{out} = G\,P_{in}$ is written in **decibels** as:

$$P_{out} = G\,P_{in}$$

$$\frac{P_{out}}{1mW} = \frac{G\,P_{in}}{1mW}$$

$$10\log_{10}\left[\frac{P_{out}}{1mW}\right] = 10\log_{10}\left[\frac{G\,P_{in}}{1mW}\right]$$

$$10\log_{10}\left[\frac{P_{out}}{1mW}\right] = 10\log_{10}[G] + 10\log_{10}\left[\frac{P_{in}}{1mW}\right]$$

$$P_{out}(dBm) = G(dB) + P_{out}(dBm)$$

It is evident that "deebees" are **not** a unit! The units of the result can be found by **multiplying** the units of each term in a **summation** of decibel values.

For example, say some power $P_1 = 6\,dBm$ is combined with power $P_2 = 10\,dBm$. What is the resulting total power $P_T = P_1 + P_2$?

Q: *This result really **is** obvious—of **course** the total power is:*

$$\begin{aligned} P_T(dBm) &= P_1(dBm) + P_2(dBm) \\ &= 6\ dBm + 10\ dBm \\ &= 16\ dBm \end{aligned}$$

A: **NO!** Never do **this** either!

Logarithms are very helpful in expressing **products** or **ratios** of parameters, but they are **not** much help when our math involves sums and differences!

$$10\log_{10}[x+y] = ????$$

So, if you wish to add P_1 =6 dBm of power to P_2 =10 dBm of power, you must first **explicitly** express power in Watts:

P_1 =10 dBm = 10 mW and P_2 =6 dBm = 4 mW

Thus, the total power P_T is:

$$
\begin{aligned}
P_T &= P_1 + P_2 \\
&= 4.0\ mW + 10.0\ mW \\
&= 14.0\ mW
\end{aligned}
$$

Now, we can express this total power in *dBm*, where we find:

$$P_T\left(dBm\right) = 10\ log_{10}\left(\frac{14.0\ mW}{1.0\ mW}\right) = 11.46\ dBm$$

The result is **not** 16.0 *dBm* !.

We **can** mathematically add 6 *dBm* and 10 *dBm*, but we must understand what result means (nothing useful!).

$$
\begin{aligned}
6\, dBm\ + 10\ dBm &= 10 \log_{10}\left[\frac{4 mW}{1 mW}\right] + 10 \log_{10}\left[\frac{10 mW}{1 mW}\right] \\
&= 10 \log_{10}\left[\frac{40\ mW^2}{1 mW^2}\right] \\
&= 16\ dB \text{ relative to } 1\ \text{mW}^2
\end{aligned}
$$

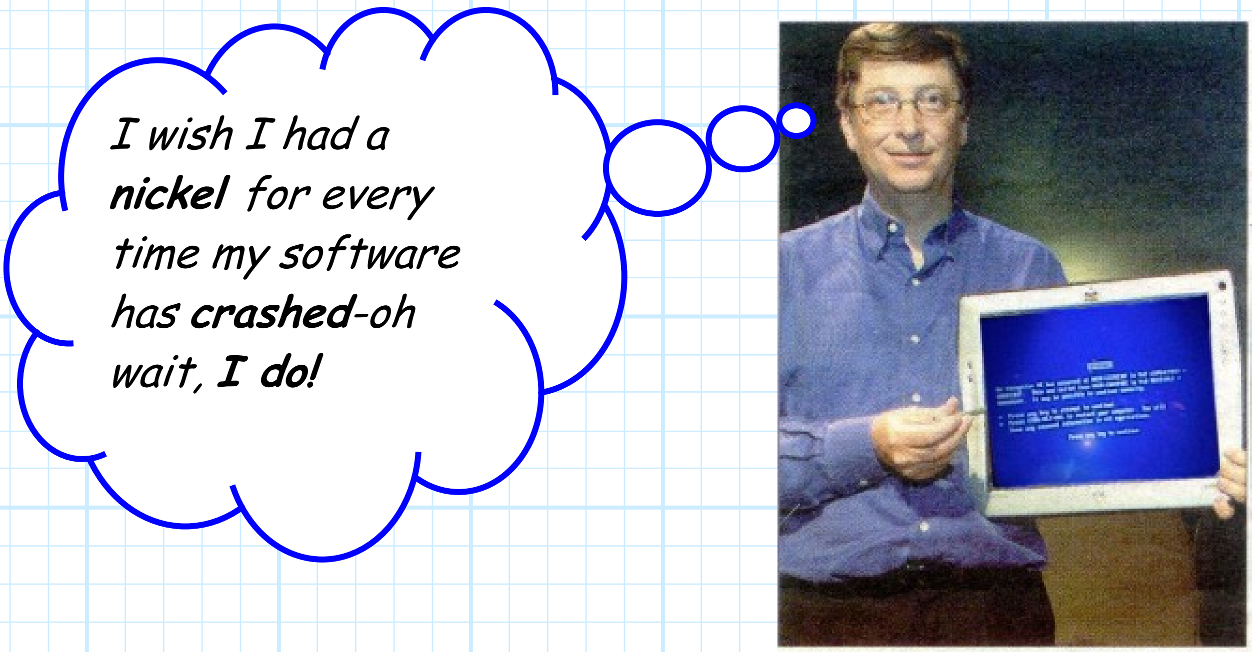
Thus, mathematically speaking, 6 *dBm* + 10 *dBm* implies a multiplication of power, resulting in a value with units of **Watts squared** !

A few more **tidbits** about decibels:

1. $1.0 \quad \leftrightarrow \quad 0\ dB$

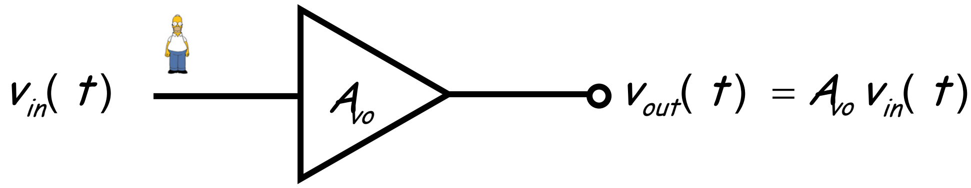
2. $0.0 \quad \leftrightarrow \quad -\infty\ dB$

3. $5^n \quad \leftrightarrow \quad \simeq 7n\ dB$ (can **you** show why?)



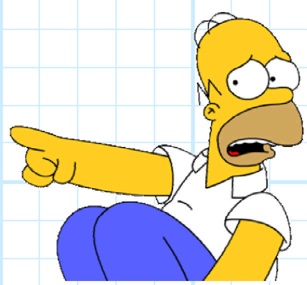
Amplifiers

An **ideal** amplifier takes an input signal and reproduces it **exactly** at its output, only with a **larger** magnitude!



where A_{vo} is the open-circuit **voltage gain** of the amplifier.

Now, let's express this result using our knowledge of **linear circuit theory**!



Recall, the output $v_{out}(t)$ of a linear device can be determined by **convolving** its input $v_{in}(t)$ with the device **impulse response** $g(t)$:

$$v_{out}(t) = \int_{-\infty}^{t} g(t-t')\, v_{in}(t')\, dt'$$

The impulse response for the **ideal** amplifier would therefore be:

$$g(t) = A_{vo}\,\delta(t)$$

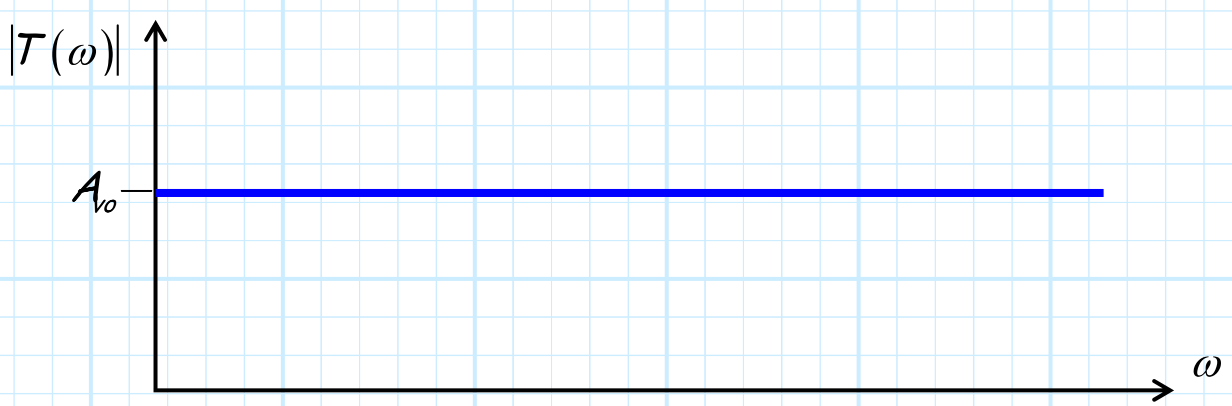
so that:

$$\begin{aligned} v_{out}(t) &= \int_{-\infty}^{t} g(t-t')\,v_{in}(t')\,dt' \\ &= \int_{-\infty}^{t} A_{vo}\,\delta(t-t')\,v_{in}(t')\,dt' \\ &= A_{vo}\,v_{in}(t) \end{aligned}$$

We can alternatively represent the ideal amplifier response in the **frequency domain**, by taking the **Fourier Transform** of the impulse response:

$$\begin{aligned} T(\omega) &= \int_{-\infty}^{\infty} g(t)\,e^{-j\omega t}\,dt \\ &= \int_{-\infty}^{\infty} A_{vo}\,\delta(t)\,e^{-j\omega t}\,dt \\ &= A_{vo} + j0 \end{aligned}$$

This result, although simple, has an interesting interpretation. It means that the amplifier exhibits gain of A_{vo} for sinusoidal signals of **any** and **all** frequencies!



