E. Received Power

In addition to using an antenna as a **transmitter**, we of course can likewise use it as a **receiver**.

The important parameter for describing an antenna's behavior on receive is its **effective aperture**.

HO: Effective Aperture

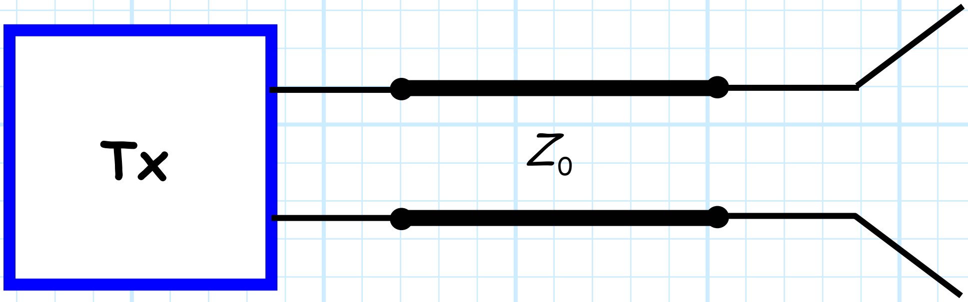
Q: *So an antenna has **both** gain and effective aperture. Are these values in any way **related**?*

A: They are **completely** and directly related. If we know one, then we know the other!

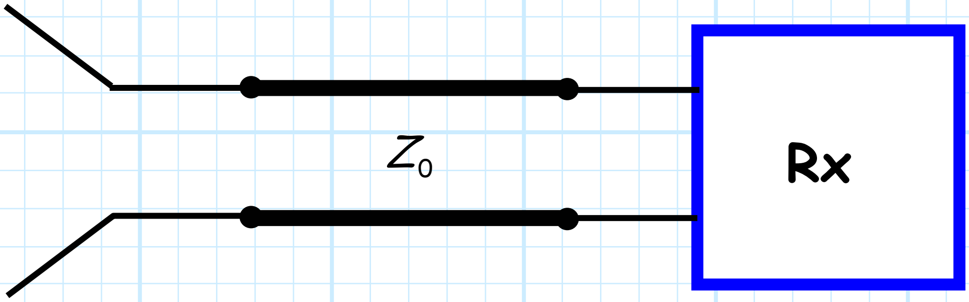
HO: Gain and Effective Aperture

Effective Aperture

Q: *We keep discussing an antenna as a **radiating** device, examining the situation when it is connected to the output of a **transmitter**.*



*But we likewise know that an antenna is used at the input of a **receiver**. This receiver antenna is **not** radiating. When are we going to learn about **receive antennas**?*

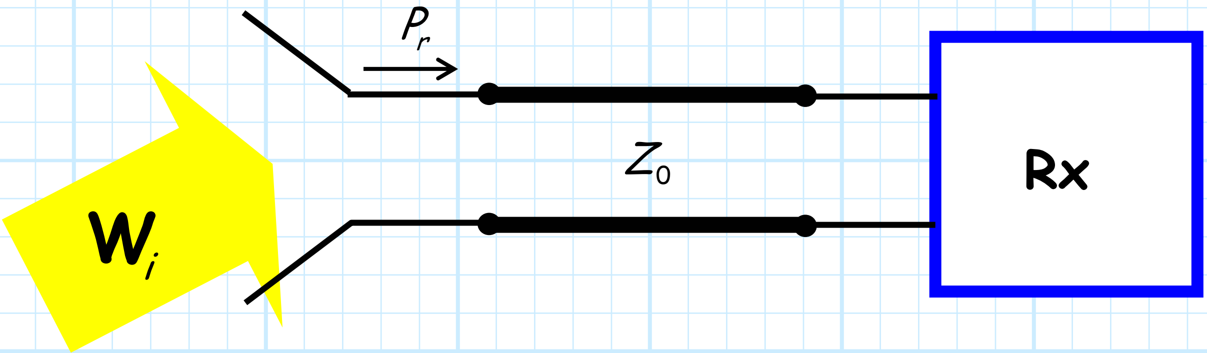


A: Right **now**!

The receive antenna acts as a device for **coupling** a propagating electromagnetic wave into the receiver input. The **two** important quantities are therefore:

1. W_i - The **power density** of the incident electromagnetic wave (assumed to be a constant in the region of the antenna).

