Interference Analysis for Millimeter-Wave Networks With Geometry-Dependent First-Order Reflections

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Abstract—Recent experimental studies reveal that urban millimeter wave (mmWave) channels are often dominated by both the line-of-sight (LoS) and geometry-dependent first-order reflection paths. In such environments, it is possible that any LoS and/or reflection components from surrounding interferers can critically deteriorate the link quality. Analyzing such impacts is, however, challenging in the absence of appropriate mmWave network interference models. In this correspondence paper, we address this challenge by providing an analytical stochastic geometry model that quantifies the total amount of network interference. Our study reveals that dense mmWave networks are often interference limited. The geometry-dependent first-order reflection interference is a dispensable component and must be encountered when analyzing and designing mmWave networks.

Index Terms—First-order reflection, interference analysis, millimeter wave networks, stochastic geometry.

I. INTRODUCTION

In millimeter wave (mmWave) bands, the radio channel experiences severe pathloss which is compensated for by using highly directional narrow beams. These directional beams can be blocked or reflected by obstacles. Interference occurs when the signals emitted from surrounding unintended transmitters are captured by the beam at an intended receiver via line-of-sight (LoS) and/or reflection paths. This interference event can be of a small probability. However, when it comes with a large number of unintended transmitters, their accumulated effect can critically exacerbate the link quality.

The Poisson point process (PPP) in stochastic geometry has been widely employed to accurately model the distributions of urban mmWave base stations and obstacles [1]–[4]. The work in [1] characterized the mmWave link outage due to the LoS interference. Exploiting the PPP, the mean power [2], complementary cumulative distribution function (CCDF) [3], and signal-to-interference ratio [4] of mmWave LoS interference links have been characterized. The work in [1]–[4], however, does not consider the existence of first-order reflection interference. Recently, there have been convincing measurement results showing that urban mmWave links are often dominated by both the LoS and first-order reflection paths [5].

In mmWave links, the first-order reflection paths are geometry-dependent [5], [6]. This dependency, however, makes the interference analysis challenging due to the complicated reflection and blockage mechanism. Recently, the statistics of geometry-dependent first-order reflection paths of a point-to-point (P2P) mmWave link were analyzed [6], where the random obstacles are assumed to have a common rotation angle. The P2P model in [6], however, cannot be directly extended to a network model. In practical networks, the obstacles can have random orientations. Conventional view has been pessimistic about analyzing the accumulated effect of the random LoS and geometry-dependent reflection interference.

In this paper, we provide an analytical LoS and reflection interference model and statistically quantify the accumulated effect of them in a dense urban mmWave network. We assume the obstacles and transmitters are distributed according to PPP. The obstacles have independent and identically distributed (i.i.d.) rotation angles and sizes. The distributed unintended transmitters can generate interference through either the LoS or first-order reflection paths. Under these assumptions, we find the mean and CDF of the accumulated network interference power. Our results signify that dense urban mmWave networks are often interference-limited. The reflection interference power is less than the LoS interference, but is far above the noise level. Hence, when designing and analyzing mmWave networks, the effect of the geometry-dependent reflection interference, which was often ignored, must be taken into account.

II. NETWORK SETUP

In this section, the network model under consideration and the assumptions made in subsequent derivations are provided. The key notations frequently used throughout the paper are given as below: $\overline{AB}$ is the line segment connecting points A and B, $L_{\overline{AB}}$ is the length of $\overline{AB}$, $P_{\overline{AB}}(x)$ is the probability of random event $\overline{AB}$ which is a function of $x$, $E_N(x)$ is the expectation of $N$ which is a function of $x$, and $O(r)$ is the circle centered at $(0, 0)$ with radius $r$.

A. Network Model

Consider an urban mmWave network, which consists of multiple transmitters, receivers, and obstacles. Each transmitter (receiver) employs a directional narrow beam. The obstacles in the network can block or reflect the narrow beams. Each receiver is associated with its nearest visible LoS transmitter that meets the link budget condition and regards other unintended transmitters as potential interferers. As shown in Fig. 1, we focus on the receiver placed at $(0, 0)$ in $\mathbb{R}^2$. Between the considered receiver and an unintended transmitter, there may exist an LoS or reflection path. In the case when both the beams of the considered receiver and unintended transmitter are aligned with this LoS or reflection path, interference will be imposed. As shown in Fig. 1, the reflection paths are generated by obstacles, obeying the

\footnote{1}{It may exist visible reflection paths whose lengths are smaller than the nearest visible LoS distance. Because the power along a reflection path is much weaker than that of an LoS path, we consider, in this work, LoS association only.}
specular reflection theory (i.e., the angle of incident is equal to the angle of reflection). The length of a first-order reflection path is the sum length of the two line segments constituting the reflection path.

