C. Antenna Pattern

Radiation **Intensity** is dependent on **both** the antenna and the radiated power. We can **normalize** the Radiation Intensity function to construct a result that describes the **antenna only**. We call this normalized function the **Antenna Directivity Pattern.**

HO: Antenna Directivity

The antenna directivity function essentially describes the **antenna pattern**, from which we can ascertain fundamental antenna parameters such as (maximum) **Directivity**, **beamwidth**, and **sidelobe** level.

HO: The Antenna Pattern

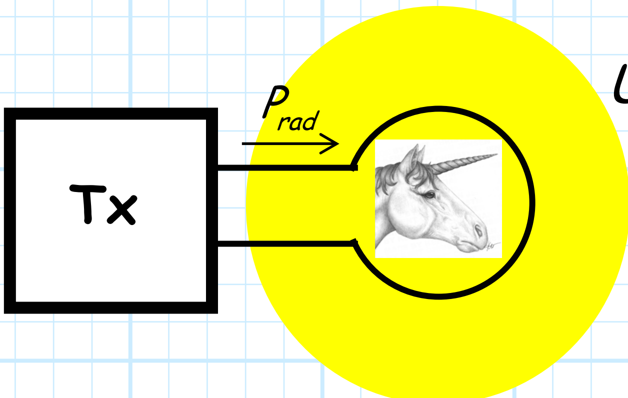
We find that conservation of energy requires a **tradeoff** between antenna (maximum) **directivity** and **beamwidth**—we increase one, we decrease the other.

HO: Beamwidth and Directivity

Antenna Directivity

Recall the **intensity** of the E.M. wave produced by the mythical **isotropic** radiator (i.e., an antenna that radiates **equally** in all directions) is:

$$U_0 = \frac{P_{rad}}{4\pi}$$

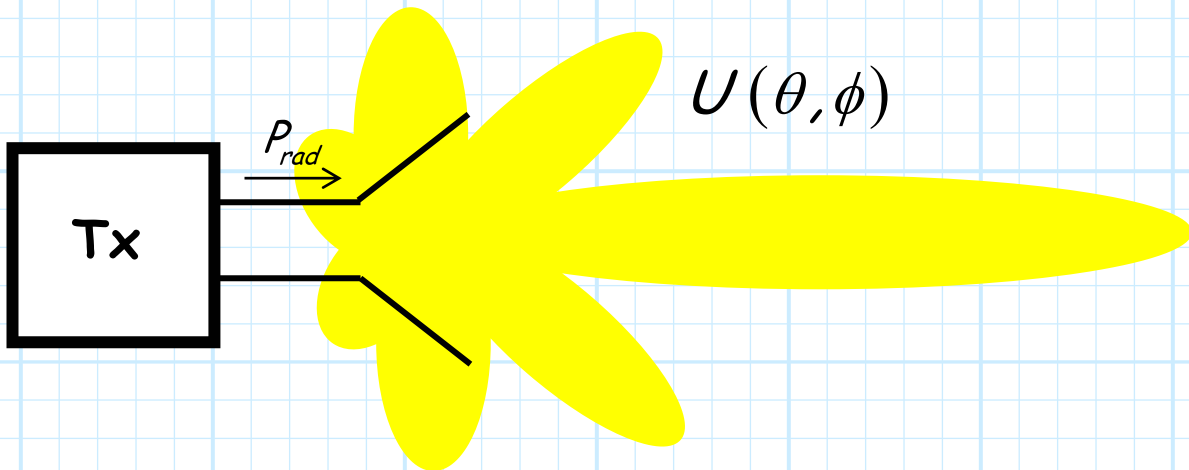


$$U(\theta,\phi) = U_0$$

But remember, and isotropic radiator is actually a physical **impossibility**!

If the electromagnetic energy is **monochromatic**—that is, it is a sinusoidal function of time, oscillating at a **one** specific frequency ω—then an antenna **cannot** distribute energy uniformly in all directions.