2. P_r - The **available power** at the output of the antenna.



As we might expect, the available power P_r is **proportional** to the magnitude of the incident power density ($|\mathbf{W}_i|$):

$$P_r \propto |\mathbf{W}_i|$$

The proportionality "constant" is denoted as A_e, meaning:

$$\boxed{P_r = A_e |\mathbf{W}_i|}$$

and:

$$\boxed{A_e \doteq \frac{P_r}{|\mathbf{W}_i|} \qquad [m^2]}$$

Q: *Just what* ***is*** *the value A_e? Does it have any* ***physical*** *meaning?*

A: Look at the **units** associated with A_e:

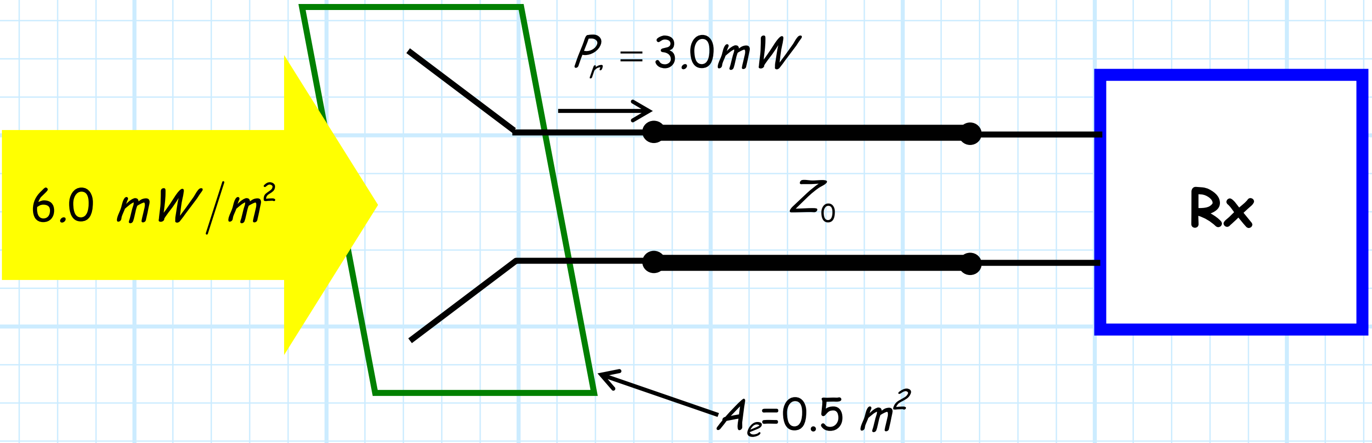
$$\frac{P_r}{|\mathbf{W}_i|} = A_e \qquad \Rightarrow \qquad \frac{Watts}{\left(W/m^2\right)} = Watts \cdot \frac{meters^2}{Watts} = meters^2$$

Q: *Square meters? That's the units of **surface area**!*

A: That's correct! The value A_e is know as the effective **aperture** of the antenna. It represents the effective size of the "**energy collecting window**" of the antenna. The **larger** the effective aperture the **more** energy will be "collected" (coupled into the receiver) from the incident electromagnetic wave

For **example**, the say that an e.m wave incident on an antenna has a power density of $|\mathbf{W}_i| = 6.0 \text{ mW/m}^2$. Some of this wave is coupled into the receiver, producing an available power at the antenna output of $P_r = 3.0\, mW$.

It is apparent that the effective collecting aperture of this antenna is 0.5 m^2.



Q: *What **determines** the size (i.e., value) of this effective aperture A_e? Is it related the **physical** size of the antenna?*

A: Certainly there is a **correlation** between the physical size of the antenna and the effective collecting aperture A_e—the **bigger** the antenna, the **bigger** the effective aperture!

However, it is generally **not** possible to **directly infer** the effective aperture of and antenna from its physical size (although there are some important **exceptions** to this!).

