D. Antenna Impedance

An antenna, like any other microwave device, has an **input impedance**. Although there are typically **no** resistors used antenna designs, an antenna impedance better have a **real** (resistive) component!

HO: Antenna Impedance

Antenna resistance has two components; the most important of which is the **radiation resistance**.

HO: Radiation Resistance

Given that antennas are **not** perfectly efficient, we find that a more useful, applicable, and measurable parameter than directivity is **antenna gain**.

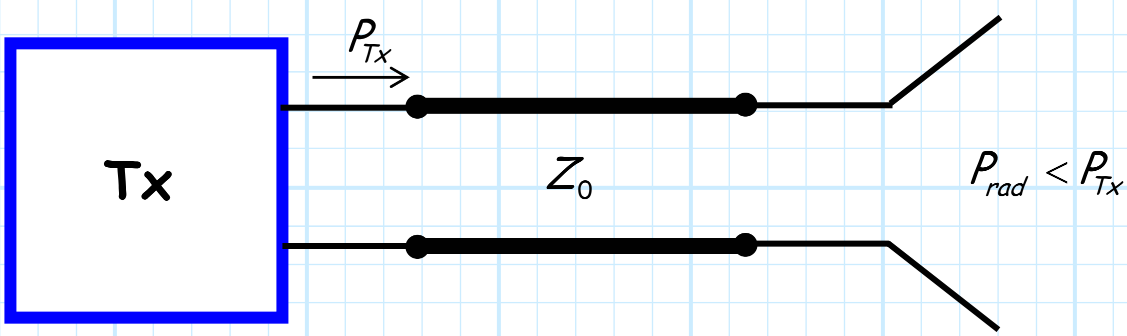
HO: Antenna Gain

Antenna Impedance

Q: *Is the radiated power **equal** to the available power (P_{Tx}) of the transmitter?*

A: **Ideally** it is! If $P_{rad} \neq P_{Tx}$, then some power is being wasted. However, the **perfectly** ideal case of $P_{rad} = P_{Tx}$ is **not** possible.

As a result, we find that P_{rad} will always be less (at least a little) than the available power P_{Tx}. However, we find for well-designed antenna that P_{rad} will be very close to available power P_{Tx}.



Q: *Why isn't the **radiated** power equal to the **available** power of the transmitter? What **happens** to this available power?*

A: One of **two things**, either:

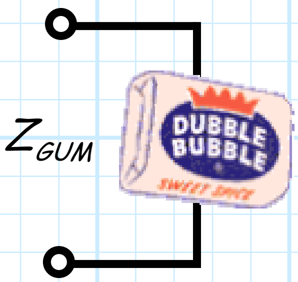
1. Power is **reflected** at the antenna.

2. Power is turned to heat in a **lossy** antenna.

Let's consider the **first** phenomenon first.

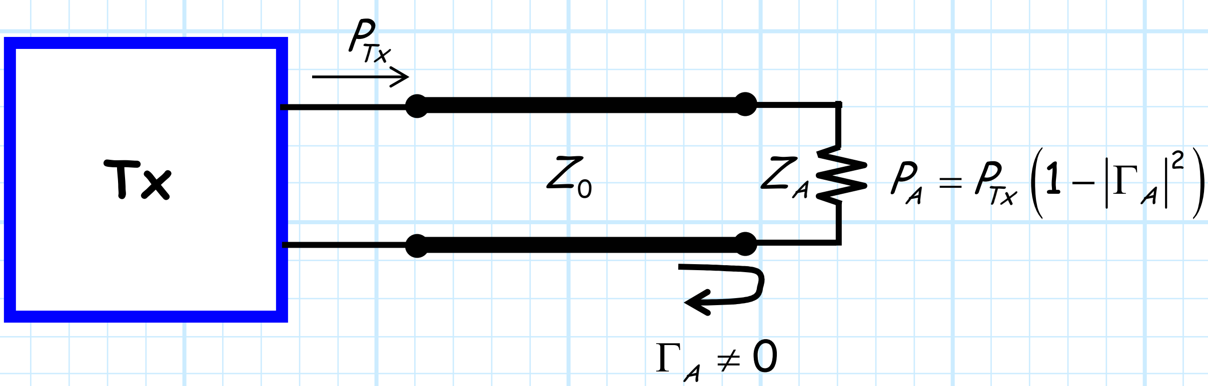
Power is **reflected** at the antenna **if** the antenna impedance Z_A is **not matched** to the transmission line.