The intensity function $U(\theta,\phi)$ thus describes this **uneven** distribution of radiated power as a function of direction, a function that is dependent on the design and construction of the **antenna** itself.



Q: *But doesn't the radiation intensity **also** depend on the power delivered to the antenna by **transmitter**?*

A: That's right! **If** the transmitter delivers **no power** to the antenna, then the resulting radiation intensity will likewise be **zero** (i.e., $U(\theta,\phi)=0$).

Q: *So is there some way to **remove** this dependence on the transmitter power? Is there some function that is dependent on the antenna **only**, and thus describes **antenna behavior** only?*

A: There sure is, and a **very important** function at that!

Will call this function $D(\theta,\phi)$—the **directivity pattern** of the antenna.

The directivity pattern is simply a **normalized** intensity function. It is the intensity function produce by an **antenna** and transmitter, normalized to the intensity pattern produced when the **same** transmitter is connected to an **isotropic** radiator.

$$D(\theta,\phi) = \frac{U(\theta,\phi)}{U_0} = \frac{\text{intensity of antenna}}{\text{intensity of isotropic radiator}}$$

Using $U_0 = P_{rad}/4\pi$, we can likewise express the directivity pattern as:

$$D(\theta,\phi) = \frac{4\pi\, U(\theta,\phi)}{P_{rad}}$$

Q: *Hey wait! I thought that this function was supposed to* ***remove*** *the dependence on transmitter power, but there is P_{rad} sitting* ***smack dab*** *in the middle of the* ***denominator****.*

A: The value P_{rad} in the denominator is necessary to **normalize** the function. The reason of course is that $U(\theta,\phi)$ (in the **numerator**) is **likewise** proportional to the radiated power.

In other words, if P_{rad} doubles then **both** numerator and denominator increases by a factor of two—thus, the **ratio** remains **unchanged**, independent of the value P_{rad}.

$$D(\theta,\phi) = \frac{U(\theta,\phi)}{U_0}$$

Another indication that directivity pattern $D(\theta,\phi)$ is independent of the transmitter power are it **units**. Note that the directivity pattern is a **coefficient**—it is unitless!

Perhaps we can rearrange the above expression to make this all more clear:

Dependent on antenna **only**.

$$U(\theta,\phi) = \frac{P_{rad}}{4\pi} D(\theta,\phi)$$

Dependent on Tx power **and** the antenna.

Dependent on Tx power

Hopefully it is apparent that the value of this function $D(\theta,\phi)$ in some direction θ and ϕ describes the intensity in that direction **relative** to that of an isotropic radiator (when radiating the same power P_{rad}).

For **example**, if $D(\theta,\phi) = 10$ in some direction, then the intensity in that direction is **10 times** that produced by an isotropic radiator in that direction.

If in another direction we find $D(\theta,\phi) = 0.5$, we conclude that the intensity in that direction is **half** the value we would find if an isotropic radiator is used.

Q: *So, can the directivity function take* ***any*** *form? Are there any* ***restrictions*** *on the function* $D(\theta,\phi)$*?*

A: Absolutely! For example, let's **integrate** the directivity function over **all directions** (i.e., over 4π steradians).

$$\int_0^{2\pi}\int_0^{\pi} D(\theta,\phi)\, \sin\theta\, d\theta\, d\phi = \int_0^{2\pi}\int_0^{\pi} \frac{U(\theta,\phi)}{U_0}\, \sin\theta\, d\theta\, d\phi$$

$$= \frac{1}{U_0}\int_0^{2\pi}\int_0^{\pi} U(\theta,\phi)\, \sin\theta\, d\theta\, d\phi$$

$$= \frac{4\pi}{P_{rad}}\int_0^{2\pi}\int_0^{\pi} U(\theta,\phi)\, \sin\theta\, d\theta\, d\phi$$

$$= \frac{4\pi}{P_{rad}}\left(P_{rad}\right)$$

$$= 4\pi$$

Thus, we find that the directivity pattern $D(\theta,\phi)$ of **any** and **all** antenna must satisfy the equation:

$$\int_0^{2\pi}\int_0^{\pi} D(\theta,\phi)\, \sin\theta\, d\theta\, d\phi = 4\pi$$

