The Statistics of Noise

Noise is a random process, so we must describe it statistically.

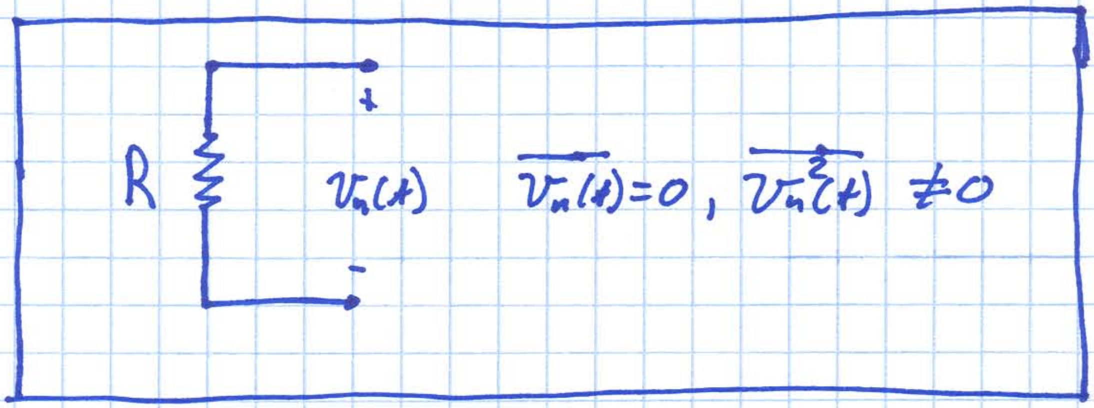
{i.e., its average power spectral density}

- Let's define the average power spectral density of noise as $N(f)$ (units of Watts/Hz).

- Consider now a resistor R at temperature T. We think of a resistor as a passive device that generates no power.

- ⇒ Not quite true!!
  Since the resistor is "warm", the free electrons in the device will be moving, causing a random electric field, and so a tiny.

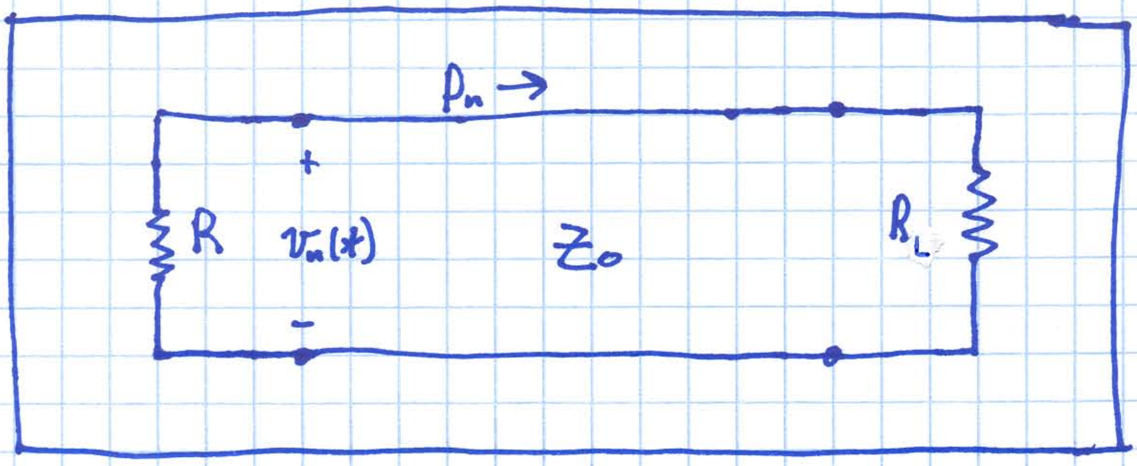
Voltage across the resistor!



This value $v_n(t)$ is a <u>random process</u> with time. Its average, or <u>mean</u> value is zero ($\overline{v_n(t)} = 0$), <u>but</u> its <u>variance</u> is <u>not</u> zero ($\overline{v_n^2(t)} \neq 0$)!

$$\left\{ \begin{array}{l} \therefore \text{ the resistor generates } \underline{\underline{\text{power}}}! \\ \qquad \Rightarrow \quad P_n \doteq \text{noise power} \\ \qquad\qquad\qquad \propto \overline{v_n^2(t)} \end{array} \right\}$$

If we connect this resistor to a load, we can transfer this power:



If $Z_o = R$ + $R_L = R$, then the power aborbed by R_L is:

$$\left\{ \frac{\overline{v_n^2(t)}}{R_L} = \frac{\overline{v_n^2(t)}}{R} = P_n \right\}$$

Recall that noise power P_n can also be found by integrating $N(f)$ over all frequencies:

$$\boxed{P_n = \int_0^{\infty} N(f)\, df}$$

Q: What is the average spectral power density $N(f)$?

