

# Total Radiated Power

Consider an antenna that propagates a spherical wave of the form:

$$\vec{W}(\theta, \phi, r) = U(\theta, \phi) \frac{\hat{r}}{r^2}$$

Recall  $\vec{W}(\theta, \phi, r)$  has units of Watts/m<sup>2</sup>, so the total radiated power can be found by integrating over any surface surrounding the antenna:

$$P_{\text{rad}} = \oiint_S \vec{W}(\theta, \phi, r) \cdot d\vec{s}$$

Makes sense!

$$\left\{ \begin{array}{l} \left( \frac{\text{Watts}}{\text{m}^2} \right) (\text{m}^2) = \text{Watts} \\ |\vec{W}| A = P_{\text{rad}} \end{array} \right\}$$

Where  $P_{\text{rad}} = \text{Total Radiated Power}$

The easiest surface to use in evaluating this integral is a sphere.

i.e.

$$d\vec{s} = r^2 \sin\theta d\theta d\phi \hat{r}$$

where

$$0 < \phi < 2\pi$$

$$0 < \theta < \pi$$

$$\begin{aligned} \circ \circ \quad P_{\text{rad}} &= \iint_S \vec{W}(\theta, \phi, r) \cdot d\vec{s} \\ &= \int_0^{2\pi} \int_0^{\pi} u(\theta, \phi) \frac{\hat{r} \cdot \hat{r}}{r^2} r^2 \sin\theta d\theta d\phi \end{aligned}$$

$$P_{\text{rad}} = \int_0^{2\pi} \int_0^{\pi} U(\theta, \phi) \sin\theta \, d\theta \, d\phi$$

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**Q:** This function  $U(\theta, \phi)$  seems to be very important. Just what the heck is it?!?

**A:** The  $U(\theta, \phi)$  describes the antennas' radiation intensity !!