We can slightly **rearrange** this integral to find:

$$\boxed{\frac{1}{4\pi}\int_0^{2\pi}\int_0^{\pi} D(\theta,\phi)\, \sin\theta\, d\theta\, d\phi = 1.0}$$

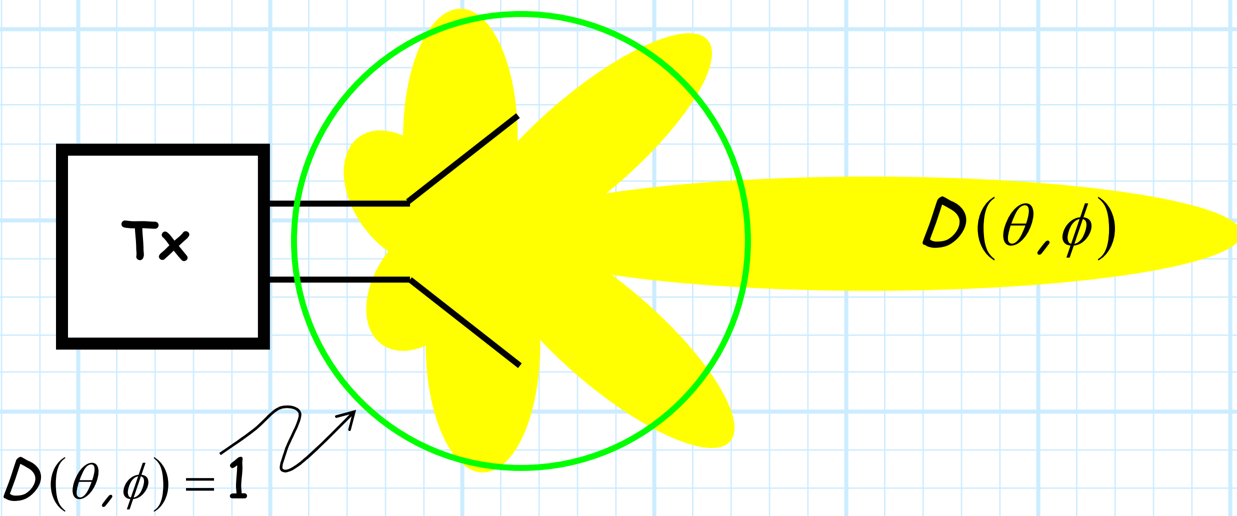
The left side of the equation is simply the **average** value of the directivity pattern (D_{ave}), when averaged over **all directions**—over **4π steradians**!

The equation thus says that the **average** directivity of **any** and **all** antenna **must** be equal to **one**.

$$D_{ave} = 1.0$$

This means that—on average—the intensity created by an **antenna** will equal the intensity created by an **isotropic radiator**.

→ In some directions the intensity created by any and all antenna will be **greater** than that of an isotropic radiator (i.e., $D > 1$), while in other directions the intensity will be **less** than that of an isotropic resonator(i.e., $D < 1$).



Q: *Can the directivity pattern $D(\theta,\phi)$ equal one for **all directions** θ and ϕ? Can the directivity pattern be the **constant** function $D(\theta,\phi) = 1.0$?*

A: Nope! The directivity function **cannot** be isotropic.

In other words, since:

$$U(\theta,\phi) \neq U_0$$

then:

$$U(\theta,\phi) \neq U_0 \quad \Rightarrow \quad \frac{U(\theta,\phi)}{U_0} \neq \frac{U_0}{U_0} \quad \Rightarrow \quad D(\theta,\phi) \neq 1.0$$

Q: *Does this mean that there is* ***no*** *value of θ and ϕ for which $D(\theta,\phi)$ will equal 1.0?*

A: NO! There will be **many** values of θ and ϕ (i.e., directions) where the value of the directivity function will be equal to **one**!