Q: *Antenna impedance? Does an antenna **have** an impedance?*



A: An antenna is a one-port device—**every** one-port device has an impedance!

The antenna impedance acts as the **load** at the end of a transmission line. If $Z_A \neq Z_0$, then power will be **reflected**, and the power delivered to the antenna (P_A) will be **less** than the transmitter available power:



Thus, **all** the available power is delivered to the antenna **only** if its impedance is:

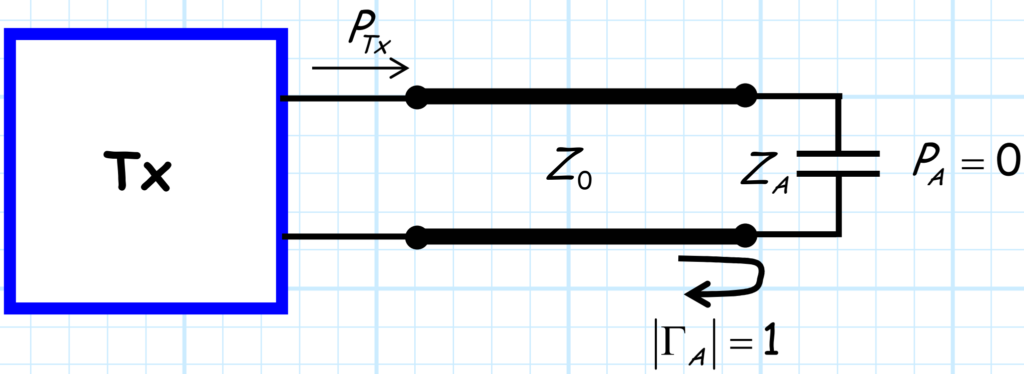
$$Z_A = Z_0 \quad \Rightarrow \quad \Gamma_A = 0$$

Q: *Huh?? Characteristic impedance is a **real** value. If $Z_A = Z_0$, then the antenna impedance is purely resistive. Wouldn't a resistor make a particularly **bad** antenna?*

A: A **resistor** actually would make a particularly **lousy** antenna. **Yet**, the impedance of an **ideal** antenna is **purely resistive**.

→ These statements are **not** contradictory!

Remember, a real load can absorb incident energy, whereas a **purely reactive** load cannot. For a reactive impedance, all incident power would be **reflected**—a purely reactive Z_A would result in $P_A = 0$.



Thus, it is imperative that the impedance of an antenna have a **real** component if we wish for it to **absorb** energy, with maximum power transfer occurring when $Z_A = Z_0$.

The **difference** between a resistor and an antenna, however, is what it **does** with this absorbed power.

* A **resistor** will convert its absorbed power into **heat**.

* An **antenna** will (ideally) convert its absorbed power into a propagating, spherical, **electromagnetic wave**!

In other words, an antenna dissipates its absorbed power by **radiating** it into space.

Q: *So does this mean that an antenna will reflect **no** power?*

A: Generally speaking, antenna impedance will posses both a **real** and **reactive** component:

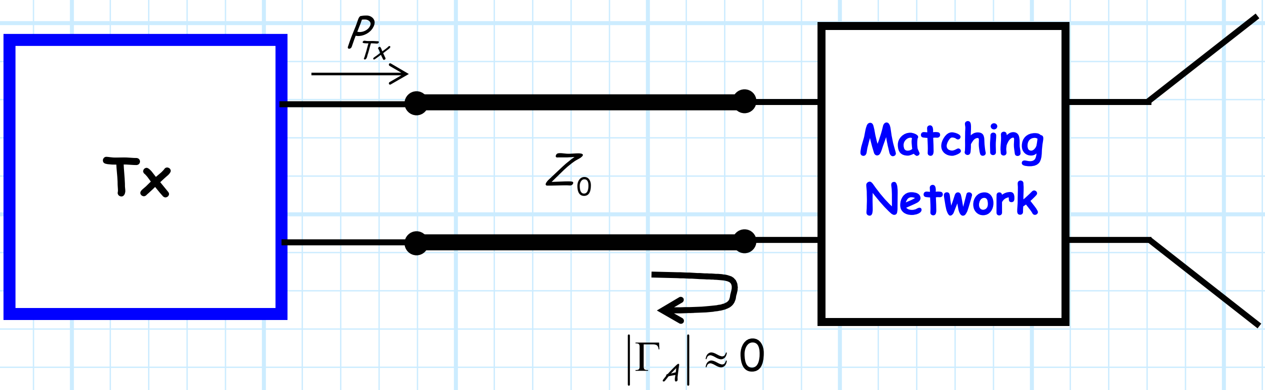
$$Z_A = R_A + jX_A$$

Thus, we find antenna impedance—like **all** other antenna parameters—is **frequency dependent**.

Q: *So how do we eliminate (or at least **minimize**) the **reflected** power??*

A1: Design the antenna such that $R_A = Z_0$ (e.g., 50Ω, 75Ω and then operate at a frequency ω such that $X_A = 0$.

A2: Implement a **matching network**!



Radiation Resistance

Q: *Does **all** the power **absorbed** by R_A get **radiated** (i.e., is P_{rad} equal to P_A)?*

A: Generally speaking, **no**!

Remember, there were **two** reasons why radiated power P_{rad} is less than the available transmitter power P_{Tx}.

1. Power is **reflected** at the antenna.

2. Power is turned to heat in a **lossy** antenna.

From the first reason we have **already** determined that:

$$P_A = P_{Tx}\left(1-\left|\Gamma_A\right|^2\right)$$

But because of the **second** reason we find that:

$$P_A < P_{rad}$$

Ideally, all of the power delivered to the antenna (P_A) is radiated ($P_{rad} = P_A$). However, antennas are made of materials with **finite** conductivity. Therefore they exhibit **Ohmic losses**!

In other words, **most** of the absorbed power is radiated, but **some** of the absorbed power is converted to **heat**.

Thus, we find absorbed power consists of **two components**:

$$P_A = P_L + P_{rad}$$

where:

$$P_A = \text{Power delivered to the antenna}$$

$$P_L = \text{Power converted to heat}$$

$$P_{rad} = \text{Radiated Power}$$

Now, the power delivered to the antenna is the power absorbed by the antenna resistance R_A. We can likewise divide this resistance into **two components**:

$$R_A = R_L + R_{rad}$$

so that:

$$Z_A = R_L + R_{rad} + jX_A$$

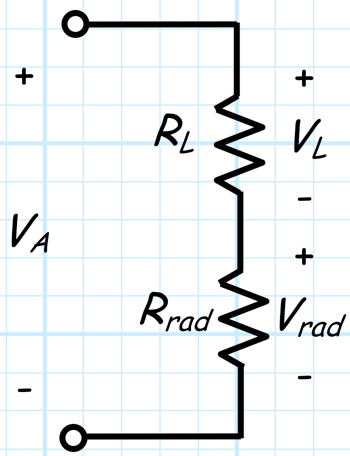
where:

$$R_L \doteq \text{Ohmic Loss Resistance}$$

$$R_{rad} \doteq \text{Radiation Resistance}$$

* The **radiation resistance** is defined such that **radiated** power is equal to the power **absorbed** by R_{rad}.

* The **Ohmic loss resistance** is defined such that the power converted to **heat** is equal to the power **absorbed** by R_L.



Using our basic circuit theory we find:

$$P_A = \frac{|V_A|^2}{2R_A} = \frac{1}{2}\frac{|V_A|^2}{(R_L + R_{rad})}$$

$$P_L = \frac{|V_L|^2}{2R_L} \qquad P_{rad} = \frac{|V_{rad}|^2}{2R_{rad}}$$

And from KCL:

$$V_L = V_A \frac{R_L}{R_L + R_{rad}} = V_A \frac{R_L}{R_A} \qquad V_{rad} = V_A \frac{R_{rad}}{R_L + R_{rad}} = V_A \frac{R_{rad}}{R_A}$$