Moreover, the ideal amplifier does not alter the **relative phase** of the sinusoidal signal (i.e., no phase shift).

In other words, if:

$$v_{in}(t) = \cos(\omega t)$$

then at the output of the ideal amplifier we shall see:

$$\begin{aligned} v_{out}(t) &= |T(\omega)| \cos(\omega t + \angle T(\omega)) \\ &= A_{vo} \cos(\omega t) \end{aligned}$$

BUT, there is one **big** problem with an ideal amplifier:

➡ They are **impossible** to build !!

Q: *Why is that ??*

A: Two reasons:

a) An ideal amplifier has **infinite** bandwidth.

b) An ideal amplifier has **zero** delay.

Not gonna happen !

Let's look at this **second** problem first. The ideal amplifier impulse response $g(t) = A_{vo}\,\delta(t)$ means that the signal at the output occurs **instantaneously** with the signal at the input.

This of course **cannot** happen, as it takes some small, but non-zero amount of **time** for the signal to propagate through the amplifier. A more **realizable** amplifier impulse response is:

$$g(t) = A_{vo}\,\delta(t-\tau)$$

resulting in an amplifier output of:

$$\begin{aligned} v_{out}(t) &= \int_{-\infty}^{t} g(t-t')\, v_{in}(t')dt' \\ &= \int_{-\infty}^{t} A_{vo}\,\delta(t-\tau-t')\, v_{in}(t')dt' \\ &= A_{vo}\, v_{in}(t-\tau) \end{aligned}$$

In other words, the output is both an amplified and **delayed** version of the input.

* Ideally, this delay does not **distort** the signal, as the output will have the same form as the input.

* Moreover, the delay for electronic devices such as amplifiers is **very small** in comparison to human time scales (i.e., $\tau \ll 1$ second).

* Therefore, propagation delay τ is generally **not** considered a **problem** for most amplifier applications.

Let's examine what this delay means in the **frequency domain**.

Evaluating the Fourier Transform of this **modified** impulse response gives:

$$\begin{aligned} T(\omega) &= \int_{-\infty}^{\infty} g(t)\, e^{-j\omega t} dt \\ &= \int_{-\infty}^{\infty} A_{vo}\, \delta(t-\tau)\, e^{-j\omega t} dt \\ &= A_{vo} \cos(\omega\tau) + j A_{vo} \sin(\omega\tau) \\ &= A_{vo}\, e^{j\omega\tau} \end{aligned}$$

We see that, as with the ideal amplifier, the magnitude $|T(\omega)| = A_{vo}$. However, the relative **phase** is now a linear function of frequency:

$$\angle T(\omega) = \omega\tau$$

As a result, if $v_{in}(t) = \cos(\omega t)$, the output signal will be:

$$\begin{aligned} v_{out}(t) &= |T(\omega)| \cos(\omega t - \angle T(\omega)) \\ &= A_v \cos(\omega t - \omega\tau) \end{aligned}$$

In other words, the output signal of a **real** amplifier is **phase shifted** with respect to the input.

In general, the amplifier phase shift $\angle T(\omega)$ will not be a perfectly linear function (i.e., $\angle T(\omega) \neq \omega\tau$), but instead will be a more general function of frequency ω.

However, if the phase function $\angle T(\omega)$ becomes too "non-linear", we find that signal **dispersion** can result—the output signal can be **distorted**!

Now, let's examine the **first problem** with the ideal amplifier. This problem is best discussed in the **frequency** domain.

We discovered that the **ideal** amplifier has a frequency response of $|T(\omega)| = A_{vo}$. Note this means that the amplifier gain is A_{vo} for **all** frequencies $0 < \omega < \infty$ (D.C. to daylight !).

The bandwidth of the ideal amplifier is therefore **infinite** !

* Since every electronic device will exhibit **some** amount of inductance, capacitance, and resistance, every device will have a **finite** bandwidth.

* In other words, there will be frequencies ω where the device does **not work** !

* From the standpoint of an amplifier, "not working" means $|T(\omega)| \ll A_{vo}$ (i.e., **low gain**).

* Amplifiers will therefore have **finite** bandwidths.

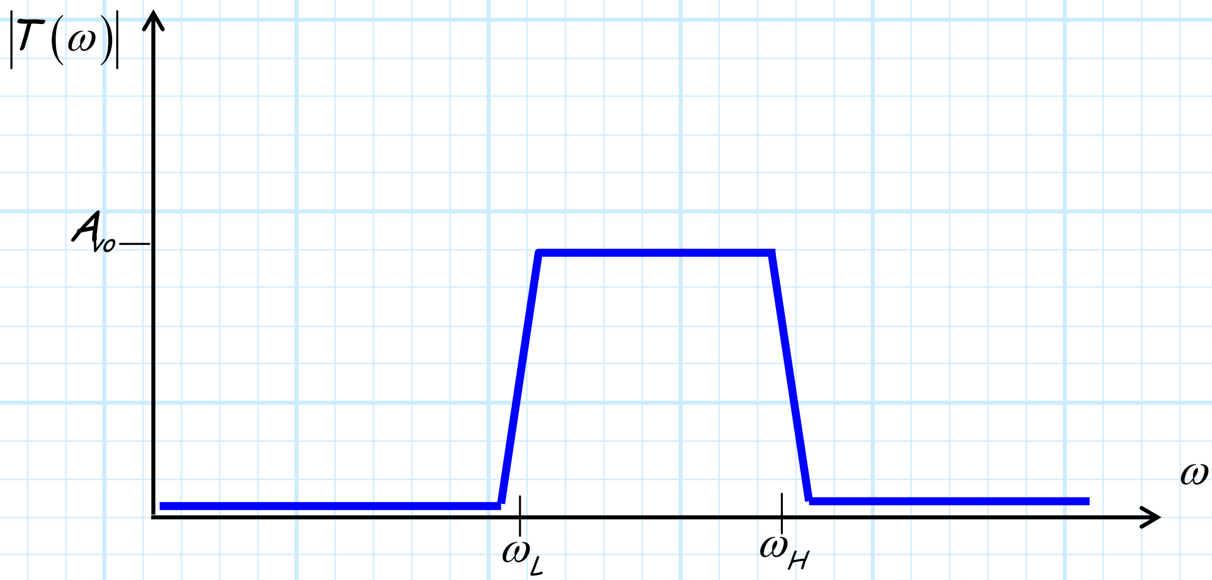
There is a range of frequencies ω between ω_L and ω_H where the gain will (approximately) be A_{vo}. For frequencies outside this range, the gain will typically be small (i.e. $\left|T(\omega)\right| \ll A_{vo}$):

$$\left|T(w)\right| = \begin{cases} \approx A_{vo} & \omega_L < \omega < \omega_H \\ \ll A_{vo} & \omega < \omega_L, \omega > \omega_H \end{cases}$$

The **width** of this frequency range is called the amplifier **bandwidth**:

$$Bandwidth \doteq \omega_H - \omega_L \quad \text{(radians/sec)}$$

$$\doteq f_L - f_H \quad \text{(cycles/sec)}$$



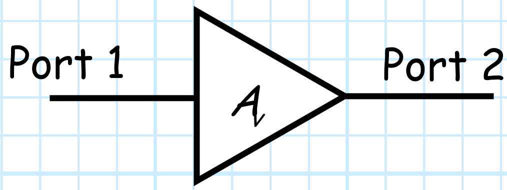
One result of having a **finite bandwidth** is that the amplifier impulse response is **not** an impulse function !

$$g(t) = \int_{-\infty}^{\infty} T(\omega)\, e^{+j\omega t} dt \neq A_{vo}\, \delta(t - \tau)$$

The **ideal** amplifier is not possible!