Instead, when we say that:

$$D(\theta,\phi) \neq 1.0$$

we mean that the directivity function **cannot** be a **constant** (with value 1.0) with respect to θ and ϕ.

The Antenna Pattern

Another term for the directivity pattern $D(\theta,\phi)$ is the **antenna pattern**. Again, this function describes how a specific antenna distributes energy as a function of direction.

An **example** of this function is:

$$D(\theta,\phi) = c\,(1+\cos\phi)^2 \sin^2\theta$$

where c is a constant that **must** be equal to:

$$c = \frac{4\pi}{\int_0^{2\pi}\int_0^{\pi}(1+\cos\phi)^2 \sin^2\theta \sin\theta \, d\theta \, d\phi}$$

Do **you** see why c must be equal to this value?

Q: *How can we **determine** the antenna pattern of given antenna? How do we **find** the explicit form of the function $D(\theta,\phi)$?*

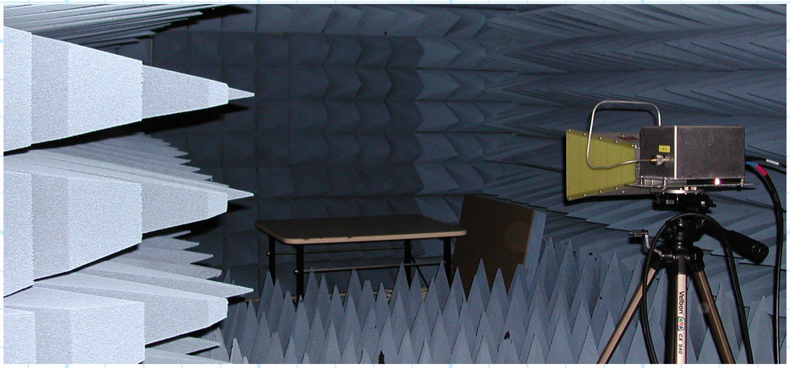
A: There are **two ways** of determining the pattern of a given antenna

1. By electromagnetic analysis - Given the size, shape, structure, and material parameters of an antenna, we can use Maxwell's equations to determine the **function** $D(\theta,\phi)$.

However, this analysis often must resort to **approximations** or assumption of ideal conditions that can lead to some **error**.

$$
\begin{aligned}
&\mathbf{E}^{\mathbf{s}}_{\mathbf{n}}(\bar{r}) = \\
&\quad \frac{(\epsilon_r - 1)}{4\pi} f(z_n) \int_{-k\Delta\ell}^{k\Delta\ell} \int_0^{2\pi} \int_0^{ka} \frac{\hat{x}2}{(\epsilon_r + 1)} \frac{e^{\imath\sqrt{k\rho'^2 + (ku - k\delta)^2}}}{\sqrt{k\rho'^2 + (ku - k\delta)^2}} \, dk\rho' \, d\phi' \, dk\delta' \\
&\quad -\imath \frac{(\epsilon_r - 1)}{4\pi} f(z_n) \int_{-k\Delta\ell}^{k\Delta\ell} \int_0^{2\pi} (\cos\phi'\hat{x} + \sin\phi'\hat{y}) \cdot \frac{\hat{x}2}{(\epsilon_r + 1)} \frac{e^{\imath\sqrt{k\rho'^2 + (ku - k\delta)^2}}}{(\sqrt{k\rho'^2 + (ku - k\delta)^2})^2} \\
&\quad \Big(- ka\cos\phi'\hat{x} - ka\sin\phi'\hat{y} + (ku - k\delta')\hat{z} \Big) \, ka \, d\phi' \, dk\delta' \\
&\quad + \frac{(\epsilon_r - 1)}{4\pi} f(z_n) \int_{-k\Delta\ell}^{k\Delta\ell} \int_0^{2\pi} (\cos\phi'\hat{x} + \sin\phi'\hat{y}) \cdot \frac{\hat{x}2}{(\epsilon_r + 1)} \frac{e^{\imath\sqrt{k\rho'^2 + (ku - k\delta)^3}}}{(\sqrt{k\rho'^2 + (ku - k\delta)^2})^3} \\
&\quad \Big(- ka\cos\phi'\hat{x} - ka\sin\phi'\hat{y} + (ku - k\delta')\hat{z} \Big) \, ka \, d\phi' \, dk\delta'
\end{aligned}
$$