A: Using a bunch of quantum physics, we can find the answer!

For a resistor, the result is:

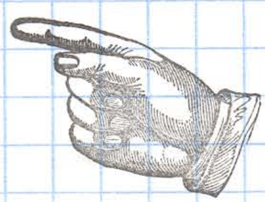
$$N(f) = \underline{kT \doteq N_0} \quad (\text{Watts}/\text{Hz})$$

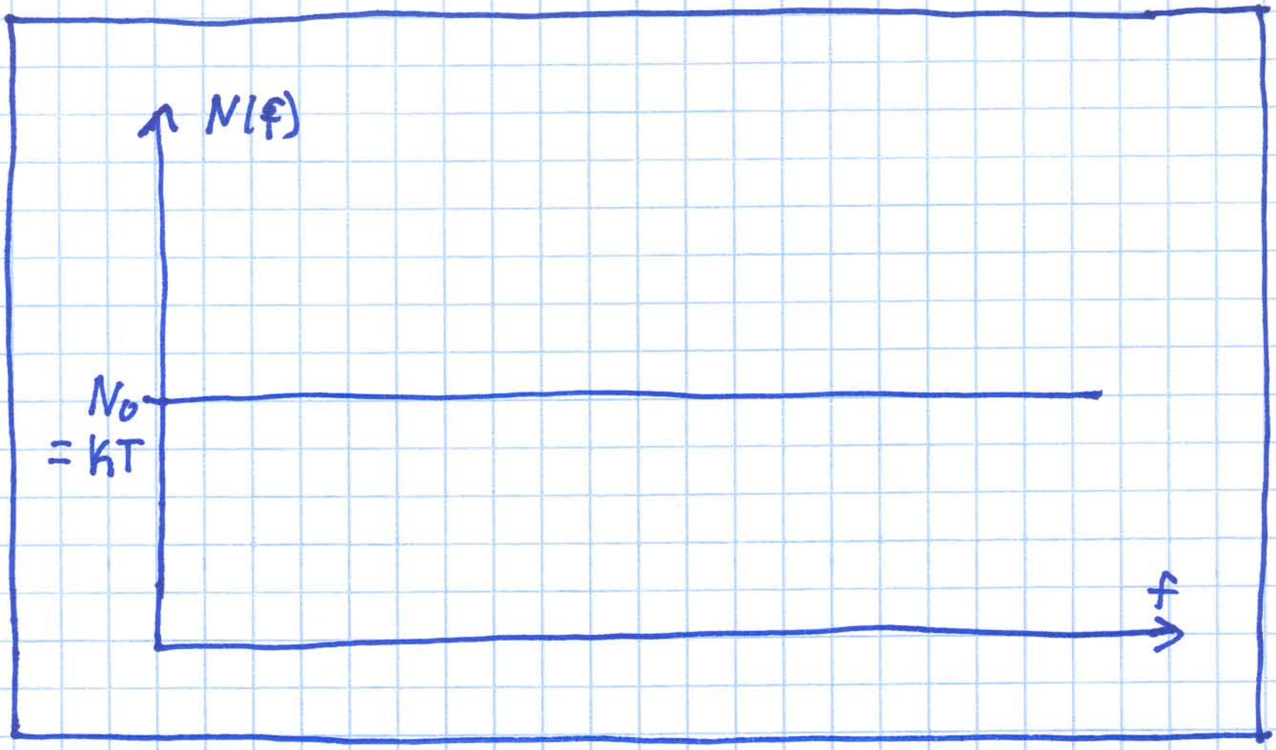
where k = Boltzman's Constant

$= 1.38 \times 10^{-23}$ (J/K°)

T = Resistor temperature in degrees <u>kelvin</u>

$\therefore$ N_0 is a <u>constant</u> wrt frequency!

