Hash Tables – Outline

- Definition
- Hash functions
- Open hashing
- Closed hashing
  - collision resolution techniques
- Efficiency

Overview

- Implementation style for the Table ADT that is good in a wide range of situations is the hash table
  - efficient Insert, Delete, and Search operations
  - difficult Sorted Traversal
  - efficient unsorted traversal
- Good approach as long as sorted output comparatively rare in the total set of hash table operations

Definition

- Hash table is defined by:
  - set of records R = { r_1, r_2, ... , r_n} stored by the table
  - set of input keys K = { k_1, k_2, ..., k_n}, n >= 0 that can be associated with records (k_x, r_y)
- Array of buckets B[0 ... m-1]: each array element is capable of holding one or more (k_x, r_y) pairs
- Hash Function H: K → {0, 1, ... , m-1}
  - for any given (k_x, r_y), B[H(k_x)] is the designated storage location for (k_x, r_y)
- Collision resolution scheme
  - when (k_x, r_y) and (k_a, r_b) map to the same bucket under H, this scheme determines where the second record is stored

Definitions

- An Array of buckets B[0 ... m-1] holds all data managed by the hash table
- Open or External Hashing
  - bucket locations store pointers (references) to record pairs (k_x, r_y)
  - colliding records stored in a linked list
- Closed or Internal Hashing
  - buckets store actual objects
  - colliding records stored in other bucket locations
- Note that the associated keys may be implicit rather than explicitly stored

Hash Functions

- H(i) = i
  - reduces the hash table to an array
- Selecting digits
  - choose some subset of digits in a large number
    - specific slice or positions
- Folding
  - take digits or slices of a number and add them together with roll-over
- H(i) = i modulo m – where m is Hash Table size
  - choosing m as a prime number is popular for an "even distribution of keys"

Hash Function – 2

- Strings are a common search key in many cases
  - convert string to an integer
  - H(string) → integer
- Approaches
  - add characters or slices of characters together as n-bit unsigned numbers with the sum rolling over within x-bits
    - bit shifting to form numbers possible
    - x-bits chose for table size or x modulo m
  - several other options possible

Open Hashing

- Example: take a hash table size of 7 (prime) and a hash function h(x) = x mod 7
  - insert 64, 26, 56, 72, 8, 36, 42
- If data set is large compared to hash table size, or the hash function clusters data, then length of the list holding the bucket contents can be significant
  - sorted list will reduce the average failure time
    - can identify failure before the end of the list
  - use binary search tree instead of list
    - why not a BST for the whole data set?
  - use second Hash table

Open Hashing – 2

- Advantages of Open Hashing with chaining
  - simple in concept and implementation
  - insertion is always possible
- Disadvantages of hashing with chaining
  - unbalanced distribution decreases efficiency
    - O(n) for a linked list, O(log n) for a BST
  - greater memory overhead
  - higher execution overhead of stepping through pointers

Closed Hashing

- Closed hashing with Open addressing
  - storing all data items within single hash table, but "open" up the address assigned to item on collision
- Hash table of size m can hold at most m items
- Only a "perfect" hash function will distribute m items to m different table elements
  - collisions will generally occur before table is full
- Collision resolution is thus crucial to efficient use of closed hash tables

Closed Hashing – Collision Resolution

- Create a sequence of collision resolution functions
  - $h_0(x)$ is base hash function
  - $h_1(x)$ used to find first alternate storage location after a collision
  - $h_2(x)$ used to find the next alternate if first alternate is occupied
- Each $h_i(x)$ must be guaranteed to choose different table locations
- Hash function series should ideally check all table locations

Collision Resolution – Linear Probing

- Search hash table sequentially starting from the original location specified by the hash function
  - $h_i(x) = (h_0(x) + i) mod\ m, \forall\ i > 0$
- Insert 64, 26, 56, 72, 8, 36, 42 in an empty table of size 7
- Fragile – causes primary clusters by occupying adjacent table locations
  - similar to long chains in open hashing

Collision Resolution – Quadratic Probing

- Spread probed locations across the table
  - $h_i(x) = (h_0(x) + i^2) mod\ m, \forall\ i > 0$
- Example: Insert 64, 26, 56, 72, 8, 36, 42
- Series of probed locations is not guaranteed to cover the whole table without duplication
- Closed hashing schemes can fail even though the
- table is not full
  - and secondary clusters may form
  - if the probing scheme will not visit all table locations and distribute probes "evenly" over 0..m