### B. Assumptions

**Assumption 1:** An obstacle \( m \) is rectangular-shaped and is defined by a quadruple \( \{ C_m = (x_m, y_m), l_m, w_m, \theta_m \} \), where the \( C_m \), \( l_m \), \( w_m \), and \( \theta_m \) denote, respectively, the center coordinates, length, width, and rotation angle of obstacle \( m \), as shown in Fig. 1. Here, the \( \theta_m \) measures the anti-clockwise rotation angle from the positive x-axis to the l-side of obstacle \( m \). The set \( \{ C_m \} \) obeys a homogeneous PPP on \( \mathbb{R}^2 \) with density \( \lambda_{\Omega, m} \). We assume \( l_m \), \( w_m \), and \( \theta_m \) are mutually independent. The random variables \( \{ l_m \} \) (respectively, \( \{ w_m \} \) and \( \{ \theta_m \} \) ) are i.i.d. according to a uniform distribution in the interval \( l_m \in [l_{Min}, l_{Max}] \) (resp., \( w_m \in [w_{Min}, w_{Max}] \) and \( \theta_m \in [0, \pi) \)).

**Assumption 2:** We set the maximum link distance as \( R_{Max} = 200 \text{m} \), which is measured at the 28 GHz band in an urban scenario with the transmit power \( P_{TX} = 30 \text{dBm} \) [7]. The received signal strengths along the paths with their lengths larger than \( R_{Max} \) become weak and negligible. Therefore, we discard any path with the length larger than \( R_{Max} \).

**Assumption 3:** Since the sizes of transmitters are much smaller than obstacles, the transmitters do not block any links. We assume the distribution of transmitters obeys a homogeneous PPP on \( \mathbb{R}^2 \) with density \( \lambda_{TX} \). The distance \( d \in (0, R_{Max}) \) between the considered receiver and its associated transmitter is then a random variable that follows the probability density function (PDF) [4, Corollary 8.1]

\[
f_d(x) = 2\pi x \lambda_{TX} \exp(-\beta x - p - 2\lambda_{TX} U(x)),
\]

where \( p = \lambda_{TX} E_l E_w \), \( \beta = 2\lambda_{TX} (E_l + E_w)/\pi \), and \( U(x) = e^{-p} \left[ 1 - (\beta x + 1)e^{-\beta x} \right]/\beta/\pi \).

**Assumption 4:** For a tractable analysis, we assume the directional beams at the transmitters and receiver in Fig. 1 obey the cone-shaped beam pattern defined by

\[
G(\phi) = \begin{cases} 
\frac{2\pi}{\chi}, & \text{if } -\frac{\chi}{2} \leq \phi \leq \frac{\chi}{2} \\
0, & \text{otherwise}
\end{cases}
\]

where \( G(\phi) \) is the array gain with boresight direction \( \phi \) uniformly distributed in \([0, 2\pi)\) and \( \Delta \) is the mainlobe beamwidth in radian. The transmitters and the receiver have different beamwidths, denoted by \( \Delta_{TX} \) and \( \Delta_{RX} \), respectively. The probability that the beams of an unintended transmitter and the receiver are aligned along a path is then given by

\[
\frac{\Delta_{TX} \Delta_{RX}}{\Delta_{TX} + \Delta_{RX}},
\]

**Assumption 5:** We admit the LoS pathloss model at 28 GHz in [8], i.e., \( P_{LoS}(r) = 61.4 + 20 \log 10(r) \) in dB, where \( r \) is the LoS path length in meter. For a first-order reflection path with length \( r \), we use the pathloss model in [9], i.e., \( P_{LoS}(r) = 61.4 + 20 \log 10(r) + RL \) in dB, where \( RL \) is the reflection loss. Once a \( q \in \{ LoS, Ref \} \) path of length \( r \) presents with which both the beams of the considered receiver and an unintended transmitter are aligned, the interference power is given by

\[
I_q(r) = P_{TX} G_{TX} G_{RX} 10^{-\frac{P_{LoS}(r)}{10}},
\]

where \( G_{TX} \) and \( G_{RX} \) are, respectively, the transmit and receiver array gains and follow (2), e.g., \( G_{TX} = \frac{2\pi}{\Delta_{TX}} \).