Amplifier Gain

Note that an amplifier is a **two-port** device.



As a result, we can describe an amplifier with a 2 x 2 **scattering matrix**:

$$\bar{\bar{\mathbf{S}}}(\omega) = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$

Q: What is the scattering matrix of an **ideal** amplifier??

A: Let's start with S_{11} and S_{22}.

To insure maximum power transfer, the input and output ports would ideally be matched:

$$S_{11} = S_{22} = 0$$

Now, let's look at scattering parameter S_{21}. We know that:

$$P_2^- = |S_{21}|^2 P_1^+$$

or, stated **another** way:

$$P_{out} = \left|S_{21}\right|^2 P_{in}$$

Therefore, we can **define** the amplifier **power gain** as:

$$G \doteq \frac{P_{out}}{P_{in}} = \left|S_{21}\right|^2$$

As the purpose of an amplifier is to boost the signal power, we can conclude that **ideally**:

$$\left|S_{21}\right| \gg 1$$

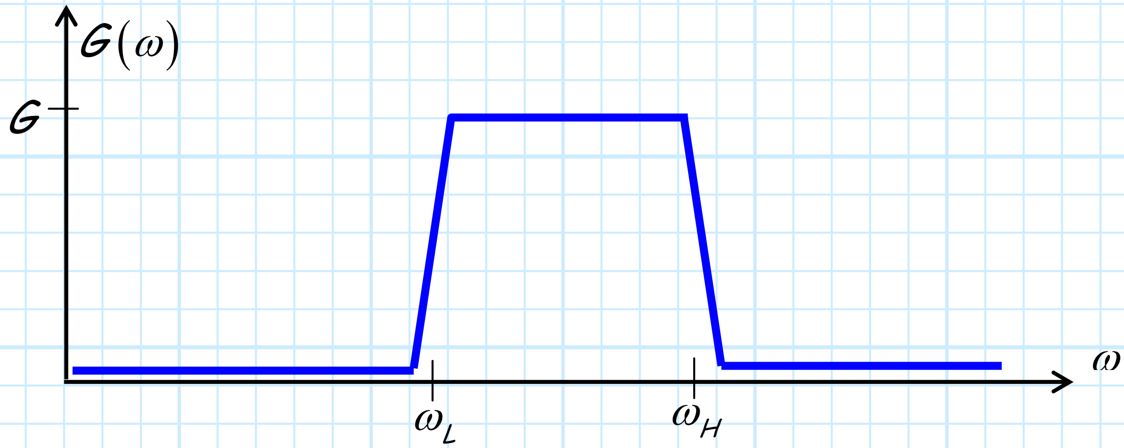
Clearly, an amplifier must be an **active** device!

As discussed earlier, the gain of an amplifier will change with signal frequency:

$$G(\omega) = \left|S_{21}(\omega)\right|^2$$

When radio engineers speak of amplifier **gain**, they almost always are speaking of this **power gain G**. However, they do not generally state it as a specific function of frequency!

Rather, amplifier gain is typically specified as a **numeric** value such as G =20 or G =13 dB. This value is a statement of the approximate amplifier gain **within** the amplifier **bandwidth**.

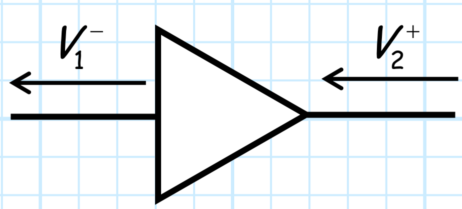


Thus, amplifier **gain** and **bandwidth** are the two most fundamental performance specifications of any microwave amplifier—together they (approximately) describe the amplifier transfer function!

Additionally, radio engineers almost always speak of amplifier gain in **decibels** (dB):

$$G(dB) = 10 \; log_{10} G$$

Finally, let's consider S_{12}. This scattering parameter relates the wave into port 2 (the output) to the wave out of port 1 (the input).



Q: *Are amplifiers **reciprocal** devices? In other words, is* $S_{12} = S_{21}$ *??*

A: No! An amplifier is strictly a **directional** device; there is a specific input, and a specific output—it does **not** work in reverse!

Ideally, $S_{12} = 0$. Any other value can just cause problems!

Typically though, S_{12} is small, but **not** zero. Generally speaking, radio engineers express S_{12} as a value called **reverse isolation**:

$$\text{reverse isolation} \doteq -10\, log_{10} \left|S_{12}\right|^2$$

Note when $S_{12} = 0$, reverse isolation will be **infinite**. Thus, the **larger** the reverse isolation, the **better**!

Summarizing, we find that the scattering matrix of the **ideal amplifier** is:

$$\bar{\bar{\mathbf{S}}}_{ideal} = \begin{bmatrix} 0 & 0 \\ S_{21} & 0 \end{bmatrix} \qquad \text{where } \left|S_{21}\right| \gg 1$$

Sort of like an **isolator** with gain!

The **non-ideal** reality is that the zero valued terms will be **small**, but not **precisely** zero. Moreover, each scattering parameter will change with signal **frequency**—although they remain **approximately** constant within the amplifier **bandwidth**.

Amplifier Output Power

Say we have an amplifier with gain G = 30 dB (i.e., G = 1000).

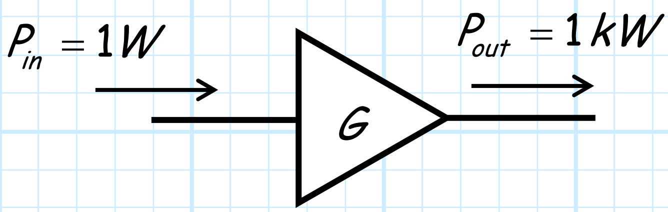
If the input power to this amplifier is 0 dBw (i.e., P_{in} = 1W), then the output power is:

$$P_{in} G = P_{out}$$

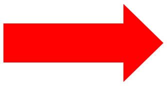
$$(1\text{ W})\, 1000 = 1000\text{ W}$$

Or, in dB:

$$0\text{ dBw} + 30\text{ dB} = 30\text{ dBw}$$



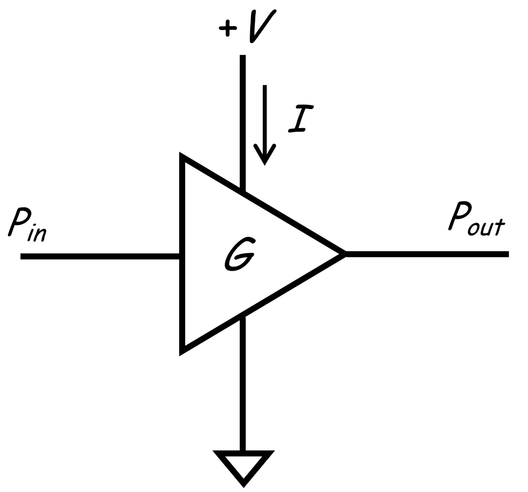
WOW! We created 999 Watts !

 The energy crisis is solved !

Of course, the amplifier cannot **create** energy.

Q: *Then, where does the power come from ???*

A: The **D.C.** power supply ! (Every amplifier has one).