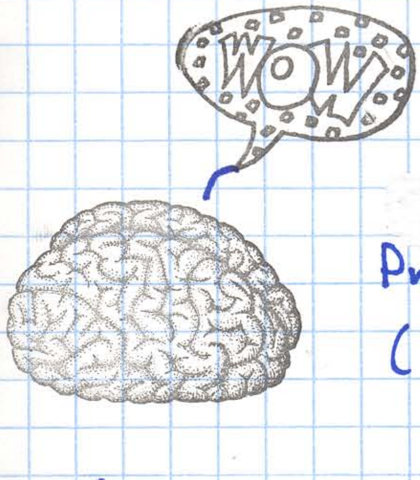




$\therefore$ $N_0 = KT$ means $N(f)$ has equal magnitude for all frequencies !!

$\Rightarrow$ called <u>white</u> <u>noise</u>.

Q: If $N(f) = N_0 = KT$, then noise power would be:



$$P_n = \int_0^{\infty} N_0 \, df = N_0 \int_0^{\infty} df = \underline{\underline{\infty}} \, !$$

P_n = <u>infinity</u> $\Leftarrow$ that's alot !
(Again, the energy crisis is solved !!)

A: Actually, as $f \to \infty$, $N(f)$ will approach $\underline{\underline{0}}$.

$$\left\{ \therefore \quad P_n \int_0^\infty N(f)\, df < \infty \right\}$$

$N(f) = N_0 = kT$ is an <u>approximation</u>, valid in the <u>RF</u> and <u>μwave</u> region for all but the very <u>coldest</u> resistors (i.e., all but small values of T).

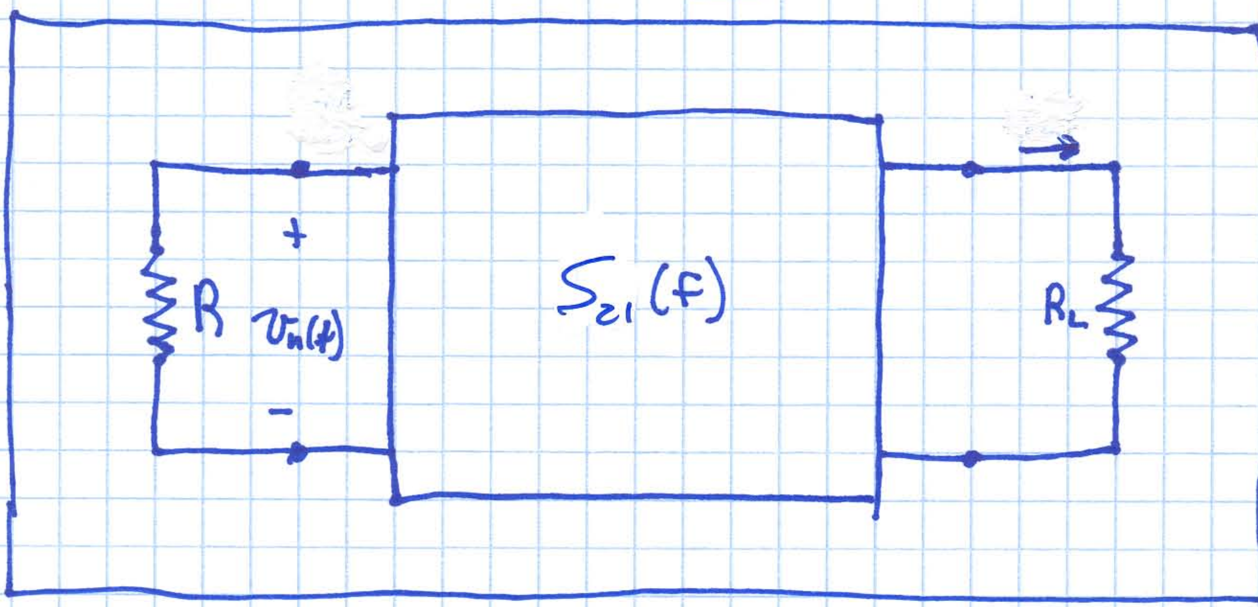
Q: Still, wouldn't the value:

$$P_n = \int_0^\infty N_0\, df$$

be <u>very</u> large??

A: Mathematicaly speaking <u>yes</u>. <u>But</u>, remember P_n is the noise power <u>delivered</u> to a load by the resistor. <u>Although</u>

the noise spectral power density $N(f)$ of the resistor may be a <u>constant</u> with frequency, the response of the <u>circuit</u> it is attached to will <u>not</u> be!!