### III. PRELIMINARIES

In this section, we summarize two existing stochastic models for a P2P mmWave link, and based on these, derive useful statistics that will be used in the sequel. We focus on the P2P link from the unintended transmitter at \((D, 0)\) to the receiver at \((0, 0)\) in Fig. 2. For ease of description, we denote the unintended transmitter as \( T \) and the intended receiver as \( R \).

#### A. First-Order Reflection: Ellipse Model

The *Ellipse Model* in [6] can be exploited to characterize the average number of reflection paths of a P2P link. The *Ellipse Model* is based on the fact that the reflection points of all the first-order reflection paths with the same length \( r \in (D, D_{Max}) \) reside on a common ellipse, defined by

\[
\Omega(r) = \left\{ (x, y) : \frac{(x - D/2)^2}{r^2/4} + \frac{y^2}{(r^2 - D^2)/4} = 1 \right\},
\]

where the foci of \( \Omega(r) \) are the \( T \) and \( R \), as shown in Fig. 2(a). For obstacles with an arbitrary rotation angle \( \theta \), there are four possible reflection points \( Z_i, i \in \{1, 2, 3, 4\} \). The coordinate of \( Z_i = (x_i, y_i) \) can be computed by using [6, (12) and (13)]. The first-order reflection path at \( Z_i \) exists if and only if the following two events occur: (i) \( \text{Event } A_i \): The reflection path is created at \( Z_i \) by the obstacles with arbitrary \( l \) and \( w \), but a fixed \( \theta \) and (ii) \( \text{Event } B_i \): The reflection path at \( Z_i \) is not blocked by any other obstacles.

The \( \text{Event } A_i \) occurs when the center of an obstacle resides on the \( A_i B_i \) (e.g., \( A_1 B_1 \) in Fig. 2(a)). The \( L_{AB_i} \) can be \( l \) or \( w \), depending on which side of the obstacle tangent to the ellipse. However, because the area of a line is zero, the \( \text{Prob } A_i (\theta, r, D) = 0 \). Instead, finding the PDF of \( A_i \) is of interest. To this end, we introduce the reflection points \( \hat{Z}_i, i \in \{1, 2, 3, 4\} \) (e.g., \( \hat{Z}_i \) in Fig. 2(a)) for the obstacles with the same orientation \( \theta \), but placed on ellipse \( \Omega(r + dr) \) in Fig. 2(a), where \( dr \) is an infinitesimal increment of \( r \). We denote the counterpart of \( A_i B_i \) for \( \hat{Z}_i \) as \( \hat{A}_i \hat{B}_i \) (e.g., \( \hat{A}_1 \hat{B}_1 \)). The region \( \hat{A}_i \hat{B}_i B_i A_i \) is then a parallelogram that corresponds to the probability that obstacles with the fixed \( \theta \) create
reflection paths at the reflection points on $Z_1 Z_2$. One can first compute this probability. As the $dr$ tends to 0, the PDF of $A_i$ is attained by normalizing the probability by $dr$. This has previously been found in [7, (19)] as

$$f_{A_i} (\theta, r, D) = \lambda_{O_{b_1}} \frac{\gamma_i}{2} \sqrt{r^2 - D^2 \cos^2 \theta},$$

(5)

where $\gamma_i = E_i$ when $i = 1$ and $3$ and $\gamma_2 = E_w$ otherwise. The PDF of the first-order reflection path at $Z_i$ is then obtained by

$$f_{Z_i} (\theta, r, D) = f_{A_i} (\theta, r, D) Pr_{R_{B_i} | A_i} (\theta, r, D).$$

(6)

Provided (6), one can compute the average number of reflection paths of the P2P link $\overline{TR}$ with length $D$ as

$$E_N (D) = \sum_{i=1}^{4} \int_0^\pi \int_D f_{Z_i} (\theta, r, D) d\theta d\phi.$$

(7)

Finding $E_N (D)$ in (7) requires the expression for $Pr_{R_{B_i} | A_i} (\theta, r, D)$ in (6). Although this has previously been studied in [1, 6, 10], their analytical models are tailored for circular obstacles [1], rectangular obstacles with common orientation [6], and line-shaped obstacles [10]. We will investigate, in Section IV, the model for the rectangular obstacles with random orientation in mmWave interference networks by exploiting the LoS blockage model in the below.