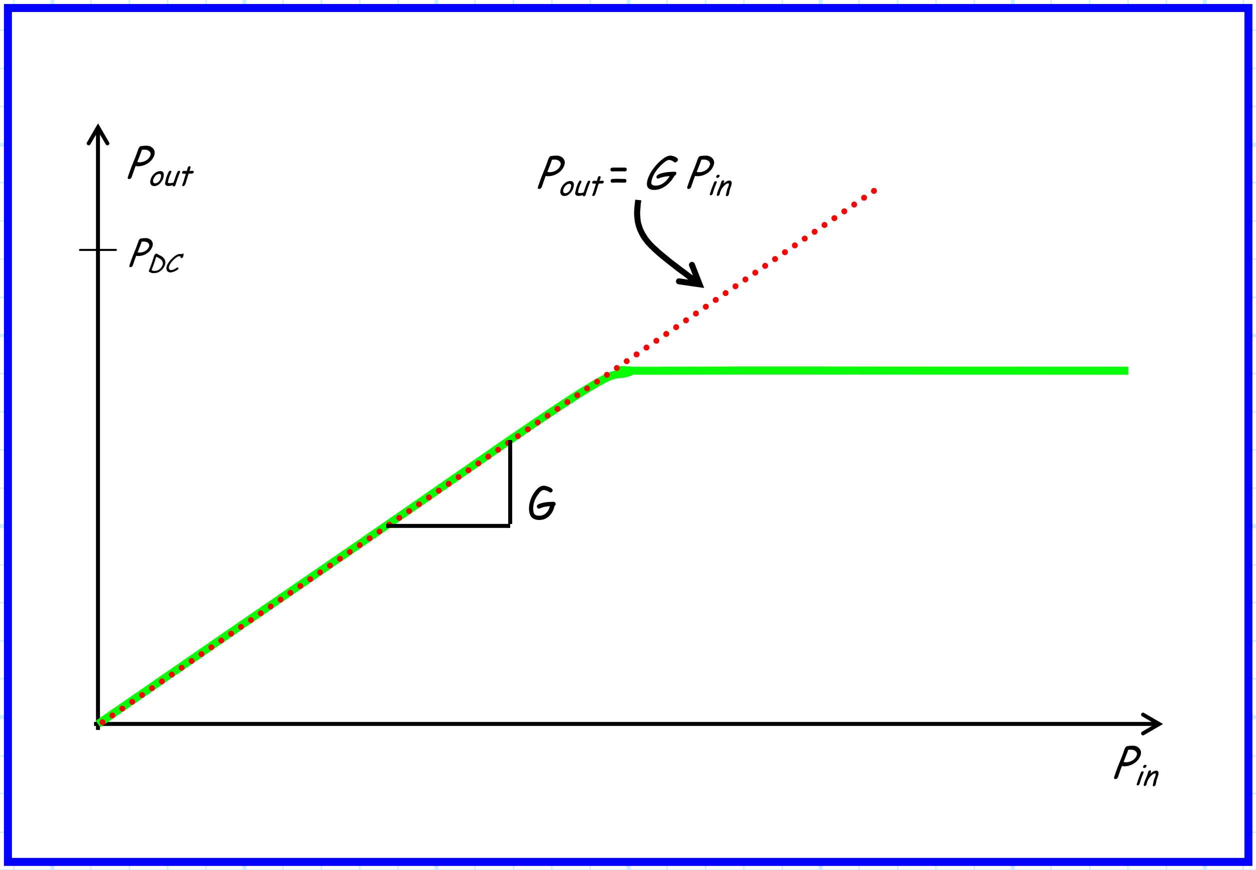
D. C. Power $= P_{DC} = V\,I > P_{out}$

The output power P_{out} cannot exceed the power delivered by the D.C. supply.

Q: *What happens to the D.C. power not converted to signal power P_{out} ??*

A:

So, if we were to plot P_{out} vs. P_{in} for a microwave amplifier, we would get something like this:



We notice that the output power **compresses**, or saturates.

Note there is **one** point on this curve where the amplifier output power P_{out} is 1 dB less than its ideal value of $G\,P_{in}$. In other words, there is one (and only one!) value of P_{in} and P_{out} that will satisfy the equation:

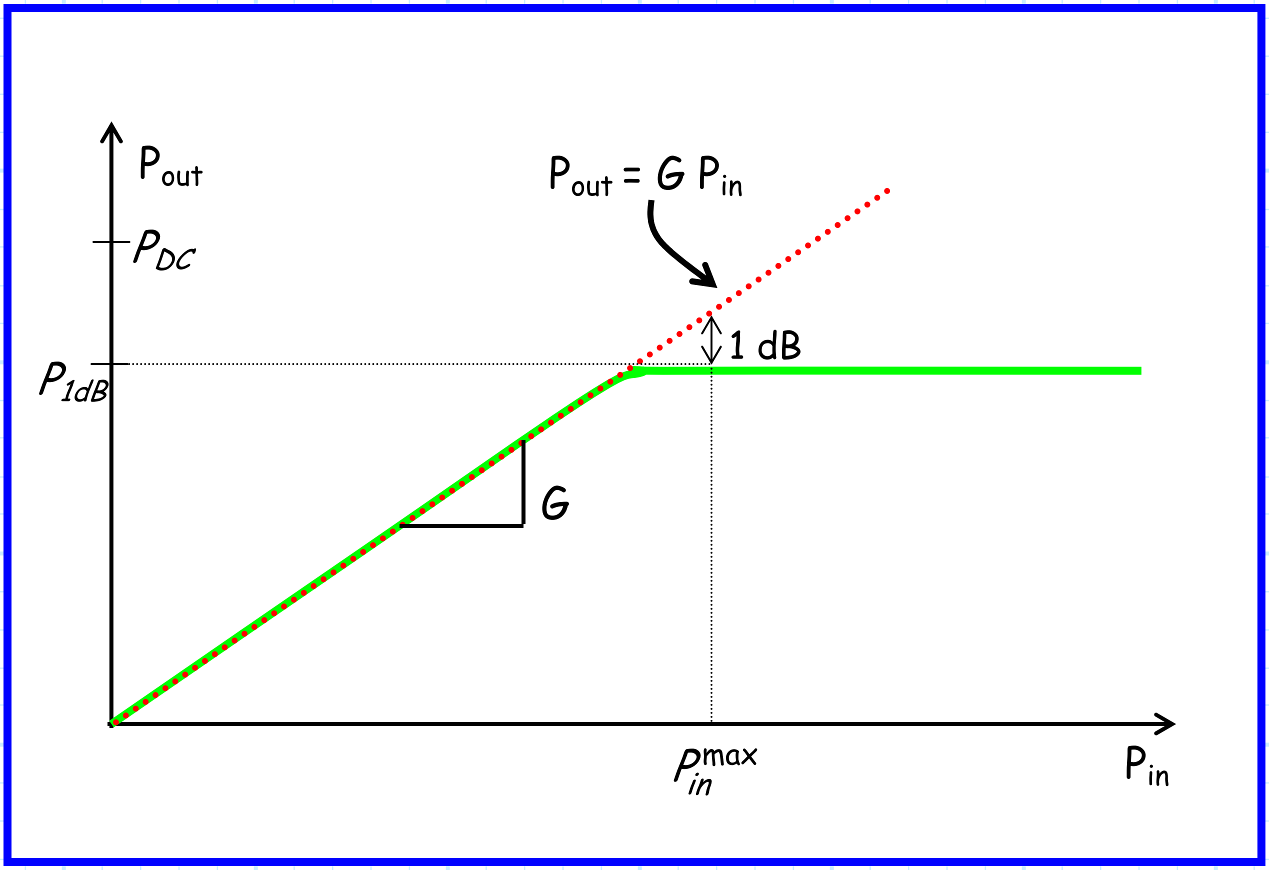
$$P_{out}(dB) = \left[P_{in}(dB) + G(dB)\right] - 1\ dB$$

At this point, the amplifier is said to be compressed 1 dB. Therefore, a 10 dB amplifier would appear to be a 9 dB amplifier!

The output power when the amplifier has compressed 1dB is called the **1 dB compression point** $\doteq$ $\mathbf{P_{1dB}}$ of the amplifier.

The 1 dB compression point is generally considered to be the **maximum power output** of the amplifier.

The input power at the 1 dB compression point is said to be the **maximum input power** (P_{in}^{max}) of the amplifier. We of course **can** put more than P_{in}^{max} into the amplifier—but we **won't** get much more power out!



Note the equation $P_{out}(dB) = \left[P_{in}(dB) + G(dB)\right] - 1\ dB$ alone is **not sufficient** to determine the 1 dB compression point, as we have two uknowns (P_{in} and P_{out}). We need **another** equation!

This second "equation" is the actual **curve** or **table** of data relating P_{in} to P_{out} for a **specific** amplifier.

Amplifier Efficiency

We can define **amplifier efficiency** e as the ratio of the maximum output power (P_{1dB}) to the D.C. power:

$$e = \frac{P_{1dB}}{P_{DC}}$$

(**don't** use decibels here!)

For example, if e=0.4, then up to 40% of the D.C. power **can** be converted to **output power**, while the remaining 60% is converted to **heat**.

➡ We require **high power** amps to be **very** efficient!

Intermodulation Distortion

The 1 dB compression curve shows that amplifiers are only **approximately** linear.

Actually, this should be **obvious**, as amplifiers are constructed with transistors—**non-linear** devices!