Note the average spectral power at the load is:

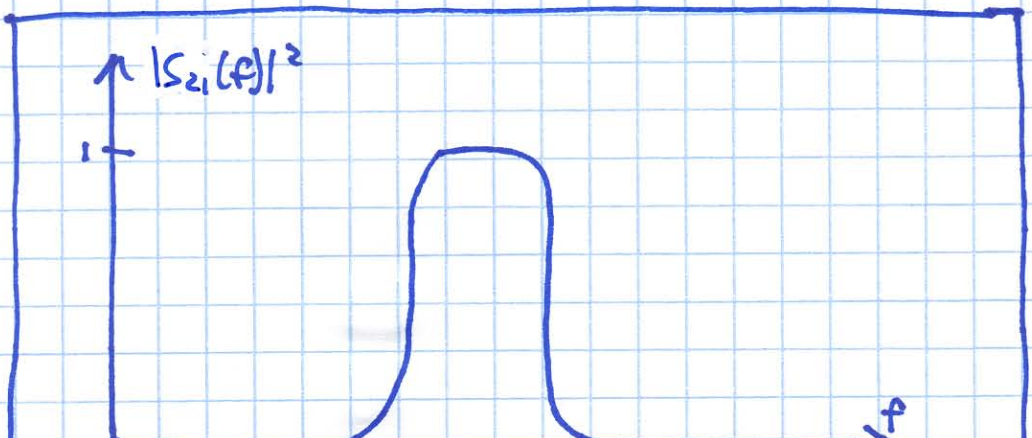
$$\boxed{N(f) = |S_{21}(f)|^2 N_0 = \quad (f)|^2 kT}$$

$\therefore$ the noise power at the load is:

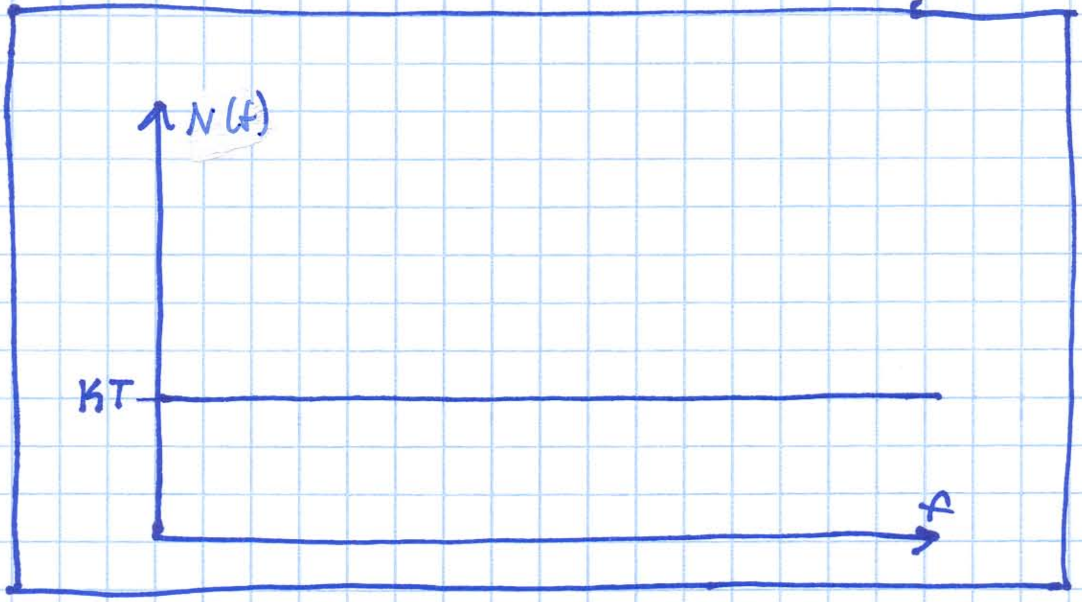
$$\boxed{P_n = \int_0^{\infty} |S_{21}(f)|^2 N_0 \, df}$$

The circuit described by $S_{21}(f)$ will have some finite bandwidth, so the noise delivered to the load (e.g. a detector) will be limited, however still annoyingly large!