B. LoS Blockage Model

We introduce the LoS blockage model in [4] to compute the unblockage probability of an LoS link. According to [4, Theorem 1], the LoS path $\overline{TR}$ in Fig. 2(b) is blocked when the center of an obstacle with $l$, $w$, and $\theta$ falls in the region EFGHJ, where the area of EFGHJ is given by $S_{\overline{TR}} (l, w, \theta, D)$. The $h(l, w, \theta)$ is the height of the parallelogram EFHJ in Fig. 2(b) and the $l w$ is the sum area of two right triangles EIJ and FGH in Fig. 2(b). The expectation of $S_{\overline{TR}} (l, w, \theta, D)$ over $l$, $w$, and $\theta$ is then given by $E_{S_{\overline{TR}}} (D) = E_h D + E_I E_w$, where 

$$E_h = \frac{2 (E_I + E_w)}{\pi}.$$ 

The number of the obstacles that block the $\overline{TR}$ follows the Poisson distribution with mean $\lambda_{O_{b_1}} E_{S_{\overline{TR}}} (D)$. Hence, the unblockage probability of the path $\overline{TR}$ is given by

$$Pr(D) = \exp (- \lambda_{O_{b_1}} E_{S_{\overline{TR}}} (D)),$$

(8)

where $C$ denotes the unblockage event of an LoS path.

IV. REFLECTION PATH BLOCKAGE ANALYSIS

We now derive an expression for the first-order reflection path blockage probability $Pr_{R_{B_i} | A_i} (\theta, r, D)$ in (6), associated with the reflection point $Z_i = (x_i z_i, \theta, r, D)$ in Fig. 2(a), in order to compute the $E_N (D)$ in (7). First, define $E_{S_{R_{B_i} | i}} (\theta, r, D)$ as the average area where the centers of obstacles falling in and those obstacles block the reflection path at $Z_i$. Then, the number of the obstacles blocking the reflection path is Poisson distributed with mean $\lambda_{O_{b_1}} E_{S_{R_{B_i} | i}} (\theta, r, D)$. If $E_{S_{R_{B_i} | i}} (\theta, r, D)$ is known, we have

$$Pr_{R_{B_i} | A_i} (\theta, r, D) = \exp (- \lambda_{O_{b_1}} E_{S_{R_{B_i} | i}} (\theta, r, D)).$$

(9)

Hence, in what follows, we focus on finding the $E_{S_{R_{B_i} | i}} (\theta, r, D)$ associated with $Z_i$ in Fig. 2(a), keeping in mind that the exactly same analysis applies to compute the $E_{S_{R_{B_i} | i}} (\theta, r, D), i \in \{2, 3, 4\}$. For ease of notation, we will omit the $(\theta, r, D)$ from $E_{S_{R_{B_i} | i}} (\theta, r, D)$ in the sequel.

The reflection path at $Z_i$, which has two constituent line segments, i.e., $\overline{T Z_1}$ and $\overline{R Z_1}$, is illustrated in Fig. 2(c). The average area $E_{S_{\overline{T Z_1}}} (r, \theta)$ (resp., $E_{S_{\overline{R Z_1}}}$) characterizing the blockage of $\overline{T Z_1}$ (resp., $\overline{R Z_1}$) follows the LoS blockage model in Fig. 2(b). There is non-negligible chance that an obstacle near $Z_1$ can simultaneously block the $\overline{T Z_1}$ and $\overline{R Z_1}$. Then simply adding $E_{S_{\overline{T Z_1}}}$ and $E_{S_{\overline{R Z_1}}}$ to obtain the $E_{S_{R_{B_1} | i}}$ is of low accuracy because of the overlap. On the other hand, analyzing the various cases of overlap events is prohibitively complicated. We overcome this difficulty by employing approximations.