So, instead of the **ideal** case:

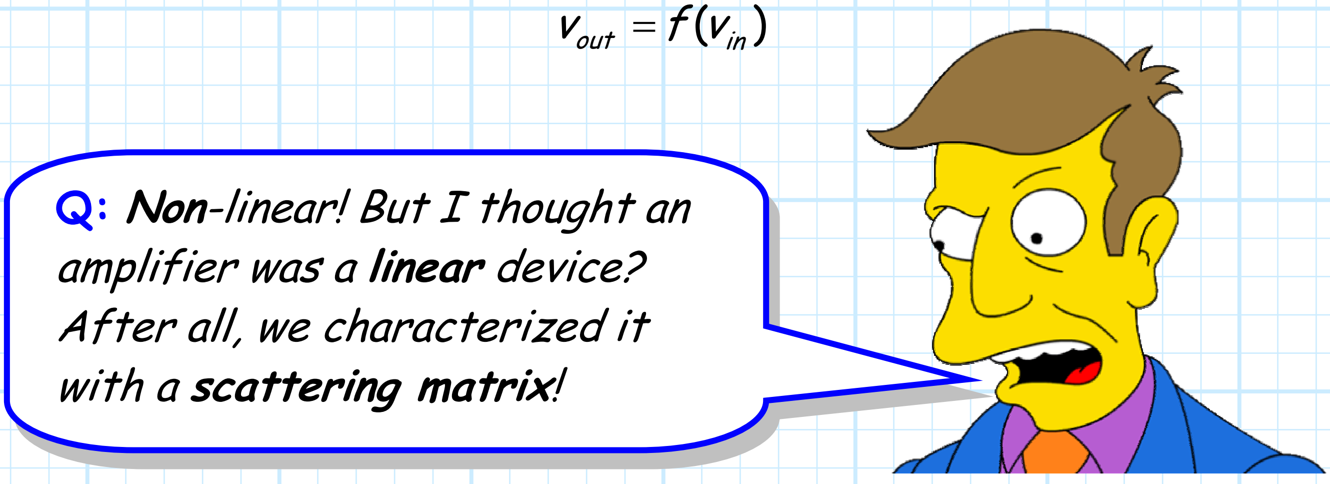
$$v_{out} = A_v \, v_{in}$$

Actual amplifier behavior requires more terms to describe!

$$v_{out} = A_v \, v_{in} + B \, v_{in}^2 + C \, v_{in}^3 + \cdots$$

This representation is simply a **Taylor Series** representation of the **non-linear** function:

$$v_{out} = f(v_{in})$$



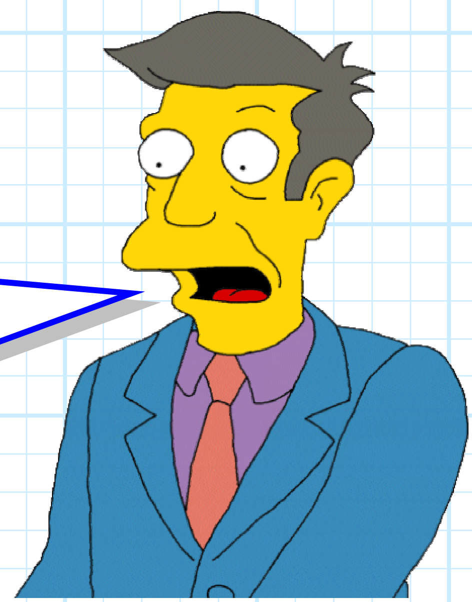
A: Generally speaking, the constants B, C, D, etc. are **very** small compared to the voltage gain A_v. Therefore, **if** v_{in} is likewise small, we can **truncate** the Taylor Series and **approximate** amplifier behavior as the linear function:

$$v_{out} \approx A_v v_{in}$$

BUT, as v_{in} gets large, the values v_{in}^2 and v_{in}^3 will get **really** large! In that case, the terms $B v_{in}^2$ and $C v_{in}^3$ will become **significant**.

As a result, the output will not simply be a larger version of the input. The output will instead be **distorted**—a phenomenon known as **Intermodulation Distortion.**

Q: *Good heavens! This sounds terrible. What exactly is **Intermodulation Distortion,** and what will it do to our signal output?!?*



A: Say the input to the amplifier is sinusoidal, with magnitude a:

$$v_{in} = a \cos \omega t$$

Using our knowledge of trigonometry, we can determine the result of the second term of the output Taylor series:

$$\begin{aligned} B\,v_{in}^2 &= B\,a^2 \cos^2 \omega t \\ &= \frac{B\,a^2}{2} + \frac{B\,a^2}{2} \cos 2\omega t \end{aligned}$$

We have created a **harmonic** of the input signal!

In other words, the input signal is at a frequency ω, while the output includes a signal at **twice** that frequency (2ω).

We call this signal a **second** order product, as it is a result of **squaring** the input signal.

Note we also have a **cubed** term in the output signal equation:

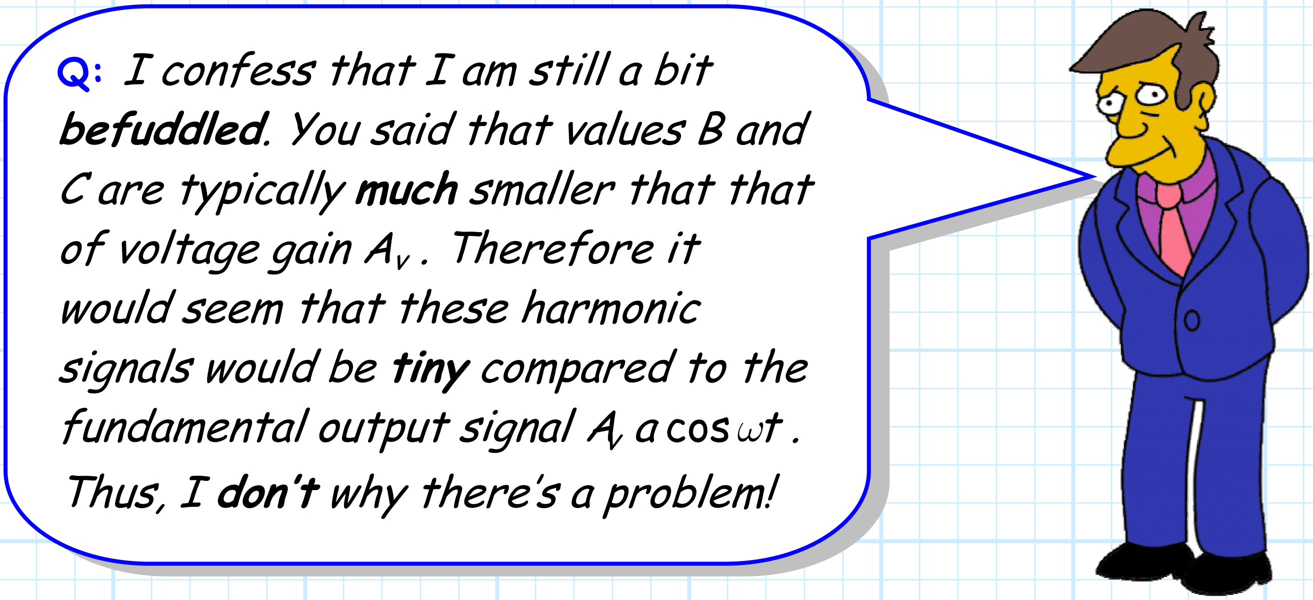
$$v_{out} = A_v\, v_{in} + B\,v_{in}^2 + C\,v_{in}^3 + \cdots$$

Using a trig identity, we find that:

$$\begin{aligned} C\,v_{in}^3 &= C\,a^3 \cos^3 \omega t \\ &= \frac{C\,a^3}{2} \cos \omega t + \frac{C\,a^3}{4} \cos 3\omega t \end{aligned}$$

Now we have produced a **second harmonic** (i.e., 3ω)!

As you might expect, we call this harmonic signal a **third-order** product (since it's produced from v_{in}^3).



To understand why intermodulation distortion can be a problem in amplifiers, we need to consider the **power** of the output signals.