Combining the above:

$$P_L = \frac{|V_L|^2}{2R_L} = \frac{|V_A|^2}{2R_L}\left(\frac{R_L}{R_A}\right)^2 = \frac{|V_A|^2}{2R_A}\frac{R_L}{R_A} = P_A \frac{R_L}{R_A}$$

$$P_{rad} = \frac{|V_{rad}|^2}{2R_{rad}} = \frac{|V_A|^2}{2R_{rad}}\left(\frac{R_{rad}}{R_A}\right)^2 = \frac{|V_A|^2}{2R_A}\frac{R_{rad}}{R_A} = P_A \frac{R_{rad}}{R_A}$$

Note then, as **expected**:

$$
\begin{aligned}
P_L + P_{rad} &= P_A \frac{R_L}{R_A} + P_A \frac{R_{rad}}{R_A} \\
&= P_A \left(\frac{R_L}{R_A} + \frac{R_{rad}}{R_A} \right) \\
&= P_A \left(\frac{R_L + R_{rad}}{R_A} \right) \\
&= P_A \left(\frac{R_A}{R_A} \right) \\
&= P_A
\end{aligned}
$$

Thus, rearranging the above results, we can determine resisitances R_L and R_{rad}:

$$
P_L = P_A \left(\frac{R_L}{R_A} \right) \qquad \Rightarrow \qquad R_L = R_A \left(\frac{P_L}{P_A} \right)
$$

$$
P_{rad} = P_A \left(\frac{R_{rad}}{R_A} \right) \qquad \Rightarrow \qquad R_{rad} = R_A \left(\frac{P_{rad}}{P_A} \right)
$$

Now, we define **antenna efficiency** as:

$$
e = \frac{P_{rad}}{P_A} = \text{antenna efficiency}
$$

* Note then if $e = 1$, then $P_{rad} = P_A$ and so $P_L = 0$. We say this antenna is **100% efficient**.

* And if $e = 0\ e = 1$, then $P_{rad} = 0$ and so $P_L = P_A$. We say this antenna is **0% efficient**.

We likewise find we can write the important antenna parameters in terms of this efficiency:

$$P_{rad} = e\, P_A \qquad\qquad R_{rad} = e\, R_A$$

$$P_L = P_A\,(1 - e) \qquad\qquad R_L = (1 - e)\, R_A$$

So, in **summary**:

$$P_A = \left(1 - |\Gamma_A|^2\right) P_{Tx}$$

$$P_{rad} = e\, P_A$$

$$P_{rad} = e\left(1 - |\Gamma_A|^2\right) P_{Tx}$$

Antenna Gain

Recall that the **directivity pattern** of an antenna is:

$$D(\theta,\phi) = \frac{4\pi\, U(\theta,\phi)}{P_{rad}}$$

The problem with this definition is in **determining** (measuring) the radiated power P_{rad}. Recall that it was ideally found by **integrating** the antenna intensity pattern across **all directions**:

$$P_{rad} = \int_0^{2\pi}\int_0^{\pi} U(\theta,\phi)\, \sin\theta\, d\theta\, d\phi$$

Yuck!

A far **easier** measurement is determining the power **delivered** to the antenna (P_A). This is just a simple **transmission line** problem (i.e., no integration)!

$$P_A = P_{Tx}\left(1 - \left|\Gamma_A\right|^2\right)$$

For perfectly efficient antenna, we know $P_{rad} = P_A$, and so **if** (and **only** if) the antenna is **perfectly efficient**:

$$D(\theta,\phi) = \frac{4\pi\, U(\theta,\phi)}{P_A} \quad \text{iff } e=1$$

But, for **inefficient** antenna $(P_{rad} < P_A)$ we find:

$$D(\theta,\phi) > \frac{4\pi\, U(\theta,\phi)}{P_A} \qquad \text{for } e<1$$

Specifically, since $(P_{rad} = e\, P_A)$, we find:

$$D(\theta,\phi) = \frac{4\pi\, U(\theta,\phi)}{P_{rad}}$$

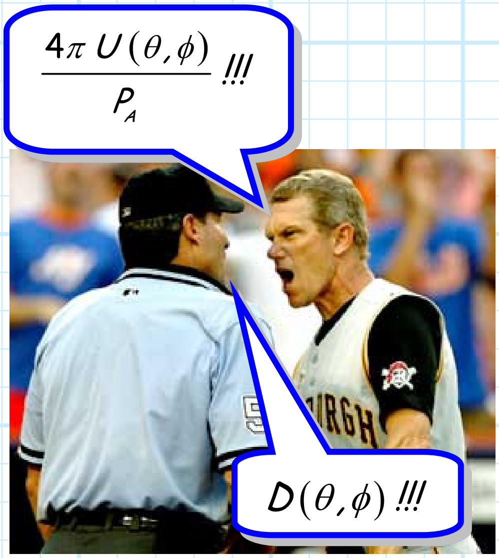
$$D(\theta,\phi) = \frac{4\pi\, U(\theta,\phi)}{e\, P_A}$$

$$e\, D(\theta,\phi) = \frac{4\pi\, U(\theta,\phi)}{P_A}$$

Therefore, the function:

$$\frac{4\pi\, U(\theta,\phi)}{P_A}$$

is one that **combines** the antenna **directivity** pattern $D(\theta,\phi)$ and the antenna **efficiency** e.



We might **argue** that **this** function is even **more useful** than the directivity pattern $D(\theta,\phi)$, as it would allow us to **directly** relate the power **delivered** to the antenna P_A to the **intensity** produced by the antenna—while taking into account its **inefficiency** (Ohmic losses)!

As a result we give this important function a name—the **gain pattern** $G(\theta,\phi)$:

$$G(\theta,\phi) = \frac{4\pi\, U(\theta,\phi)}{P_A} = e\, D(\theta,\phi)$$

Note then that the pattern $G(\theta,\phi)$ is essentially the **same** pattern as $D(\theta,\phi)$ only its **scaled** by value *e*. Or, in **decibels** we find:

$$G(\theta,\phi)[dB] = D(\theta,\phi)[dB] + 10\, log_{10}(e)$$

Recall that $e < 1$, so that the value $10\, log_{10}(e)$ will be **negative.** As a result, the **gain** pattern expressed in decibels will simply be that of the **directivity** pattern, only "**shifted down**" by a value $10\, log_{10}(e)$.

Either way, we can conclude:

$$e = \frac{G(\theta,\phi)}{D(\theta,\phi)} \qquad e[dB] = G(\theta,\phi)[dB] - D(\theta,\phi)[dB]$$

and likewise since $e < 1$, we see that the gain pattern will be less than the directivity pattern:

$$G(\theta,\phi) < D(\theta,\phi) \qquad G(\theta,\phi)[dB] < D(\theta,\phi)[dB]$$

Finally, we recall that the **peak** of the directivity pattern is a fundamental antenna parameter called **Directivity** D_0. We can now define an **equivalent** parameter called **Antenna Gain** G_0, which is simply the Directivity modified by the efficiency e:

$$G_0 = e\, D_0$$

Note then that **Gain** G_0 is equal to the **peak** value of gain pattern $G(\theta,\phi)$.

Q: *So if **gain** and gain pattern is a) **easier** to determine and b) more **useful**, why do we even **bother** with directivity and directivity pattern?*

A: Recall there were some explicit mathematical and physical **equalities** that we derived for the directivity pattern, for example:

$$\frac{1}{4\pi}\int_0^{2\pi}\int_0^{\pi} D(\theta,\phi)\, \sin\theta\, d\theta\, d\phi = 1.0$$

This says that the average value of the directivity pattern **must** be **precisely** 1.0. From this we were able to conclude the useful relationship:

$$D_0\, \Omega_A = 4\pi$$

But for gain, we can **only** conclude:

$$\frac{1}{4\pi}\int_0^{2\pi}\int_0^{\pi} G(\theta,\phi)\, \sin\theta \, d\theta \, d\phi < 1.0$$

from which we ascertain the less than helpful **inequality**:

$$G_0\, \Omega_A < 4\pi$$

Thus, **both** gain and directivity are **important** and useful antenna parameters!

Note however, that many (most) antennas are **very efficient** (e.g., $e > 0.9$). As a result, we find that:

$$G_0 \approx D_0 \quad \text{and} \quad G(\theta,\phi) \approx D(\theta,\phi) \quad \text{if} \quad e \approx 1$$

In other words, for highly **efficient** antennas, the gain and directivity are **nearly** the same, and **terms** gain and directivity are commonly used **interchangeably**.