A. Approximation of Average Area $E_{S_{R_{B_1} | i}}$

Similar to Fig. 2(b), the $\overline{T Z_1}$ in Fig. 2(c) is blocked when the center of an obstacle with $l$, $w$, and $\theta$ falls in the EFGHJ, whose average area is $E_{S_{\overline{T Z_1}}} = E_h L_{\overline{T Z_1}} + E_I E_w$. Similarly for $\overline{R Z_1}$, we have $E_{S_{\overline{R Z_1}}} = E_h L_{\overline{R Z_1}} + E_I E_w$. We now approximate the $E_{S_{R_{B_1} | i}}$ by decoupling the correlation between the blockages of $\overline{T Z_1}$ and $\overline{R Z_1}$.

To this end, we first draw a Line $c_1$ in Fig. 2(c), which is obtained by extending $\overline{H H}$. We then find a point $A_1$, the cross point between the Line $c_1$ and $\overline{T Z_1}$. One can perceive that the blockages of $\overline{R Z_1}$ and $\overline{A_1 Z_1}$ are highly correlated, while the blockages of the $\overline{T Z_1}$ and $\overline{A_1 Z_1}$ are almost independent. Exploiting the high correlation between $\overline{R Z_1}$ and $\overline{A_1 Z_1}$ allows us to approximate the average area characterizing the blockage of the path constructed by $\overline{R Z_1}$ and $\overline{A_1 Z_1}$ as $E_{S_{\overline{T Z_1}}}$. Then the near-independent blockage between $\overline{R Z_1}$ and $\overline{A_1 Z_1}$ leads to an approximation

$$E_{S_{R_{B_1} | i}} \approx E_{S_{\overline{T Z_1}}} + E_{S_{\overline{R Z_1}}},$$

(10)
where $E_{S_{\Gamma},1} \approx E_i L_{\text{TX}} + E_i E_w / 2$, obtained by only including one right triangle VSO in Fig. 2(c). This consideration is to reduce the overlap between $E_{S_{\Gamma},0}$ and $E_{S_{\Gamma},1}$. We now look at the case when the Line $c_1$ has no cross point $A_1$ with $TZ_i$. This happens when the Line $c_1$ is below the unintended transmitter $T$. This case represents highly correlated blockage between $RZ_i$ and $TZ_i$ and we have an approximation

$$E_{S_{\Gamma},1} \approx E_{S_{\Gamma},0}. \tag{11}$$

In summary, the approximation of $E_{S_{\Gamma},1}$ is obtained by

$$E_{S_{\Gamma},1} \approx \begin{cases} E_i (L_{\text{TX}} + L_{\text{RX}}) + 3E_i E_w / 2, & \text{if } A_1 \text{ exists} \\ E_i L_{\text{RX}} + E_i E_w, & \text{otherwise} \end{cases} \tag{12}$$

When computing the $L_{\text{RX}}$ in (12), the coordinates of $A_1$ must be known. This can be done by finding the equation of Line $c_1$. Note that the $RZ_i$ and Line $c_1$ have the same slope $k_1 = y_{Z_i}/x_{Z_i}$ due to parallelism. As shown in Fig. 2(c), the intersection angle and the cross point between the Line $c_1$ and $x$-axis are $\alpha = \arctan k_1$ and $B_1 = (x_{B_1}, 0)$, respectively, where $x_{B_1} = \frac{E_i}{2 \sin \alpha}$. The equation of Line $c_1$ is therefore $y = k_1 (x - x_{B_1})$. Provided the equation of $TZ_i$, $y = \frac{V_i}{x_{Z_i} - D} (x - D)$, the coordinates of $A_1$ are given by the cross point between the $TZ_i$ and Line $c_1$.

When applying the above approach to the reflection paths at $Z_i, i \in \{2, 3, 4\}$, we obtain, similar to (12), the $E_{S_{\Gamma},1}$ for $i \in \{2, 3, 4\}$. The $E_{S_{\Gamma},1}$ for $i \in \{1, 2, 3, 4\}$ allow us to compute the $\Pr_B(d, A_4)$ using (9) and this leads to (6) and, consequently, (7). It should be mentioned that the number of average reflection paths $E_{\Gamma,1}(D)$ in (7) does not admit a closed-form expression, and hence it is calculated numerically.