We know that the power of a sinusoidal signal is proportional to its **magnitude squared**. Thus, we find that the power of each output signal is related to the input signal power as:

$$\text{1rst-order output power} \doteq P_1^{out} = A_v^2 \, P_{in} = G \; P_{in}$$

$$\text{2nd-order output power} \doteq P_2^{out} = \frac{B^2}{4} P_{in}^2 = G_2 \, P_{in}^2$$

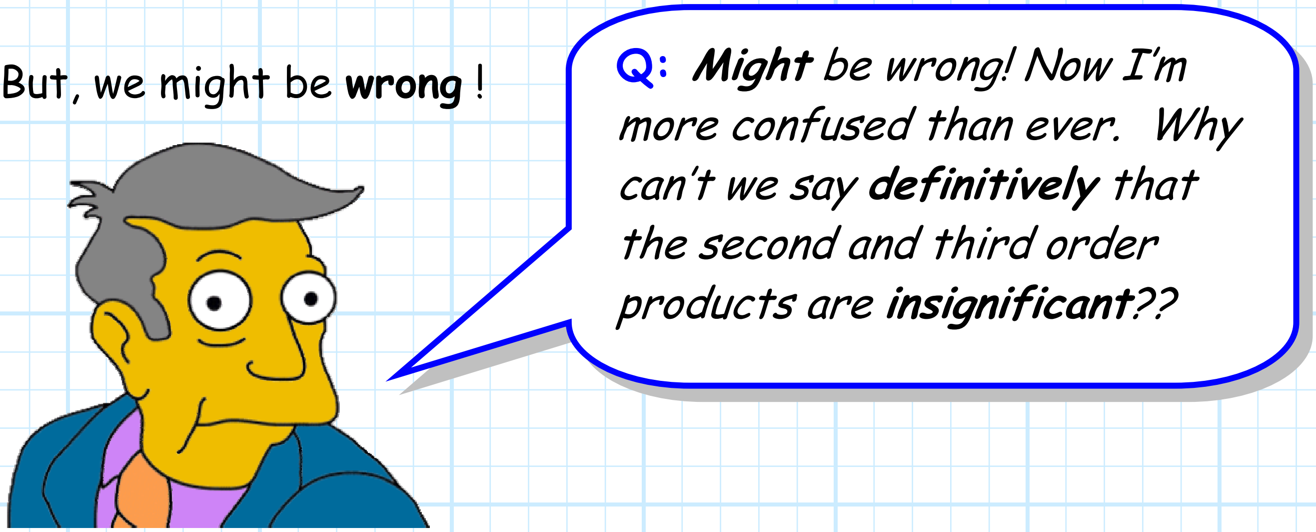
$$\text{3rd-order output power} \doteq P_3^{out} = \frac{C^2}{16} P_{in}^3 = G_3 \, P_{in}^3$$

where we have obviously **defined** $G_2 \doteq B^2/4$ and $G_3 \doteq C^2/16$.

Note that unlike G, the values G_2 and G_3 are not coefficients (i.e., not unitless!). The value G_2 obviously has units of inverse power (e.g., mW^{-1} or W^{-1}), while G_3 has units of inverse power squared (e.g., mW^{-2} or W^{-2}).

We know that typically, G_2 and G_3 are much **smaller** than G. Thus, we are **tempted** to say that P_1^{out} is much **larger** than P_2^{out} or P_3^{out}.

But, we might be **wrong** !



Look **closely** at the expressions for the output power of the first, second, and third order products:

$$P_1^{out} = G\ P_{in}$$
$$P_2^{out} = G_2\ P_{in}^2$$
$$P_3^{out} = G_3\ P_{in}^3$$

This **first** order output power is of course **directly** proportional to the input power. However, the **second** order output power is proportional to the input power **squared**, while the **third** order output is proportional to the input power **cubed**!

Thus we find that if the input power is **small**, the second and third order products **are** insignificant. But, as the input power **increases**, the second and third order products get **big** in a hurry!

For example, if we **double** the input power, the **first** order signal will of course likewise **double**. However, the **second** order power will **quadruple**, while the **third** order power will increase **8 times**.

For **large** input powers, the second and third order output products can in fact be **almost as large** as the first order signal!

Perhaps this can be most easily seen by expressing the above equations in **decibels**, e.g.,:

$$
\begin{aligned}
P_1^{out}(dBm) &= G(dB) + P_{in}(dBm) \\
P_2^{out}(dBm) &= G_2(dBm^{-1}) + 2\left[P_{in}(dBm)\right] \\
P_3^{out}(dBm) &= G_3(dBm^{-2}) + 3\left[P_{in}(dBm)\right]
\end{aligned}
$$

where we have used the fact that $\log x^n = n \log x$. Likewise, we have defined:

$$
\begin{aligned}
G_2(dBm^{-1}) &= 10\log_{10}\left[\frac{G_2}{\left(\frac{1}{1.0mW}\right)}\right] \\
&= 10\log_{10}\left[G_2\,(1.0mW)\right]
\end{aligned}
$$

and:

$$
\begin{aligned}
G_3(dBm^{-2}) &= 10\log_{10}\left[\frac{G_3}{\left(\frac{1}{1.0mW^2}\right)}\right] \\
&= 10\log_{10}\left[G_3\left(1.0mW^2\right)\right]
\end{aligned}
$$

Hint: Just express everything in milliwatts!

Note the value $2\left[P_{in}(dBm)\right]$ does **not** mean the value $2P_{in}$ expressed in decibels. The value $2\left[P_{in}(dBm)\right]$ is fact the value of P_{in} expressed in decibels—**times two**!

For **example**, if $P_{in}(dBm) = -30\,dBm$, then $2\left[P_{in}(dBm)\right] = -60\,dBm$. Likewise, if $P_{in}(dBm) = 20\,dBm$, then $2\left[P_{in}(dBm)\right] = 40\,dBm$.

What this means is that for every **1dB** increase in **input** power P_{in} the fundamental (**first-order**) signal will increase **1dB**; the **second-order** power will increase **2dB**; and the **third-order** power will increase **3dB**.

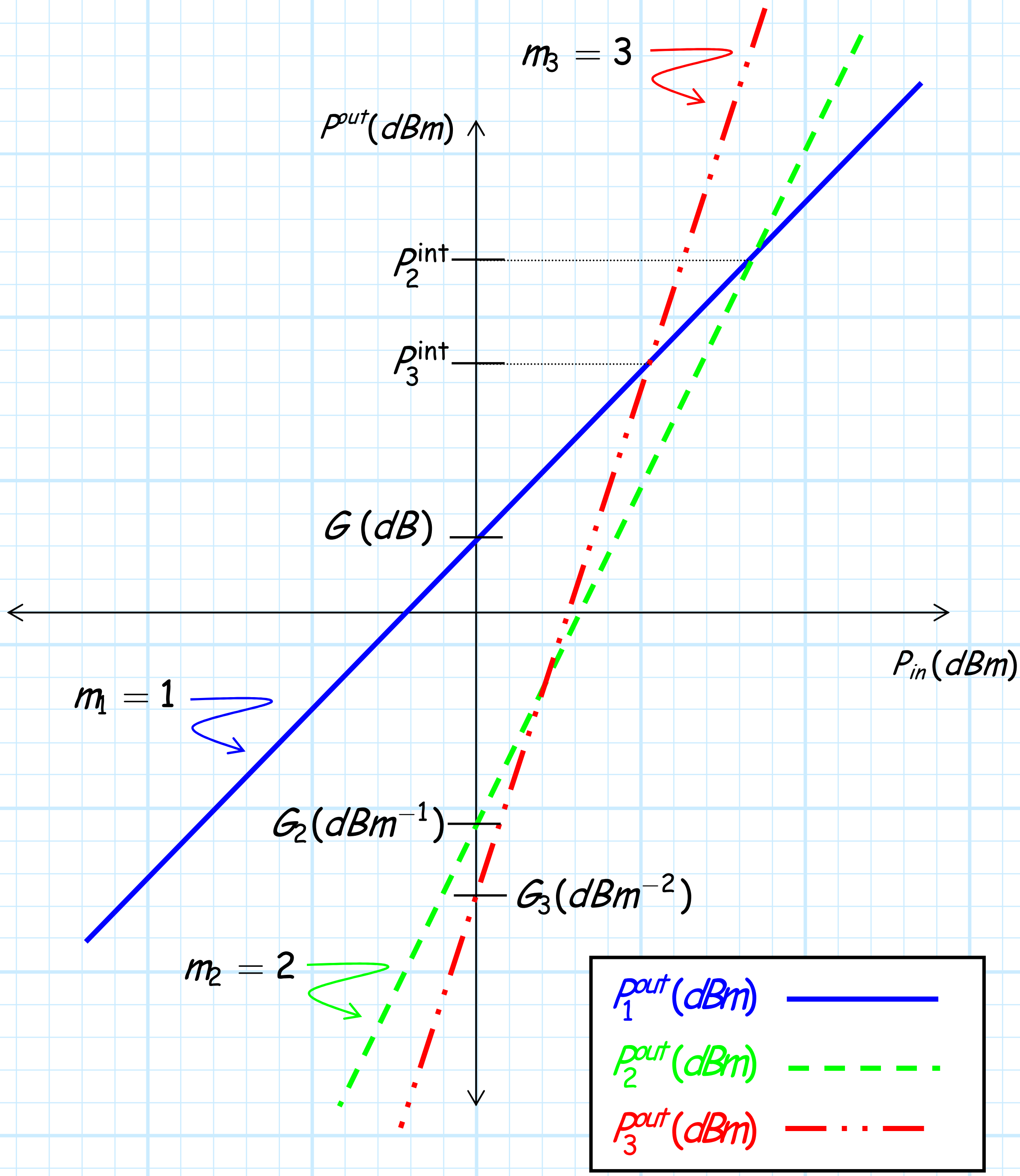
This is evident when we look at the three power equations (in decibels), as each is an equation of a **line** (i.e., $y = m\,x + b$).