2. By direct measurement - We can directly measure the antenna pattern in the laboratory. This has the advantage that it requires **no** assumptions or approximations, so it **may** be more accurate.

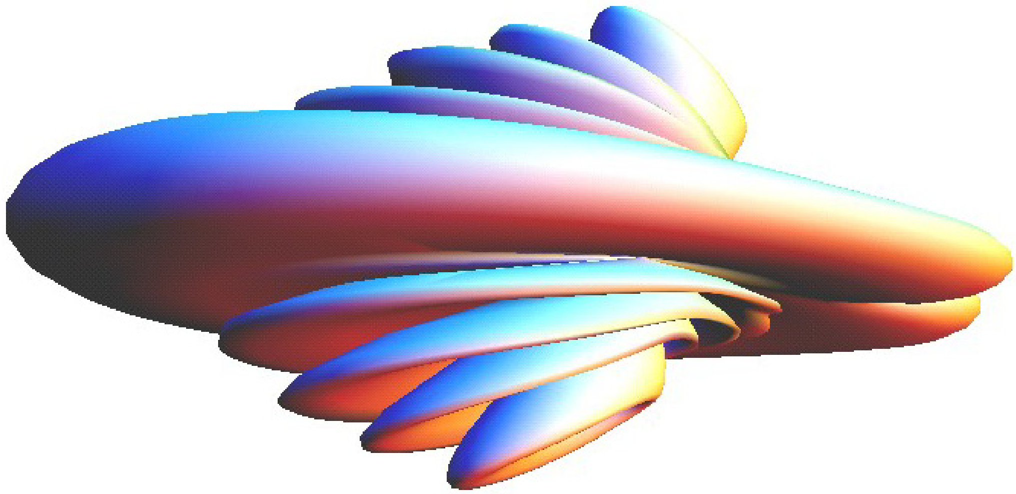


However, accuracy ultimately depends on the **precision** of your measurements, and the result $D(\theta,\phi)$ is provided as a **table** of measured data, as opposed to an explicitly mathematical function.

Q: *Functions and tables!? Isn't there some way to simply plot the antenna pattern $D(\theta,\phi)$?*

A: Yes, but there it is a bit **tricky**.

Remember, the function $D(\theta,\phi)$ describes how an antenna distributes energy in **three** dimensions. As a result, it is difficult to plot this function on a **two-dimensional** sheet (e.g., a page of your notes!).