V. NETWORK INTERFERENCE ANALYSIS

In this section, we find the statistics of the number of interferers and derive the mean and CDF of the accumulated network interference power. These results are on the basis of the following independent blockage model.

A. Independent Blockage Model

When multiple links are close in location, one obstacle can simultaneously block some of them, implying that link blockage events are correlated in general. Nevertheless, in mmWave, ignoring the latter imposes less degradation on the accuracy of the analysis. This can be justified if there exist only a few interference paths without considering the blockage effect [4]. Unlike [4], where the independent blockage model was validated via simulation studies, we analytically verify the latter model based on the results in Section III-A in the below.

Denote the numbers of LoS and reflection interference paths without considering the blockage effect as $\Gamma_{\text{LoS}}$ and $\Gamma_{\text{Ref}}$, respectively. We first focus on the $\Gamma_{\text{LoS}}$. Given the distance $d$ of the intended LoS path, LoS interferers can only reside within the region outlined by $O(d)$ and $O(R_{\text{Max}})$. Since the average number of transmitters on a $O(D), D \in [d, R_{\text{Max}}]$, is $2\pi D \lambda_{TX}$, the average total number of LoS interference paths without considering the blockage effect, for a given $d$, is given by

$$E_{\Gamma_{\text{LoS}}}(d) = \int_d^{R_{\text{Max}}} 2\pi D \lambda_{TX} \frac{\Delta f_{\text{TX}} \Delta f_{\text{RX}}}{4\pi^2} dD = \int_d^{R_{\text{Max}}} \xi dD, \tag{13}$$

where $\xi \triangleq \lambda_{TX} \frac{\Delta f_{\text{TX}} \Delta f_{\text{RX}}}{4\pi^2}$. Taking the expectation of $E_{\Gamma_{\text{LoS}}}(d)$ with respect to the intended path distance $d$ leads to

$$E_{\Gamma_{\text{LoS}}} = \int_0^{R_{\text{Max}}} \int_0^{R_{\text{Max}}} \xi D dD f_d(x) dx. \tag{14}$$

In our work, the reflection interferers are independent of $d$ and can reside on any $O(D)$ with $D \in [0, R_{\text{Max}}]$. Therefore, the total number of reflection interference paths is computed as

$$E_{\Gamma_{\text{Ref}}} = \int_0^{R_{\text{Max}}} \xi D f_d(x) dx. \tag{15}$$

The total number of interference paths without considering the link blockage is then $E_{\Gamma_{\text{Total}}} = E_{\Gamma_{\text{LoS}}} + E_{\Gamma_{\text{Ref}}}$ in (13) and (14).

To evaluate the $E_{\Gamma_{\text{Total}}}$, we set $l \in [10, 14]$ m, $w \in [5, 10]$ m, $\Delta_{TX} = 13$ degree, $\Delta_{RX} = 20$ degree, $\lambda_{TX} = 0.005$, and $\lambda_{RX} \leq 0.0014$. Under this parameterization, there are on average 628 transmitters inside the $O(R_{\text{Max}}), R_{\text{Max}} = 200$ m, and the $E_{\Gamma_{\text{Total}}}$ is upper-bounded by $E_{\Gamma_{\text{Total}}} \leq 3.8$. Compared to the simulation studies in [4] where the average number of interference paths guaranteeing the independent link blockage is found to be below 6, the $E_{\Gamma_{\text{Total}}} \leq 3.8$ suffices to statistically ensure the independent blockage. Therefore, in the sequel, we focus on the network under the above parameterization and treat the blockage of individual link to be independent.

B. Statistics of Number of Interferers

In this subsection, we characterize the numbers of LoS and reflection interferers, denoted by $K_{\text{LoS}}$ and $K_{\text{Ref}}$, respectively. The analysis is based on the derivations of $E_{\Gamma_{\text{LoS}}}$ in (13) and $E_{\Gamma_{\text{Ref}}}$ in (14), while considering the blockage effect.