The effective collecting aperture of an antenna is instead dependent on a **number** of factors:

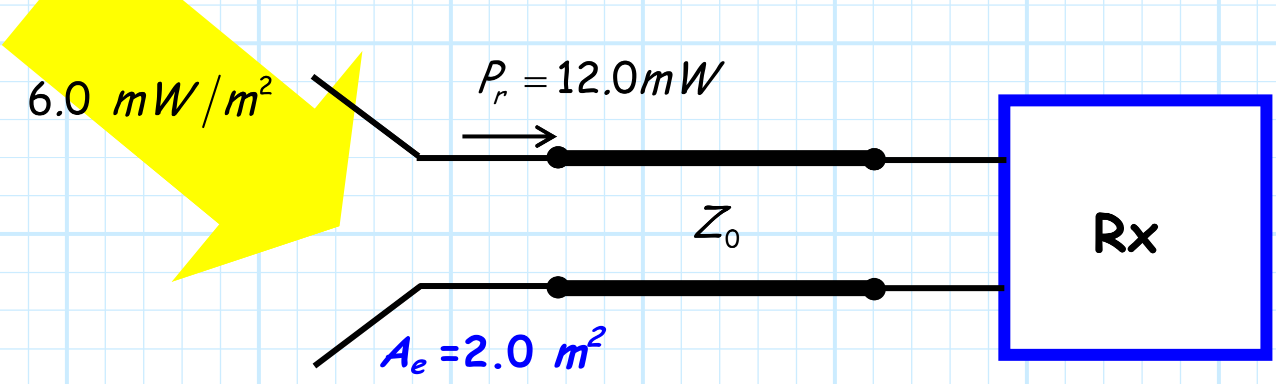
1. The antenna **design**
2. The **frequency** of the e.m. wave.
3. Wave **polarization**
4. The **direction** of the propagating wave!

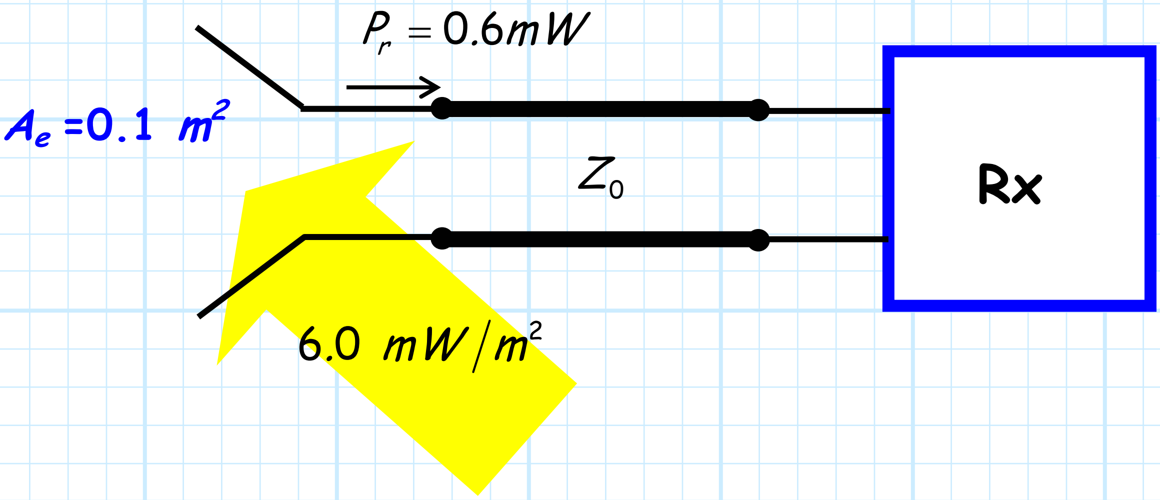
Q: *Yikes! Number 4 is **disturbing**. Are you saying that effective aperture is **not** a constant with respect to propagation **direction**?*

A: That's **precisely** what I'm saying!

It turns out that the effectiveness of an antenna in collecting electromagnetic energy is greatly affected by the **direction** at which that energy is **incident** on the antenna.

If the e.m. wave is propagating in **one** direction, the effective collecting aperture of the antenna maybe **large**. If propagating in some **other** direction, the effective collecting aperture might be very **small** (or even zero!).





Thus, effective aperture is a **function** of direction, a function expressed in terms of **spherical coordinates** θ and ϕ:

$$A_e(\theta,\phi)$$

Often, however, radio engineers are most concerned with the **maximum** (i.e., peak) value of this function, and thus generally refer to that **value** when expressing the effective aperture of an antenna.

$$\boxed{A_{em} \doteq \max_{\theta,\phi} A_e(\theta,\phi)}$$

Gain and Effective Aperture

Q: *How can we* ***determine/measure*** *the effective aperture* $A_e(\theta,\phi)$ *of a specific antenna?*

A: It turns out that if we know the antenna's **gain pattern** $G(\theta,\phi)$, we can **easily** determine effective aperture $A_e(\theta,\phi)$!