For example, the equation:

$$P_3^{out}(dBm) = 3\left[P_{in}(dBm)\right] + G_3(dBm^{-2})$$
$$y = m\,x + b$$

describes a line with **slope** $m = 3$ and "y **intercept**" $b = G_3(dBm^{-2})$ (where $x = P_{in}(dBm)$ and $y = P^{out}(dBm)$).

Plotting each of the three equations for a typical amplifier, we would get something that looks like this:



Note that for $P_{in}(dBm) < 0\,dBm$ (the left side of the plot), the second and third-order products are small compared to the fundamental (first-order) signal.

However, when the input power increases **beyond** 0 dBm (the right side of the plot), the second and third order products rapidly **catch up**! In fact, they will (theoretically) become **equal** to the first order product at some large input power.

The point at which each higher order product **equals** the first-order signal is defined as the **intercept point**. Thus, we define the **second order intercept** point as the output power **when**:

$$P_2^{out} = P_1^{out} \doteq P_2^{int} \quad \textit{Second - order intercept power}$$

Likewise, the **third order intercept** point is defined as the third-order output power **when**:

$$P_3^{out} = P_1^{out} \doteq P_3^{int} \quad \textit{Third - order intercept power}$$

Using a little algebra **you** can show that:

$$P_2^{int} = \frac{G^2}{G_2} \quad \text{and} \quad P_3^{int} = \sqrt{\frac{G^3}{G_3}}$$

Or, expressed in **decibels**:

$$P_2^{int}(dBm) = 2\,G(dB) - G_2(dBm^{-1})$$

$$P_3^{int}(dBm) = \frac{3\,G(dB) - G_3(dBm^{-2})}{2}$$

* Radio engineers specify the intermodulation distortion performance of a specific amplifier in terms of the **intercept points**, rather than values G_2 and G_3.

* Generally, only the **third-order** intercept point is provided by amplifier manufactures (we'll see why later).

* **Typical** values of P_3^{int} for a **small-signal** amplifier range from +20 dBm to +50 dBm

* Note that as G_2 and G_3 **decrease**, the intercept points **increase**.

Therefore, the **higher** the intercept point of an amplifier, the better the amplifier !

One other **important point**: the **intercept points** for most amplifiers are much **larger** than the **compression point**! I.E.,:

$$P^{int} > P_{1dB}$$

In other words the intercept points are "theoretical", in that we can **never**, in fact, increase the input power to the point that the higher order signals are **equal** to the fundamental signal power.

All signals, including the higher order signals, have a **maximum** limit that is determined by the amplifier **power supply**.

Two-Tone Intermodulation

Q: *It doesn't seem to me that this* ***dad-gum*** *intermodulation distortion is really that much of a problem.*

I mean, the first and second ***harmonics*** *will likely be well* ***outside*** *the amplifier bandwidth, right?*

A: True, the **harmonics** produced by intermodulation distortion typically are **not** a problem in radio system design. There is a problem, however, that is **much worse** than **harmonic** distortion!

This problem is called **two-tone** intermodulation distortion.

Say the input to an amplifier consists of **two** signals at **dissimilar** frequencies:

$$v_{in} = a\cos\omega_1 t + a\cos\omega_2 t$$

Here we will assume that both frequencies ω_1 and ω_2 are within the **bandwidth** of the amplifier, but are **not** equal to each other $(\omega_1 = \omega_2)$.

This of course is a much more **realistic** case, as typically there will be **multiple** signals at the input to an amplifier!

For example, the two signals considered here could represent two **FM radio stations**, operating at frequencies within the FM band (i.e., $88.1 \text{ MHz} \leq f_1 \leq 108.1 \text{ MHz}$ and $88.1 \text{ MHz} \leq f_2 \leq 108.1 \text{ MHz}$).

Q: *My point exactly! Intermodulation distortion will produce those **dog-gone** second-order products:*

$$\frac{a^2}{2}\cos 2\omega_1 t \quad \text{and} \quad \frac{a^2}{2}\cos 2\omega_2 t$$

*and **gul-durn** third order products:*

$$\frac{a^3}{4}\cos 3\omega_1 t \quad \text{and} \quad \frac{a^3}{4}\cos 3\omega_2 t$$

*but these harmonic signals will lie well **outside** the FM band!*

A: True! Again, the **harmonic** signals are **not** the problem. The problem occurs when the **two input** signals combine together to form **additional** second and third order products.

Recall an amplifier output is accurately described as:

$$v_{out} = A_v\, v_{in} + B\, v_{in}^2 + C\, v_{in}^3 + \cdots$$

Consider first the **second-order** term if **two** signals are at the input to the amplifier:

$$\begin{aligned} v_2^{out} &= B\, v_{in}^2 \\ &= B\left(a\cos\omega_1 t + a\cos\omega_2 t\right)^2 \\ &= B\left(a^2\cos^2\omega_1 t + 2a^2\cos\omega_1 t\ \cos\omega_2 t + a^2\cos^2\omega_2 t\right) \end{aligned}$$

Note the first and third terms of the above expression are **precisely** the same as the terms we examined on the previous handout. They result in **harmonic** signals at frequencies $2\omega_1$ and $2\omega_2$, respectively.

The **middle** term, however, is something **new**. Note **it** involves the product of $\cos\omega_1 t$ and $\cos\omega_2 t$. Again using our knowledge of **trigonometry**, we find:

$$2a^2\cos\omega_1 t\ \cos\omega_2 t = a^2\cos\left(\omega_2 - \omega_1\right)t + a^2\cos\left(\omega_2 + \omega_1\right)t$$

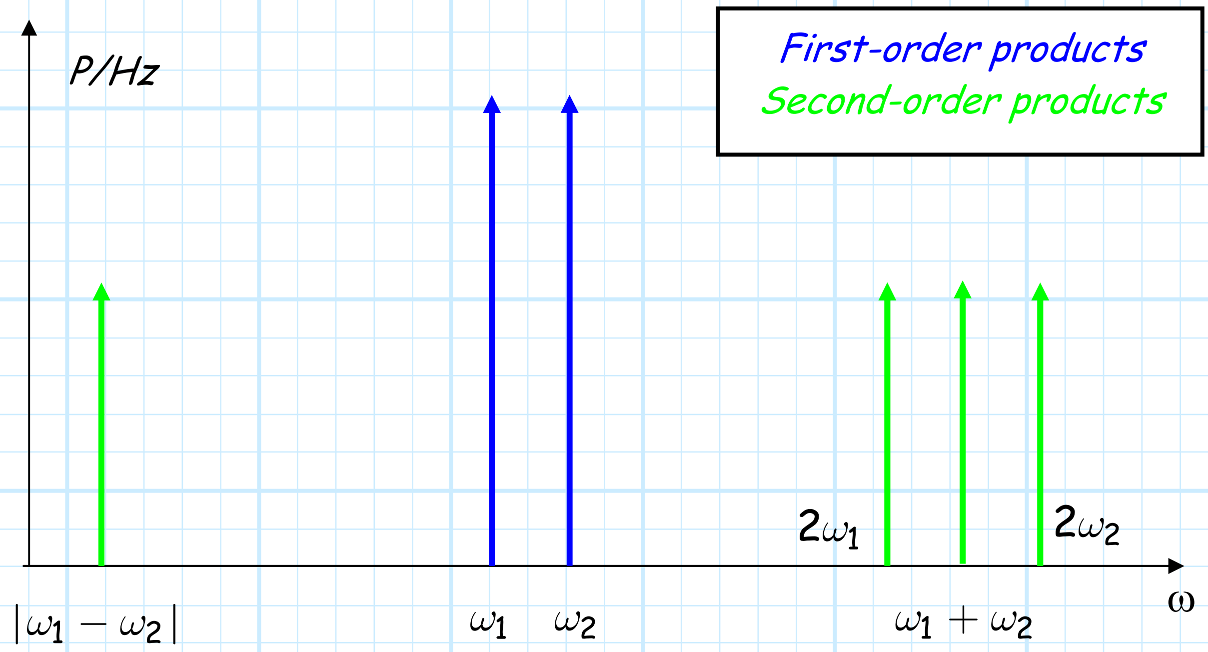
Note that since $\cos(-x) = \cos x$, we can **equivalently** write this as:

$$2a^2\cos\omega_1 t\ \cos\omega_2 t = a^2\cos\left(\omega_1 - \omega_2\right)t + a^2\cos\left(\omega_1 + \omega_2\right)t$$

Either way, the result is obvious—we produce **two new signals**!