With the above independent blockage model, the average number of LoS interferers on $O(D)$ is given by $v_{\text{LoS}}(D) = \xi D \Pr_{\text{LoS}}(D)$, where $\Pr_{\text{LoS}}(D)$ is the LoS unblockage probability in (8). Referring to (13), the average number of LoS interferers in the network is obtained by

$$E_{K_{\text{LoS}}} = \int_0^{R_{\text{Max}}} \int_0^{R_{\text{Max}}} v_{\text{LoS}}(D) dD f_d(x) dx. \tag{16}$$

According to (15), the number $K_{\text{LoS}}(d)$ for a fixed $x = d$ follows a non-homogeneous Poisson distribution with the density $v_{\text{LoS}}(D), D \in [d, R_{\text{Max}}]$, and the mean $E_{K_{\text{LoS}}}(d) = \int_d^{R_{\text{Max}}} v_{\text{LoS}}(D) dD$, namely,

$$\Pr(K_{\text{LoS}}(d) = k) = e^{-E_{K_{\text{LoS}}}(d)} \left(E_{K_{\text{LoS}}}(d)\right)^k / k!. \tag{17}$$

Hence, the probability $\Pr(K_{\text{LoS}} = k)$, is computed as

$$\Pr(K_{\text{LoS}} = k) = \int_0^{R_{\text{Max}}} \Pr(K_{\text{LoS}}(x) = k) f_d(x) dx. \tag{18}$$
Similarly, from (14), the average number of reflection interferers on \( \Omega(D) \) is given by
\[
E_{K_{\text{Ref}}} = \int_0^{R_{\text{Max}}} \nu_{\text{Ref}}(D) dD. 
\] (17)

Analogous to the above LoS case, the number of reflection interferers \( K_{\text{Ref}} \) in the network follows a non-homogeneous Poisson distribution with the density \( \nu_{\text{Ref}}(D) \), \( D \in [0, R_{\text{Max}}] \), and the mean \( E_{K_{\text{Ref}}} \). Thus,
\[
\text{Pr}(K_{\text{Ref}} = k) = e^{-E_{K_{\text{Ref}}}} \left( E_{K_{\text{Ref}}} \right)^k / k!. 
\]

Remark: A beam of an interferer may be aligned with the beam at the receiver through both the LoS and reflection paths or two reflection paths when: (i) The two paths exist and (ii) The two paths simultaneously reside in the mainlobe of the beams. In general, the joint probability of these two events is very low due to the path blockage, random reflection paths, and small \( \Delta \)s. Because the placements of interferers, obstacles, and beams are all i.i.d., we treat the LoS and reflection interference events independently.

C. Mean of Accumulated Interference Power

The sum \( q \in \{\text{LoS, Ref}\} \) interference can be written as
\[
\Lambda_q = \sum_{i=1}^{K_q} I_q(r_i),
\] (18)
where \( I_q(r_i) \) is the \( i \)th \( q \in \{\text{LoS, Ref}\} \) interference power with the link length \( r_i \) and is computed by (3). Based on (18), the accumulated network interference power is given by
\[
\Lambda_{\text{Tot}} = \Lambda_{\text{LoS}} + \Lambda_{\text{Ref}}
\] (19)
with the mean \( E_{\Lambda_{\text{Tot}}} = E_{\Lambda_{\text{LoS}}} + E_{\Lambda_{\text{Ref}}} \). The \( E_{\Lambda_q} \) for \( q \in \{\text{LoS, Ref}\} \) can be written as
\[
E_{\Lambda_q} = E_{K_q E_{I_q}} = E_{K_q E_{I_q}},
\] (20)
where \( (a) \) is due to the i.i.d. \( (I_q(r_i)) \).

