The 90° Hybrid Coupler

The 90° Hybrid Coupler, otherwise known as the **Quadrature Coupler**, has the **same** symmetric form as the directional coupler:

$$\bar{\bar{\mathcal{S}}} = \begin{bmatrix} 0 & \alpha & j\beta & 0 \\ \alpha & 0 & 0 & j\beta \\ j\beta & 0 & 0 & \alpha \\ 0 & j\beta & \alpha & 0 \end{bmatrix}$$

However, for **this** coupler we find that

$$\alpha = \beta = \frac{1}{\sqrt{2}}$$

Therefore, the scattering matrix of a quadrature coupler is:

$$\bar{\bar{\mathcal{S}}} = \begin{bmatrix} 0 & 1/\sqrt{2} & j/\sqrt{2} & 0 \\ 1/\sqrt{2} & 0 & 0 & j/\sqrt{2} \\ j/\sqrt{2} & 0 & 0 & 1/\sqrt{2} \\ 0 & j/\sqrt{2} & 1/\sqrt{2} & 0 \end{bmatrix}$$

It is evident that, just as with the directional coupler, the ports are **matched** and the device is **lossless**. Note also, that if a signal is incident on one port only, then there will be a port from which **no** power will exit (i.e., an **isolation** port).

Unlike the directional coupler, the power that is flows into the input port will be **evenly** divided between the two non-isolated ports.

For example, if 10 mW is incident on port 3 (and all other ports are matched), then 5 mW will flow out of **both** port 1 and port 4, while no power will exit port 2 (the isolated port).

Note however, that the although the **magnitudes** of the signals leaving ports 1 and 4 are **equal**, the relative **phase** of the two signals are separated by **90 degrees** ($e^{j\pi/2} = j$).

We find, therefore, that if in **real** terms the voltage out of port 1 is:

$$v_1(z,t) = \frac{|V_{03}^-|}{\sqrt{2}} \cos(\omega_0 t + \beta z)$$

then the signal form port 4 will be:

$$v_4(z,t) = \frac{|V_{03}^-|}{\sqrt{2}} \sin(\omega_0 t + \beta z)$$

There are **many** useful applications where we require both the **sine** and **cosine** of a signal!