Antenna are generally **passive** devices made of **simple**, linear materials (e.g., conductors and dielectrics). Just as with our microwave components, this means that our antenna will be a **reciprocal** device.

Reciprocal devices result from a fundamental concept in electromagnetics called **Reciprocity**. With respect to an antenna, reciprocity requires that an antenna behave essentially in the **same way** as **either** a transmit **or** a receive antenna!

As a result, we find that the gain pattern $G(\theta,\phi)$ and the effective aperture are **directly proportional**:

$$A_e(\theta,\phi) \propto G(\theta,\phi)$$

This means that the two patterns $A_e(\theta,\phi)$ and $G(\theta,\phi)$ are essentially **identical**! For example, the direction and size of the **main lobe** for each pattern is precisely the **same**.

The **proportionality constant** can be found taking the ratio of the two functions $A_e(\theta,\phi)$ and $G(\theta,\phi)$. The result is provided without proof or explanation:

$$\frac{A_e(\theta,\phi)}{G(\theta,\phi)} = \frac{\lambda^2}{4\pi} \qquad \left[m^2\right]$$

where λ is the **wavelength** of the propagating electromagnetic wave (i.e., $\lambda = 2\pi c/\omega$).

Thus, we find that the relationship between **effective aperture** and **gain** is:

$$\boxed{A_e(\theta,\phi) = \frac{\lambda^2}{4\pi} G(\theta,\phi)}$$

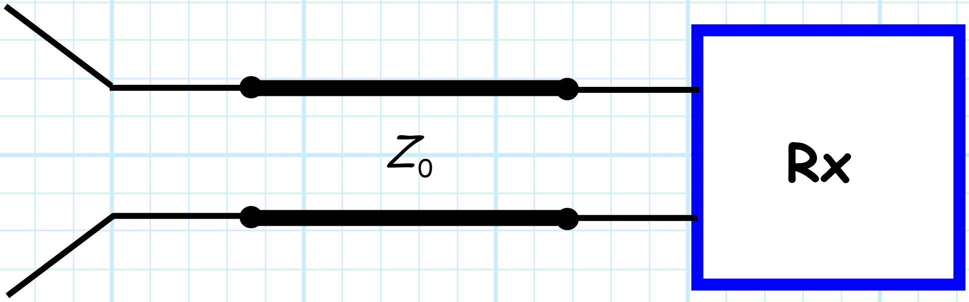
This relationship means that the ratio of A_e to G is a **constant** value $\lambda^2/4\pi$ for any and **all** directions θ and ϕ.

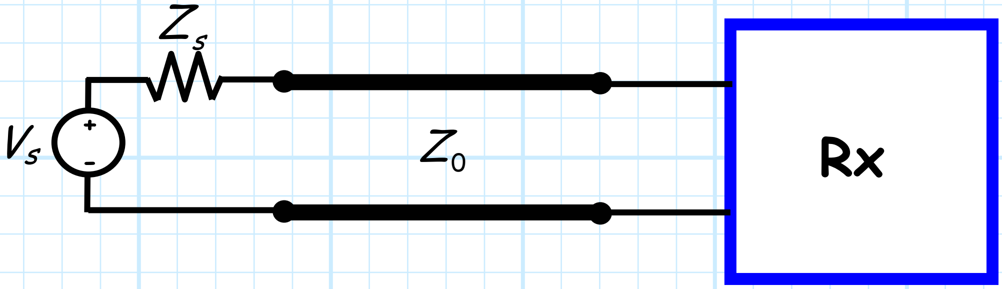
This includes of course the (same) direction in which both $A_e(\theta,\phi)$ and $G(\theta,\phi)$ are **maximum**. In other words, we find the relationship between antenna **gain** G_0 and **maximum** effective aperture A_{em} is:

$$\boxed{A_{em} = \frac{\lambda^2}{4\pi} G_0}$$

One more comment about the reciprocity and antennas:

With respect the receiver, the antenna acts as a **source** of electromagnetic energy (as opposed the transmitter, where is acts as a load). Like any source, a receive antenna has a **source impedance** Z_s.





From reciprocity, we find that the source impedance of an antenna is its impedance Z_A! I.E.,

$$Z_s = Z_A$$

We found that the **available power** of the transmitter is delivered **to** the antenna only if the antenna impedance Z_A is **matched** to the transmitter.

Likewise, we find that the **available power** of the receive antenna is delivered **to** the receiver only if the receiver input impedance is **matched** to the antenna (i.e., source) impedance Z_A.