These new **second-order** signals oscillate at frequencies $(\omega_1 + \omega_2)$ and $|\omega_1 - \omega_2|$.

Thus, if we looked at the **frequency spectrum** (i.e., signal power as a function of frequency) of an amplifier **output** when two sinusoids are at the input, we would see something like this:



Note that the new terms have a frequency that is either much **higher** than both ω_1 and ω_2 (i.e., $(\omega_1 + \omega_2)$), or much **lower** than both ω_1 and ω_2 (i.e., $|\omega_1 - \omega_2|$).

Either way, these new signals will typically be **outside** the amplifier bandwidth!

Q: *I thought you said these "two-tone" intermodulation products were some "**big problem**". These sons of a gun appear to be **no more** a problem than the harmonic signals!*

A: This observation is indeed correct for **second**-order, two-tone intermodulation products. But, we have **yet** to examine the **third**-order terms! I.E.,

$$\begin{aligned} v_3^{out} &= C\, v_{in}^3 \\ &= C\left(a\cos\omega_1 t + a\cos\omega_2 t\right)^3 \end{aligned}$$

If we multiply this all out, and again apply our trig knowledge, we find that a **bunch** of new **third-order** signals are created.

Among these signals, of course, are the second **harmonics** $\cos 3\omega_1 t$ and $\cos 3\omega_2 t$. Additionally, however, we get these **new** signals:

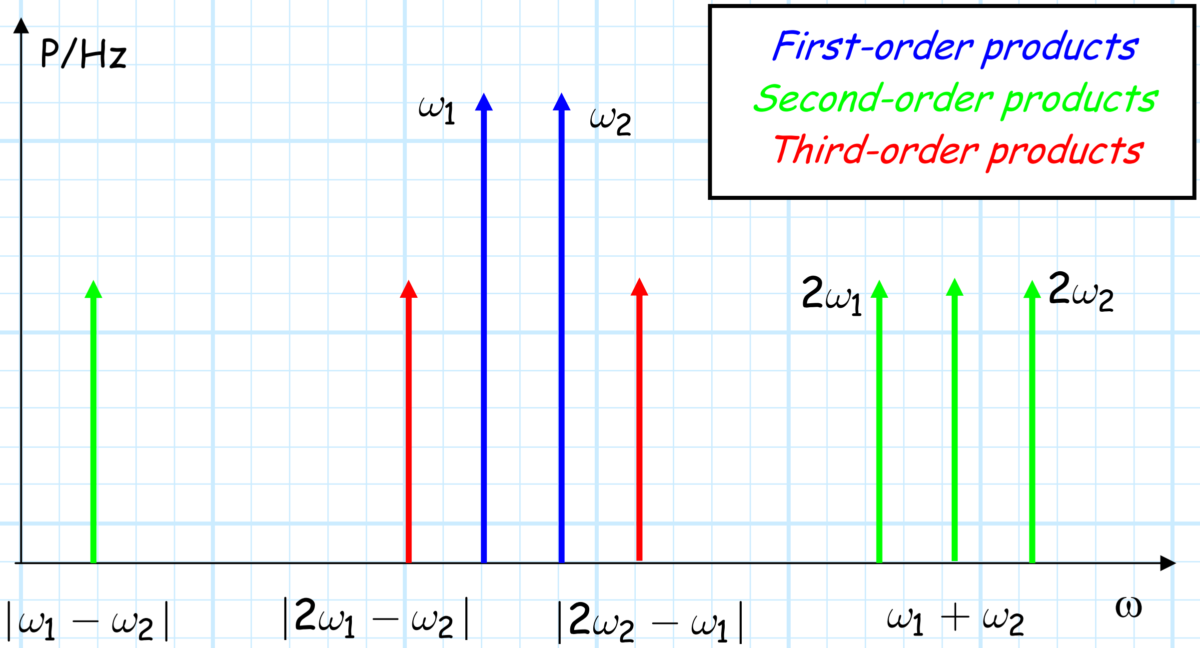
$$\cos(2\omega_2 - \omega_1)t \quad \text{and} \quad \cos(2\omega_1 - \omega_2)t$$

Note since $\cos(-x) = \cos x$, we can **equivalently** write these terms as:

$$\cos(\omega_1 - 2\omega_2)t \quad \text{and} \quad \cos(\omega_2 - 2\omega_1)t$$

Either way, it is apparent that the **third-order** products include signals at frequencies $|\omega_1 - 2\omega_2|$ and $|\omega_2 - 2\omega_1|$.

Now lets look at the output spectrum with **these new** third-order products included:



Now **you** should see the problem! **These** third-order products are very **close** in frequency to ω_1 and ω_2. They will likely lie **within** the bandwidth of the amplifier!

For example, if f_1=100 MHz and f_2=101 MHz, then $2f_2$-f_1=102 MHz and $2f_1$-f_2= 99 MHz. All frequencies are **well** within the FM radio bandwidth!

Thus, these **third-order**, **two-tone** intermodulation products are the **most significant** distortion terms.

This is why we are most concerned with the **third-order intercept point** of an amplifier!

The Amplifier Spec Sheet

Here's a list of some of the most **important** amplifier specifications:

Gain (dB)

A numeric value that specifies the **average gain** over the amplifier bandwidth. Note that this value is for the case where both the source connected to input, and the load connected to the output, are **matched** to Z_0.

Typical values for single-stage amplifiers are **8 dB to 25 dB**.

Bandwidth (Hz)

Typically, bandwidth is defined as the frequencies where the amplifier will produce gain within **3 dB** of the nominal gain value expressed above.

For example, if the gain of an amplifier is stated as 17dB, then the bandwidth would specify the range of frequencies for which the gain is 14 dB or greater.

Generally speaking then, the **lower** and **upper** frequency values are provided (e.g., amplifier bandwidth is 2.2 GHz to 4.7 GHz).

Amplifiers can have an **extremely** wide bandwidth (e.g., multiple **octaves**), but **generally** we find a **trade** between **gain and**

bandwidth (sound familiar?)—the **wider** the bandwidth, the **lower** the gain (and vice versa).

Gain Variance(dB)

Be careful! This parameter can have a variety of names (e.g., gain ripple) and definitions.

Generally it describes the **gain flatness** over the middle portion of the amplifier bandwidth. For example, if our 17dB amplifier has a gain variance of +/- 1.0 dB, then the gain might vary from 16dB to 18dB across the amplifier bandwidth.

This parameter sometimes also refers to the variation in gain a function of **temperature**, or specifies the variation in the **manufacturing process**.

Typical values are +/- 0.5 dB to +/- 2.0 dB

Input Impedance (S_{11}, Z_{in}, Γ, return loss, VSWR)

Amplifiers are generally well-matched over their operating bandwidth. There are (as we have discovered) a variety of ways to express this match. Often the **worst-case** value over the operating bandwidth is provided (e.g., return loss > 30 dB over the operating frequency).

Typically, an amplifier input port return loss is **30 dB or** more, although this value typically gets worse and bandwidth increases.

Output Impedance (S_{11}, Z_{in}, Γ, return loss, VSWR)

See above.

Reverse Isolation (dB)

This value can change markedly (in dB)over the amplifier bandwidth, and so a worst-case value is often provided (e.g. >40dB over the operating bandwidth).

Typical values are **35 dB** or greater.

D.C. Power (Nominal D.C. voltage and current)

Generally, a microwave amplifier requires a **regulated DC voltage supply** (e.g., 15.0 V). The DC current can vary, and typically a maximum value is given. This leads to a maximum DC power requirement (I'll leave the description of this value up to **you**).

A fairly **standard** supply voltage for low-power amplifiers is +15.0 V DC.

1 dB compression point (Watts, dBm, dBw)

This again is determined in the operating bandwidth of the amplifier. This value is considered to be the **largest output** power the amplifier can provide.

For low-power (i.e., small-signal) amplifiers, typical values range from +10 dBm to +25 dBm.

3rd order intercept point (Watts, dBm, dBw)

Remember, the **larger** this value, the **better**!

Typical values for small signal amplifiers range from **+15 dBm** to **+40 dBm**.

Noise Figure (dB)

A **very important** amplifier parameter. We will learn about this later! The smaller the noise figure, the better.

Typical values range from **1.0 dB to 6.0 dB**. Amplifiers with the best noise figure often have comparatively lower gain.

Everything Else (??)

The above list of amplifier specifications is by **no means** complete, unambiguous, or in any way **standard**.

The reason for this is that there **is no** complete, unambiguous, or **standard** list of specifications!

It is up to **YOU**—the radio engineer—to determine **if** a particular amplifier meets the needs of **your particular** radio application or design specifications.

Or, you must be able to **write** a clear, complete, **unambiguous** specification that results in an amplifier that meets your needs!