Collision Resolution – Linear Probing with Fixed Increment

- $h_i(x) = (h_0(x) + (i * FI)) mod\ m, \forall\ i > 0$
  - FI is relatively prime to m
  - linear probing will visit all table locations without repeats
- X is relatively prime to Y iff GCD(X,Y) = 1

Collision Resolution – Double Hashing

- Use a second hash function (h'(x)) to generate the probe sequence used after a collision
  - $h_i(x) = (h_0(x) + (ih'(x))) mod\ m, \forall\ i > 0$
  - Use h'(x)=R – (x mod R), where R < m is prime
- Example: m=7, R=5, insert 64,26,56,72,8,36,42

Closed Hashing -- Deletions

- Example: Insert 64, 56, 72, 8 using linear probling
  - delete 64; delete 8
- Deletion along the probing path from A → B creates a problem because the empty cell could be there for two reasons
  - no further elements exist along this probing sequence
  - deletion of an item along the sequence took place
- Two types of empty buckets
  - bucket has always been empty (AE) (flag 0)
  - bucket emptied by deletion (ED) (flag 1)

Closed Hashing -- Deletions

- During a probing sequence,
  - if an AE bucket is found, searching can stop
  - if an ED bucket is found, searching must continue
- Closed Hashing is thus subject to a form of "fatigue"
  - as cells are deleted, probing sequences generally lengthen as the probability of encountering ED cells increases
  - failed searches get more expensive because they cannot terminate until
    - an AE cell is found
    - all cells of the table can be visited

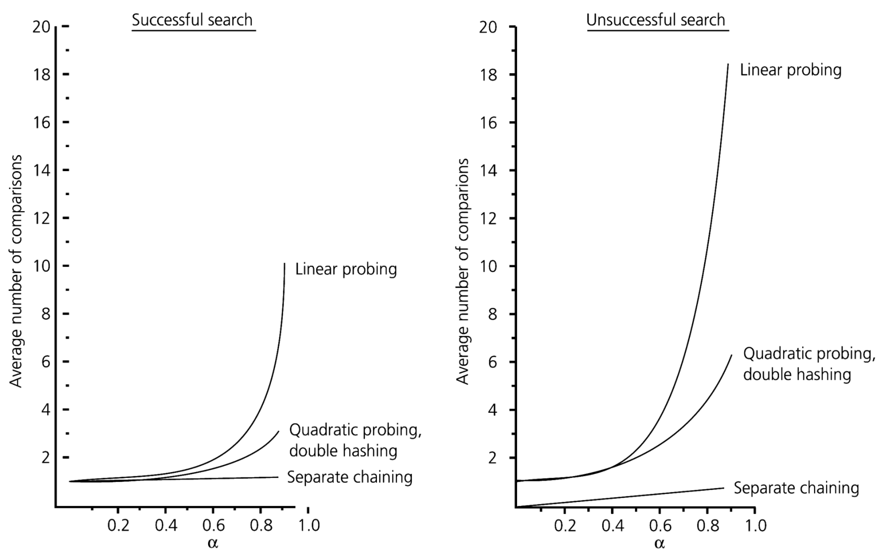
Closed Hashing

- Advantages of Closed Hashing with Open Addressing
  - lower execution overhead as addresses are calculated rather than read from pointers in memory
  - lower memory overhead as pointers are not stored
- Disadvantages
  - more complex than chaining
  - can degenerate into linear search due to primary or secondary clustering
  - Delete and Find operations are more complex
  - Insert is not always possible even though the table is not full
  - Delete can increase probe sequence length by making search termination conditions ambiguous

The Efficiency of Hashing

- An analysis of the average-case efficiency
  - Load factor α
    - ratio of the current number of items in the table to the maximum size of the array table
    - measures how full a hash table is
    - should not exceed 2/3
  - Hashing efficiency for a particular search also depends on whether the search is successful
    - unsuccessful searches generally require more time than successful searches

The Efficiency of Hashing



Summary

- Hash Tables are useful and efficient data structures in a wide range of applications
- Open hashing with chaining is simple, easy to implement, and usually efficient
  - length of the chains is key to performance
- Closed hashing with various approaches to generating a probe sequence can also be efficient
  - lower space and computation overhead
  - more complex implementation
  - performance is sensitive to probe sequence
- Monitoring load factor and other hash-table behavior parameters is important in maintaining performance