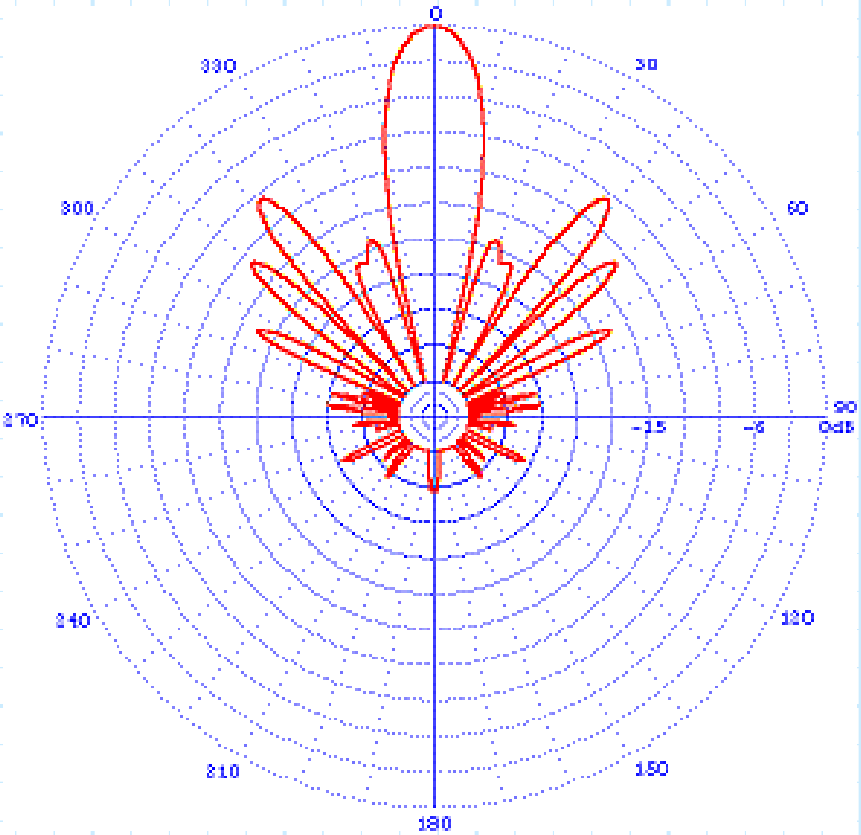


Antenna patterns are thus typically plotted as **"cuts"** in the antenna pattern—the value of $D(\theta,\phi)$ on a (two-dimensional) **plane**.

* For **example**, we might plot $D(\theta = 90^\circ,\phi)$ as a function of ϕ. This would be a plot of $D(\theta,\phi)$ on the ***x-y*** **plane**.

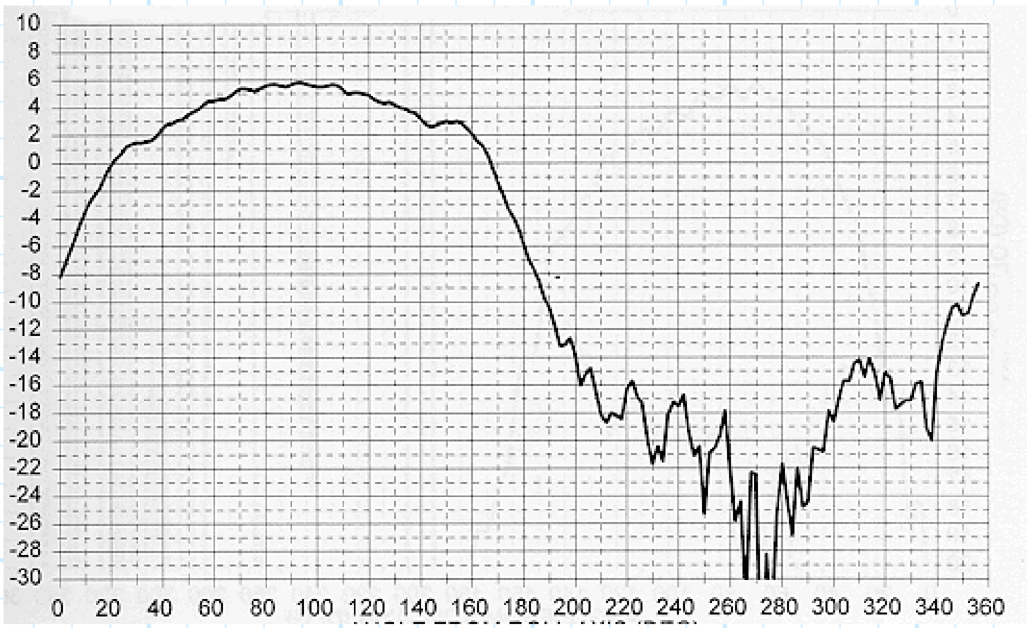
* Or, we might plot $D(\theta,\phi = 0)$ as a function of θ. This would be a plot of $D(\theta,\phi)$ along the ***x-z*** **plane**.

