

Antenna Directivity

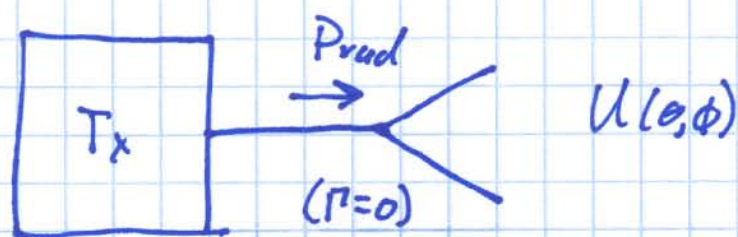
Recall the intensity of the E.M. wave produced by an isotropic radiator (i.e., an antenna that radiates equally in all directions) is:

$$U_0 = \frac{P_{\text{rad}}}{4\pi}$$

Remember, an isotropic radiator is actually a physical impossibility — real antennas propagate E.M. energy unequally as a function of direction, a fact represented by the radiation intensity function $U(\theta, \phi)$.

Note that $U(\theta, \phi)$ is dependent on both the antenna and the transmitter power. Increasing the transmitter

power will of course increase $U(\theta, \phi)$
in all directions.



Of course, this is evident when we consider that the radiated power can be determined from $U(\theta, \phi)$:

$$P_{rad} = \int_0^{2\pi} \int_0^{\pi} U(\theta, \phi) \sin\theta d\theta d\phi$$

Q: Isn't there some way to characterize the directional behaviour of the antenna only, independent of transmitter power??

A: Yes! We call it the antenna directivity pattern $D(\theta, \phi)$.

To find a function that describes the antenna behaviour only, we need to somehow normalize $U(\theta, \phi)$ with respect to P_{rad} .

We do this by comparing the radiation intensity $U(\theta, \phi)$ of the antenna to the radiation intensity produced by an isotropic radiator connected to the same transmitter!

I.E.:

$$\begin{aligned} D(\theta, \phi) &\doteq \frac{\text{intensity of antenna}}{\text{intensity of an isotropic antenna}} \\ &= \frac{U(\theta, \phi)}{U_0} \\ &= \frac{4\pi U(\theta, \phi)}{P_{\text{rad}}} \end{aligned}$$



Note $D(\theta, \phi)$ is a unitless value.

We can show that $D(\theta, \phi)$ is independent of P_{rad} by integrating $D(\theta, \phi)$ over all directions θ , and ϕ

$$\begin{aligned} & \int_0^{2\pi} \int_0^{\pi} D(\theta, \phi) \sin\theta \, d\theta \, d\phi \\ &= \int_0^{2\pi} \int_0^{\pi} \frac{4\pi U(\theta, \phi)}{P_{\text{rad}}} \sin\theta \, d\theta \, d\phi \\ &= \frac{4\pi}{P_{\text{rad}}} \int_0^{2\pi} \int_0^{\pi} U(\theta, \phi) \sin\theta \, d\theta \, d\phi \\ &= \frac{4\pi}{P_{\text{rad}}} (P_{\text{rad}}) \\ &= \underline{\underline{4\pi}} \end{aligned}$$

∴

$$\int_0^{2\pi} \int_0^{\pi} D(\theta, \phi) \sin\theta \, d\theta \, d\phi = 4\pi$$

Furthermore, we find that the average value of $D(\theta, \phi)$ across 4π steradians (i.e., across all directions θ, ϕ) is:

$$D_{\text{ave}} = \frac{1}{4\pi} \int_0^{2\pi} \int_0^{\pi} D(\theta, \phi) \sin\theta \, d\theta \, d\phi$$
$$= \frac{1}{4\pi} (4\pi) = \underline{\underline{1}}$$

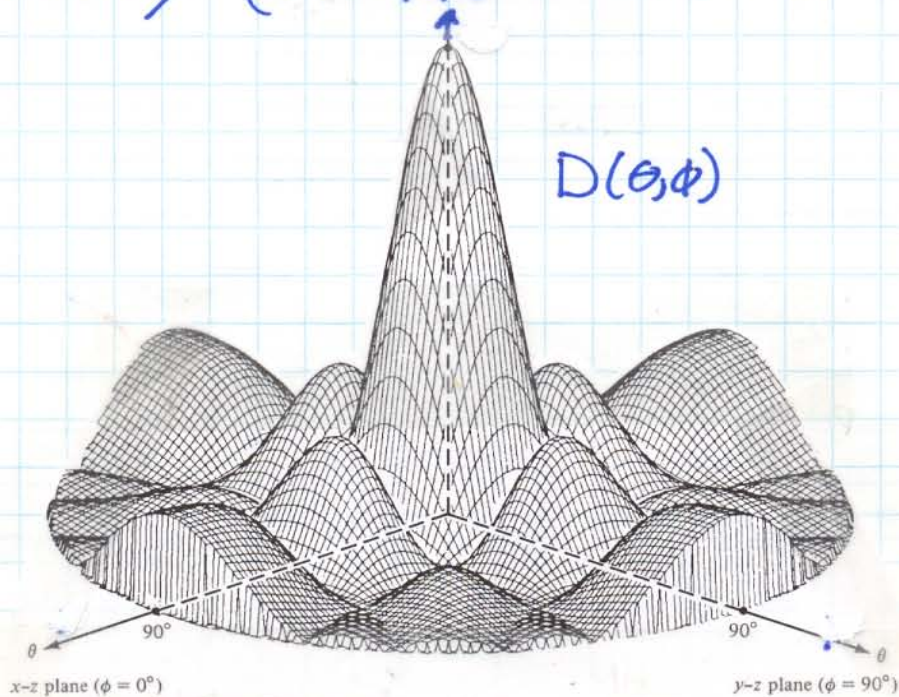
In other words, the average value of $D(\theta, \phi)$, across all directions θ, ϕ , of any and all antennas is 1!

What this means is that an antenna may produce an intensity larger than that of an isotropic radiator (U_0) in some directions, but then it must produce an intensity less than an isotropic radiator in other directions.

Likewise, if the intensity is much, much larger than U_0 in a specific direction, then the intensity must be very small in many other directions.

⇒ In other words, the antenna cannot produce above average intensity in all directions!!

Typically an antenna will produce very large intensity (i.e., $D(\theta, \phi) \gg 1$) in one general direction, and very small intensity ($D(\theta, \phi) \ll 1$) in the rest.



Note that there will typically be one direction where the function $D(\theta, \phi)$ is its maximum value.

This maximum value is defined as the antennas directivity :

$$\text{Directivity} \doteq D_0 = D(\theta, \phi) \Big|_{\text{max}}$$

Note D_0 is a number (often expressed in dB) while directivity pattern $D(\theta, \phi)$ is a function of θ and ϕ .