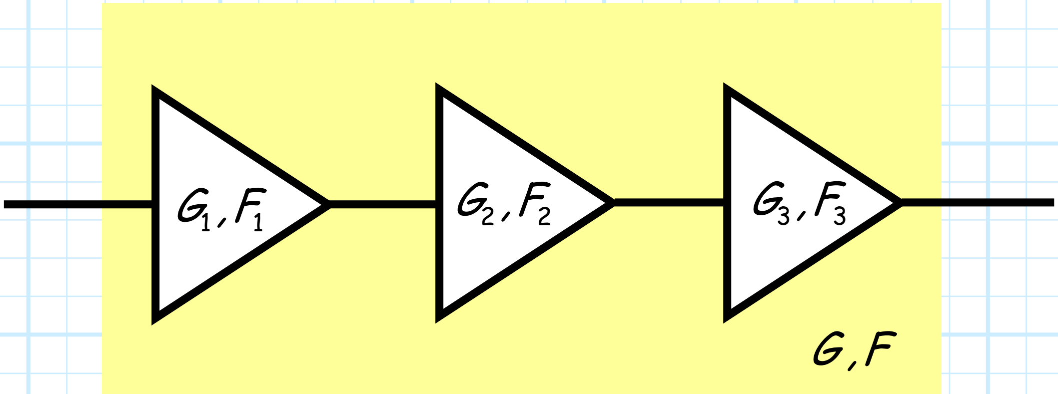
System Noise Figure

Say we **again** cascade three microwave devices, each with a different **gain** and **noise figure**:



These three devices together can be thought of as **one** new microwave device.

Q: *What is the noise temperature of this **overall** device?*

A: Recall that we found the overall **equivalent noise temperature** of this system to be:

$$T_e = T_{e1} + \frac{T_{e2}}{G_1} + \frac{T_{e3}}{G_1 G_2}$$

Likewise, the equivalent noise temperature of each device is related to its **noise figure** as:

$$T_e = (F - 1)290K^\circ$$

Combining these two expressions we find:

$$(F-1)290K^\circ = (F_1-1)290K^\circ + \frac{(F_2-1)290K^\circ}{G_1} + \frac{(F_3-1)290K^\circ}{G_1G_2}$$

and thus **solving** for F:

$$\begin{aligned} F &= \frac{1}{290K^\circ}\left((F_1-1)290K^\circ + \frac{(F_2-1)290K^\circ}{G_1} + \frac{(F_3-1)290K^\circ}{G_1G_2}\right)+1 \\ &= F_1 - 1 + \frac{F_2-1}{G_1} + \frac{F_3-1}{G_1G_2} + 1 \\ &= F_1 + \frac{F_2-1}{G_1} + \frac{F_3-1}{G_1G_2} \end{aligned}$$

Therefore, the **overall** noise figure for the "device" consisting of three cascade amplifiers can be determined **solely** from the knowledge of the gain and noise figure of **each** individual amplifier!

$$F = F_1 + \frac{F_2-1}{G_1} + \frac{F_3-1}{G_1G_2}$$

Moreover, we find that if we construct a system of N cascaded microwave components, then the **overall noise figure** of the system can be determined as:

$$F = F_1 + \frac{F_2-1}{G_1} + \frac{F_3-1}{G_1G_2} + \frac{F_4-1}{G_1G_2G_3} + \cdots + \frac{F_N-1}{G_1G_2G_3\cdots G_{N-1}}$$

* It is again evident from inspection of this equation that the **first device** in the cascaded chain will likely by the **most significant** device in terms of the overall system noise figure.

* We come to the same conclusion as for T_e—make the first device one with **low internal noise** (small noise figure F_1) and **high gain** G.

* In other words, make the **first** device in your receiver a **Low-Noise Amplifier (LNA)**!

