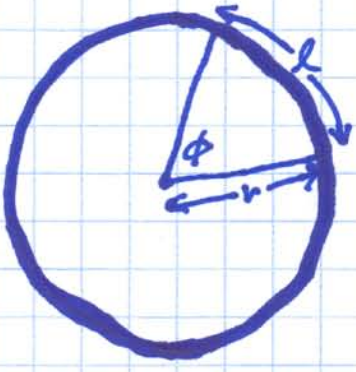
The Steradian

Q: What the heck is a steradian ?!

A: Let's first examine the radian.

Recall for a circle (a 2-D object):



r is the circle radius

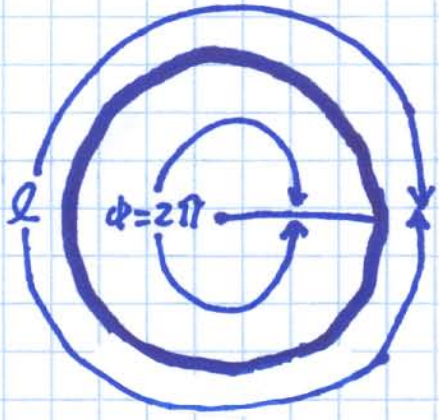
ϕ defines an arc (units of radians)

ℓ is arc length

We know that these values are related as:

$$\ell = \phi r$$

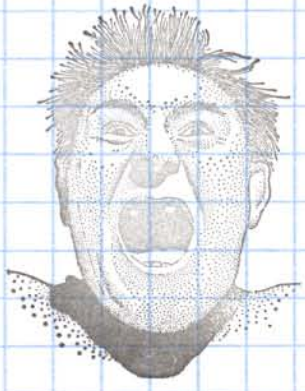
An arc that encompasses the entire circle has $\phi = 2\pi$ radians!



$\therefore \ell = 2\pi r$

Of course, ℓ is the circumference of the circle (i.e. $C = 2\pi r$)!

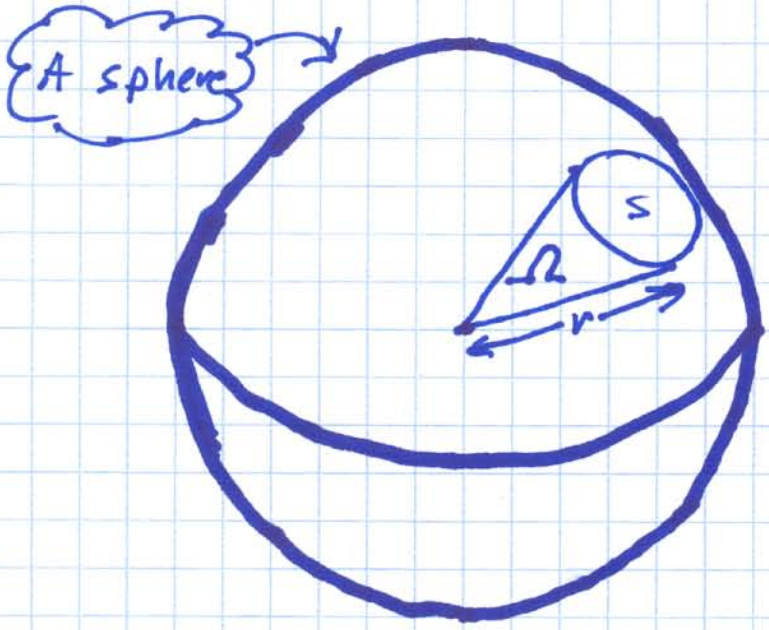
But wait! We have a Problem!! A circle is a two-dimensional object, yet we live in a three-dimensional world.



* An angle in $\underline{2}$ dimensions has units of radians.

* An angle in $\underline{3}$ dimensions has units of steradians.

For example, consider the 3-dimensional equivalent of a circle - a <u>sphere</u>.



r is the sphere radius

Ω defines the size of a <u>cone</u> (in steradians)

S is the surface <u>area</u> on the sphere surface <u>subtended</u> by the cone.

These parameters are related as:

$$S = \Omega r^2$$

The cone of Ω steradians is refered to as a <u>solid angle</u>.

If a solid angle is so large that it subtends the entire surface of a sphere, then this solid angle has a size of $\Omega = 4\pi$ steradians.

Thus, the surface area of a sphere is:

$$S = \Omega r^2$$
$$= 4\pi r^2$$

Thus, a half-sphere (a hemisphere) defines a solid angle of 2π steradians, while a quarter-sphere is subtended by a solid angle of $\Omega = \pi$ steradians.