Sometimes these cuts are plotted in **polar** format, and other times in **Cartesian**.

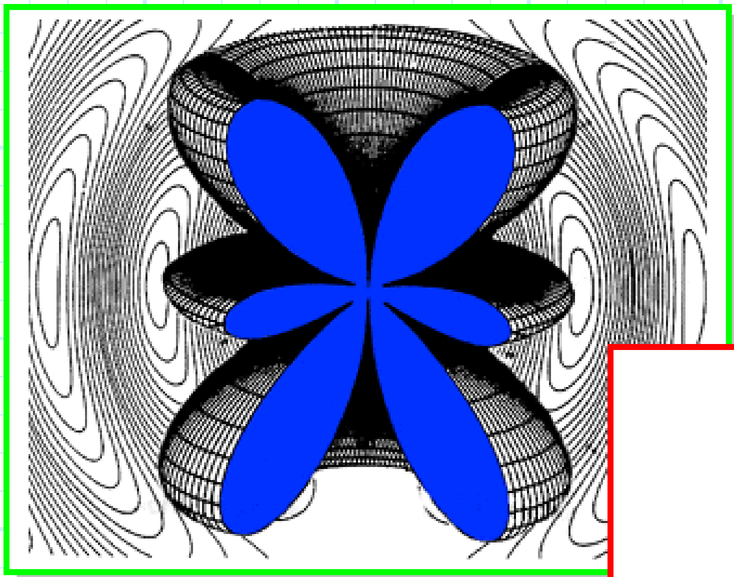


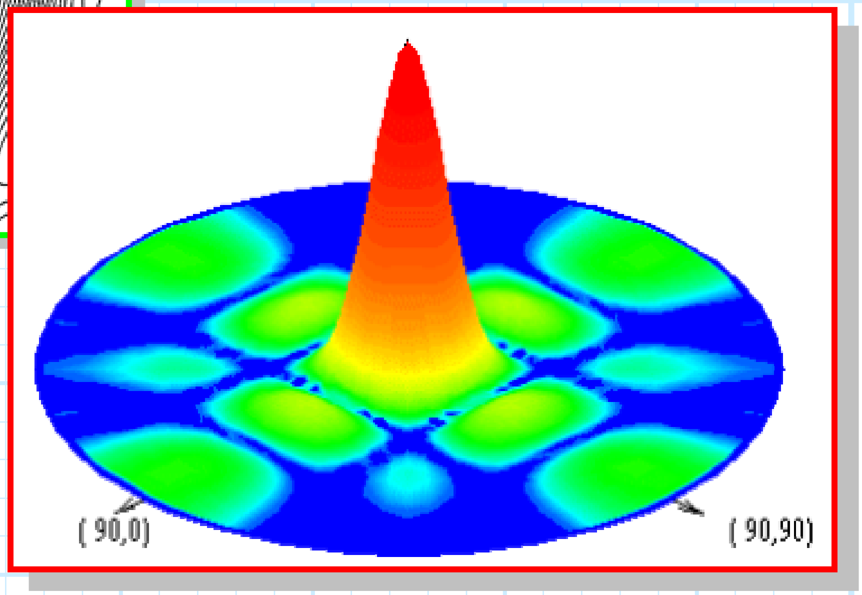
Polar plot of antenna cut $D\left(\theta = \pi/2, \phi\right)$ as a function of ϕ.

Cartesian plot of antenna cut $D\left(\theta = \pi/2, \phi\right)$ as a function of ϕ.