One other **very important** note:

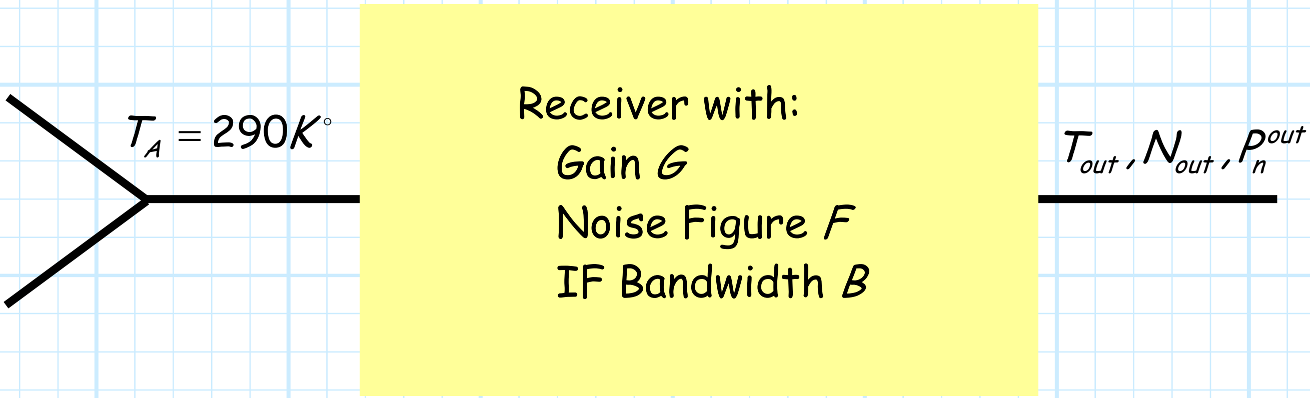
Although we used **only** amplifiers in our examples for system equivalent noise temperature and system noise figure, the results are likewise valid for **passive** devices!

Just remember, the gain G of a passive device is simply the **inverse** of its attenuation A:

$$G = \frac{1}{A}$$

Now, let's examine one important system made up of cascaded microwave components—a **receiver**!

At the **input** of every receiver is an **antenna**. This antenna, among other signals, delivers **noise power** to the input, with a temperature that is typically $T_A = 290K^\circ$.



Q: *What is the* ***output noise*** *(i.e. T_{out}, N_{out}, or P_n^{out}) of this receiver?*

A: Recall that the output noise temperature is:

$$\begin{aligned} T_{out} &= G(T_{in} + T_e) \\ &= G(T_A + T_e) \\ &= G(290\,K^{\circ} + T_e) \end{aligned}$$

and since:

$$T_e = (F-1)290K^{\circ}$$

we conclude that the **output noise temperature** is:

$$\begin{aligned} T_{out} &= G(290\,K^{\circ} + T_e) \\ &= G(290\,K^{\circ} + (F-1)290K^{\circ}) \\ &= G\,F(290K^{\circ}) \end{aligned}$$

Therefore, the **average spectral power density** at the output is:

$$\begin{aligned} N_{out} &= k T_{out} \\ &= kGF\left(290K^{\circ}\right) \end{aligned}$$

while the **output noise power** is:

$$\begin{aligned} P_n^{out} &= N_{out} B \\ &= k\,G\,F\left(290K^{\circ}\right)B \end{aligned}$$

Now, compare these values to their respective **input** values:

$$T_{in} = T_A = 290K^{\circ}$$

$$N_{in} = k\left(290K^{\circ}\right)$$

$$P_n^{in} = k\left(290K^{\circ}\right)B$$

Note for each of the values, the output is a factor ***G F*** greater than the input:

$$\frac{T_{out}}{T_{in}} = \frac{N_{out}}{N_{in}} = \frac{P_n^{out}}{P_n^{in}} = G\,F$$

→ However, I again emphasize, this expression is only valid **if** $T_{in} = 290\,K^{\circ}$!!

Of course, the ratio of the **signal** output power to the **signal** input power is:

$$\frac{P_s^{out}}{P_s^{in}} = G$$

Thus, the **signal** power is increased by a **factor** G, while the **noise** power is increased by a **factor** GF. This is why there is reduction in SNR by a **factor** F!