Example: Scattering Parameters

Consider a two-port device with a scattering matrix (at some specific frequency ω_0):

$$\bar{\bar{\mathbf{S}}}(\omega=\omega_0)=\begin{bmatrix}0.1 & j0.7\\ j0.7 & -0.2\end{bmatrix}$$

and $Z_0 = 50\Omega$.

Say that the transmission line connected to port 2 of this device is terminated in a matched load, and that the wave incident on port 1 is:

$$V_1^+(z_1) = -j2\, e^{-j\beta z_1}$$

where $z_{1P} = z_{2P} = 0$.

Determine:

1. the port voltages $V_1(z_1 = z_{1P})$ and $V_2(z_2 = z_{2P})$.
2. the port currents $I_1(z_1 = z_{1P})$ and $I_2(z_2 = z_{2P})$.
3. the net power flowing into port 1

4. the net power flowing into port 2

5. if the device is lossy, lossless, or active.

1. Since the incident wave on port 1 is:

$$V_1^+(z_1) = -j2\, e^{-j\beta z_1}$$

we can conclude (since $z_{1P} = 0$):

$$\begin{aligned} V_1^+(z_1 = z_{1P}) &= -j2\, e^{-j\beta z_{1P}} \\ &= -j2\, e^{-j\beta(0)} \\ &= -j2 \end{aligned}$$

and since port 2 is matched, we find:

$$\begin{aligned} V_1^-(z_1 = z_{1P}) &= S_{11}\, V_1^+(z_1 = z_{1P}) \\ &= 0.1\,(-j2) \\ &= -j0.2 \end{aligned}$$

The voltage at port 1 is thus:

$$\begin{aligned} V_1(z_1 = z_{1P}) &= V_1^+(z_1 = z_{1P}) + V_1^-(z_1 = z_{1P}) \\ &= -j2.0 - j0.2 \\ &= -j2.2 \\ &= 2.2\, e^{-j\pi/2} \end{aligned}$$

Likewise, since port 2 is matched:

$$V_2^+(z_2 = z_{2P}) = 0$$

And also:

$$\begin{aligned} V_2^-(z_2 = z_{2P}) &= S_{21}\, V_1^+(z_1 = z_{1P}) \\ &= j0.7\,(-j2) \\ &= 1.4 \end{aligned}$$

Therefore:

$$\begin{aligned} V_2(z_2 = z_{2P}) &= V_2^+(z_2 = z_{2P}) + V_2^-(z_2 = z_{2P}) \\ &= 0 + 1.4 \\ &= 1.4 \\ &= 1.4\, e^{-j0} \end{aligned}$$

2. The port currents can be easily determined from the results of the previous section.

$$\begin{aligned} I_1(z_1 = z_{1P}) &= I_1^+(z_1 = z_{1P}) - I_1^-(z_1 = z_{1P}) \\ &= \frac{V_1^+(z_1 = z_{1P})}{Z_0} - \frac{V_1^-(z_1 = z_{1P})}{Z_0} \\ &= -j\,\frac{2.0}{50} + j\,\frac{0.2}{50} \\ &= -j\,\frac{1.8}{50} \\ &= -j0.036 \\ &= 0.036\, e^{-j\pi/2} \end{aligned}$$

and:

$$\begin{aligned} I_2(z_2 = z_{2P}) &= I_2^+(z_2 = z_{2P}) - I_2^-(z_2 = z_{2P}) \\ &= \frac{V_2^+(z_2 = z_{2P})}{Z_0} - \frac{V_2^-(z_2 = z_{2P})}{Z_0} \\ &= \frac{0}{50} - \frac{1.4}{50} \\ &= -0.028 \\ &= 0.028\, e^{+j\pi} \end{aligned}$$

3. The net power flowing into port 1 is:

$$\begin{aligned} \Delta P_1 &= P_1^+ - P_1^- \\ &= \frac{|V_{01}^+|^2}{2Z_0} - \frac{|V_{01}^-|^2}{2Z_0} \\ &= \frac{(2)^2 - (0.2)^2}{2(50)} \\ &= 0.0396 \quad \text{Watts} \end{aligned}$$

4. The net power flowing into port 2 is:

$$\begin{aligned} \Delta P_2 &= P_2^+ - P_2^- \\ &= \frac{|V_{02}^+|^2}{2Z_0} - \frac{|V_{02}^-|^2}{2Z_0} \\ &= \frac{(0)^2 - (1.4)^2}{2(50)} \\ &= -0.0196 \quad \text{Watts} \end{aligned}$$

Note this negative value means that 0.0196 Watts of power is flowing **out** of port 2!

5. The total net power flow into the device is:

$$\begin{aligned}\sum_{n=1}^{2} \Delta P_n &= \Delta P_1 + \Delta P_2 \\ &= 0.0396 - 0.0196 \\ &= 0.02 \quad \textit{Watts}\end{aligned}$$

Thus, this device is **lossy**. For this case, it absorbs power at a rate of 0.02 Watts!