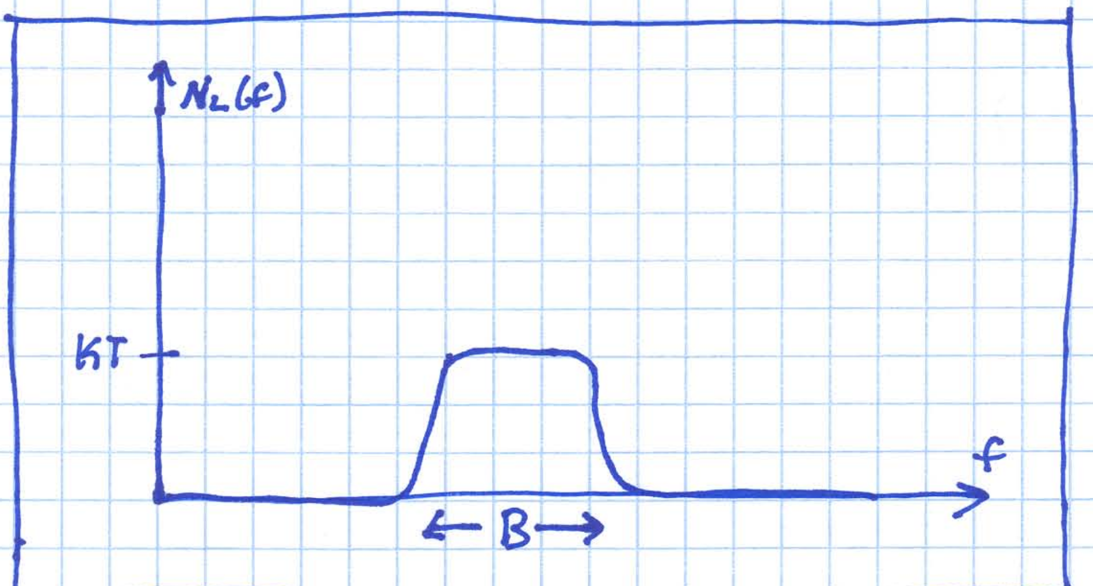
E.G., say the system described by $S_{21}(f)$ is a band-pass filter, with bandwidth $\underline{\underline{B}}$.



Since N_o is a constant with frequency,



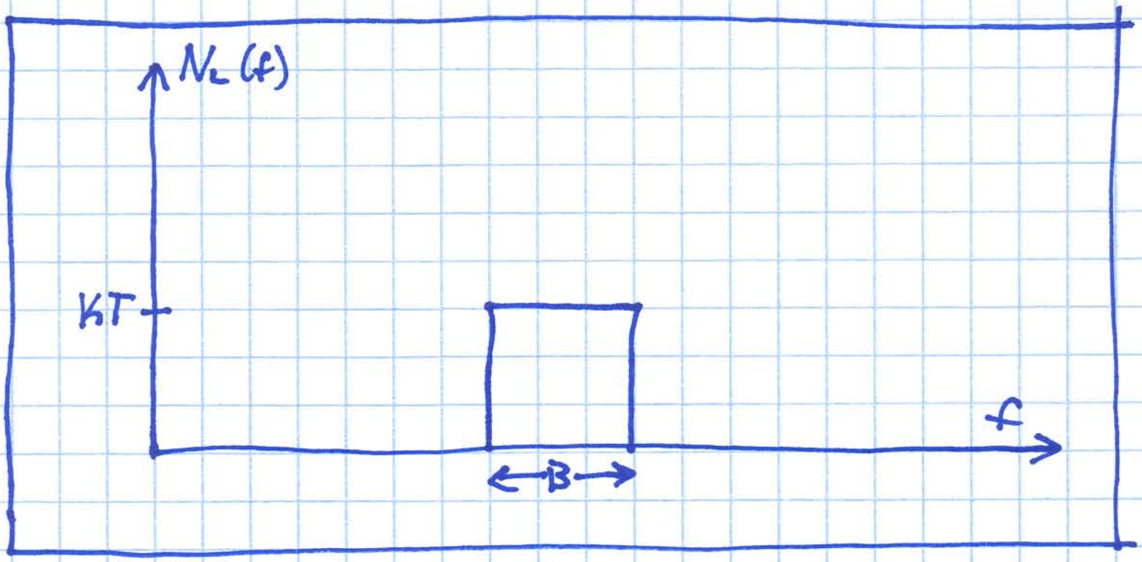
So the power spectral density of the noise at the load is:



Note for this band-pass example, we can $\approx N_L(f)$ as:

$$N_L(f) \approx \begin{cases} KT = N_0 & \underline{\text{within}} \text{ the passband} \\ 0 & \text{outside the pass band} \end{cases}$$

I.E.,



$$\therefore \; P_n = \int_0^\infty |S_{21}(f)| \, N_0 \, df$$

$$\approx \int_B N_0 \, df = B N_0$$

$\therefore$

$$\boxed{P_n = KTB = N_0 B}$$