

Open Problems in Universal Algebra

a Shanks workshop at Vanderbilt University

May 28 – June 1, 2015

	Thurs, May 28	Fri, May 29	Sat, May 30	Sun, May 31	Mon, June 1
9:00am	Registration	Coffee	Coffee	Coffee	Coffee
10:00am	L. Barto	M. Kozik	M. Valeriote	D. Zhuk	Summary discussion
11:00am	Coffee	Coffee	Coffee	Coffee	Coffee
11:30am	R. Willard	A. Wires J. Opršal	W. DeMeo J. Horowitz	Final listing of open problems	
12:30pm	Lunch	Lunch	Lunch	Lunch	
2:00pm	Moderated discussion	Moderated discussion	Open problem session	Open problem session	
3:30pm	Coffee	Coffee			
4:00pm	Open problem session				

Registration and coffee will be in the lobby of Wilson Hall.

All talks will be held in Wilson Hall, room 115. Rooms 127 and 129 are also available.

Invited talks are 50 minutes long. Contributed talks are 25 minutes long.

There will be a reception dinner at 6pm on Saturday, May 30th, details TBA.

Time: Thursday, May 28, 10:00am – 10:50am

Title: A rectangularity theorem for simple Taylor algebras

Speaker: Libor Barto

Abstract: I will present a theorem saying (roughly) that a subdirect power of simple nonabelian Taylor algebras restricted to minimal absorbing subuniverses is the full product. Universal algebraic and CSP consequences will be discussed as well.

This is a joint work with Marcin Kozik.

Time: Thursday, May 28, 11:30am – 12:20pm

Title: Maltsev constraints

Speaker: Ross Willard

Abstract: Bulatov gave a polynomial-time solution to Constraint Satisfaction Problems (CSPs) with Maltsev constraints in 2002. Three years later Bulatov and Dalmau gave a much simpler algorithm which was eventually generalized to the so-called “few subpowers algorithm.” Dissatisfied, I am seeking yet another algorithm to solve Maltsev CSPs. In this lecture I will explain the reasons for my dissatisfaction and give some conjectures which, if true, may yield the desired algorithm.

Time: Friday, May 29, 10:00am – 11:50am

Title: Congruence join semi-distributive varieties and constraint satisfaction problems

Speaker: Marcin Kozik

Abstract: I will present some new results on $\text{SD}(\vee)$ varieties: in particular a set of directed $\text{SD}(\vee)$ terms and an analogue of “graph absorbing” condition for congruence distributive varieties. I will discuss further applications of these results to solving CSPs for such algebras. Finally I will show examples of problematic algebras (in $\text{SD}(\vee)$) which present new problems while solving CSPs (compared to the CD case).

Time: Friday, May 29, 11:30am – 11:55am

Title: Cube absorption: a few properties, many questions

Speaker: Alexander Wires

Abstract: We can define the natural notion of a cube-absorbing subalgebra to complete the analogy: near-unanimity terms is to pointing terms is to absorption as cube-terms is to cubing terms is to cube-absorption. We might think of cube terms and cube-term blockers as opposing properties, but we can see them as different types of cube-absorption; that is, every finite idempotent Taylor algebra has a subalgebra with proper cube-absorption. This yields related weakened forms of directed Gumm terms and compact quasi-representations of subpowers for any finite idempotent Taylor algebra.

Time: Friday, May 29, noon – 12:25pm

Title: The lattice of linear Mal'cev conditions

Speaker: Jakub Opršal

Abstract: Vaguely speaking a Mal'cev condition (on an algebra, a variety, or a clone) is a condition of the form 'There exists some number of terms such that they satisfy some given equations.' Many of such conditions appear naturally in universal algebra. Mal'cev conditions can be ordered by implication, i.e., a stronger condition is larger than a weaker one. This order is in fact a lattice order, and it is in close connection to ordering of clones by interpretability, and the lattice of interpretability types. We will focus on part of this poset, namely those conditions that can be described by equations that do not include term composition.

Time: Saturday, May 30, 10:00am – 10:50am

Title: Deciding Maltsev conditions

Speaker: Matt Valeriote

Abstract: In this talk I will consider the problem of determining, for a given Maltsev condition, the computational complexity of deciding if a given finite algebra satisfies it. It turns out that for many familiar Maltsev conditions, the problem is EXP-TIME complete. On the other hand, if one only considers idempotent algebras, then polynomial-time algorithms have been devised that work for certain special Maltsev conditions.

I will present some recent results on testing for Maltsev conditions and will describe an approach, via “local terms”, that has been used to construct polynomial-time algorithms for a number of Maltsev conditions, in the idempotent case. There are many open problems in this area and I will discuss several of them during my talk.

The research presented in this talk was conducted with Ralph Freese and Alexandr Kazda.

Time: Saturday, May 30, 11:30am – 11:55am

Title: Which commutative idempotent binars are tractable?

Speaker: William DeMeo

Abstract: A binar is a set equipped with a binary operation. Letting A denote a finite commutative idempotent binar (CIB), and R a set of subuniverses of powers of A , we ask whether the constraint satisfaction problem $CSP(R)$ is solvable in polynomial time. It turns out that if S is the two-element semilattice, then the following are equivalent: (i) S is not a divisor of A ; (ii) $V(A)$ omits tame congruence type 5; (iii) A has an edge term. Thus, such CIBs are tractable. We will discuss some results and questions like these, and describe a few small CIBs whose tractability seems open.

This is joint work with Cliff Bergman and Jiali Li.

Time: Saturday, May 30, noon – 12:25pm

Title: Determining congruence n -permutability is hard (n at least 3?)

Speaker: Jonah Horowitz

Abstract: Building on the work of Freese and Valeriote (On the complexity of some Maltsev conditions, 2009) we reduce a restricted form of the problem of clone membership to the problem of determining whether or not a finite algebra generates a congruence n -permutable variety (for n at least 3). During this process we identify a class of Mal'cev conditions which can likewise be the target of reduction for this restricted form of the problem of clone membership. As this form of the problem of clone membership is EXPTIME-complete, this demonstrates the EXPTIME-hardness of every Mal'cev condition within the identified class, including congruence n -permutability (for n at least 3). Naturally we wish to extend this result even further if possible, with an eye towards demonstrating the EXPTIME-completeness of determining congruence 2-permutability as well.

Time: Sunday, May 31, 10:00am – 10:50am

Title: Open problems in clone theory

Speaker: Dmitriy Zhuk

Abstract: The main problem in clone theory is to get a description of all clones. In 1941 Emil Post obtained an amazing description of all clones on 2 elements, but in 1959 it was proved that there exists a continuum of clones on k elements if $k > 2$.

The first part of my talk is devoted to the efforts to describe uncountable lattices of clones, in particular, I show how uncountable lattice of all clones of self-dual operations on 3 elements can be described. However, obtaining such description for other maximal clones seems to be too cumbersome and complicated. This motivated me to focus on the following questions: how to check properties of a clone and how to find finite sublattices.

Despite the fact that clone theory has been studied for many years, we still don't know how to check simple properties of clones. For example we don't know how to check whether a clone defined by a relation is finitely generated, or how to decide how many clones contain a given operation. In the second part of the talk I try to collect such open problems and present some partial results.