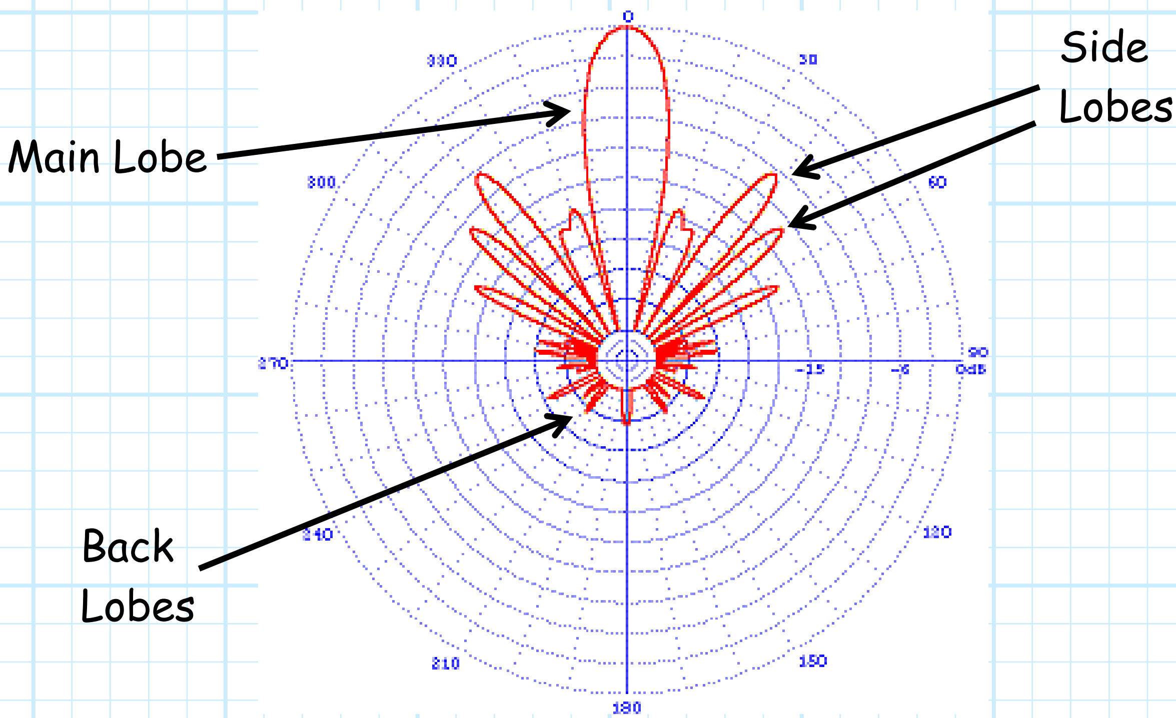


The entire function $D(\theta,\phi)$ can likewise be plotted in **3-D** for either polar or Cartesian (**if** you have the proper software!).





Note that the majority of antenna patterns consist of a number of "**lobes**".



Note these lobes have both a **magnitude** (the largest value of $D(\theta,\phi)$ within the lobe), and a **width** (the size of the lobe in steradians).

* Note that every antenna pattern has a direction(s) where the function $D(\theta,\phi)$ is at its **peak** value. The lobe associated with this peak value (i.e., the lobe with the largest magnitude) is known as the antennas **Main Lobe.**

* The main lobe is typically surrounded by **smaller** (but significant) lobes called **Side Lobes**.

* There frequently are also **very small** lobes that appear in the pattern, usually in the opposite direction of the main lobe. We call these tiny lobes **Back Lobes**.

The important characteristics of an antenna are defined by the **main lobe**. Generally, side and back lobes are **nuisance** lobes—we ideally want them to be as **small as possible**!

Q: *These plots and functions describing antenna pattern $D(\theta,\phi)$ are very **complete** and helpful, but also a bit busy and complex. Are there some **set of values** that can be used to indicate the important **characteristics** of an antenna pattern?*

A: Yes there is! The **three** most important are:

1. Antenna Directivity D_0 .
2. Antenna Beamwidth .
3. Antenna Sidelobe level.