Calculating the \( E_{I_q} \) to find the \( E_{\Lambda_q} \) in (20) is of interest. From (15), for a fixed \( d \), the PDF of an LoS interferer on \( \Omega(D) \), \( D \in [d, R_{\text{Max}}] \), is obtained by \( \nu_{\text{LoS}}(D)/E_{K_{\text{LoS}}}(d) \). Because all of the LoS interferers on \( \Omega(D) \) have the same \( I_{\text{LoS}}(D) \), we have
\[
E_{I_{\text{LoS}}(D)} = \int_0^{R_{\text{Max}}} \int_0^{R_{\text{Max}}} \frac{\nu_{\text{LoS}}(D)}{E_{K_{\text{LoS}}}(D)} I_{\text{LoS}}(D) dDf_D(x)dx.
\] (21)
Finding the \( E_{I_{\text{LoS}}} \) requires a rather refined treatment. This is because, unlike the LoS interferers, the reflection interferers on \( \Omega(D) \) can have different reflection path lengths \( r(D) \in [D, D_{\text{Max}}] \), leading to different statistics of reflection interference power \( I_{\text{Ref}}(r(D)) \) for different \( r(D) \) values. We first formulate the mean of \( I_{\text{Ref}}(r(D)) \) as
\[
E_{I_{\text{Ref}}}(D) = \int_D^{R_{\text{Max}}} I_{\text{Ref}}(z)f_r(D)(z)dz,
\] (22)
where \( f_r(D)(z) \) is the PDF of \( r(D) \). With the \( E_{I_{\text{Ref}}}(D) \) in (22), we obtain
\[
E_{I_{\text{Ref}}} = \int_0^{R_{\text{Max}}} \nu_{\text{Ref}}(D)/E_{K_{\text{Ref}}}(D) E_{I_{\text{Ref}}}(D) dD.
\] (23)

To find the \( E_{I_{\text{Ref}}}(D) \), the expression of \( f_r(D)(z) \) in (22) is needed.

Because the reflection interferers on \( \Omega(D) \) are i.i.d., we focus, without loss of generality, on a reflection interferer located at \( (D, 0) \). By the Ellipse Model in Section III-A, any point in the region outlined by the ellipse \( \Omega(R_{\text{Max}}) \) in (4) can be a reflection point of a reflection path between the reflection interferer at \( (D, 0) \) and the receiver. The area of the region is \( \pi \sqrt{\frac{L_{\text{Max}}^2}{2}} \). The contribution of an ellipse \( \Omega(z) \) to the latter area can be approximated as \( \pi \frac{z}{2} - D^2 \), where \( z \) is an infinitesimal increment of \( z \) and \( D \) is due to the first-order Maclaurin series \( x\sqrt{1-\frac{x^2}{2}} \approx x \left( 1-\frac{x^2}{2} \right) \). Hence we approximate the \( f_r(D)(z) \) in (22) as
\[
f_r(D)(z) \approx \frac{z/2 - D^2/8z}{R_{\text{Max}}^2 - D^2/4}.
\] (24)

With the \( f_r(D)(z) \) in (24), the \( E_{I_{\text{Ref}}}(D) \) in (23) is readily calculated.

Substituting (15), (17), (21), and (23) into (20) allows us to finally compute the \( E_{\Lambda_{\text{Tot}}} = E_{\Lambda_{\text{LoS}}} + E_{\Lambda_{\text{Ref}}} \).

D. CDF of Accumulated Interference Power

The approach to find the CDF \( \text{Pr}(\Lambda_{\text{Tot}} \leq x) \) is based on taking the Laplace transformation of \( \Lambda_{\text{Tot}} \).
\[
\mathcal{L}_{\Lambda_{\text{Tot}}}(s) = E^{-s \cdot \Lambda_{\text{LoS}}} E^{-s \cdot \Lambda_{\text{Ref}}} \prod_{q \in \{\text{LoS, Ref}\}} E_{E_{I_q}(s)}^{c_k},
\] (25)
where \( (c) \) is due to the i.i.d. \( \{I_q(r_i)\} \). The \( E_{\Lambda_{\text{LoS}}} \), and \( E_{\Lambda_{\text{Ref}}} \) in (25) are readily found by replacing the \( I_{\text{LoS}}(D) \) in (21) with \( e^{-s I_{\text{LoS}}(D)} \) and \( I_{\text{Ref}}(z) \) in (22) with \( e^{-s I_{\text{Ref}}(z)} \), respectively. With the \( E_{E_{I_q}} \) for \( q \in \{\text{LoS, Ref}\} \), we compute
\[
E_{E_{I_q}}^{k_q} = \sum_{k_q=0}^{\infty} (E_{E_{I_q}})^k \text{Pr}(K_q = k).
\]

This allows us to find the \( \mathcal{L}_{\Lambda_{\text{Tot}}}(s) \) in (25).

The CDF \( \text{Pr}(\Lambda_{\text{Tot}} \leq x) \) is now obtained by taking the inverse Laplace transformation of \( \mathcal{L}_{\Lambda_{\text{Tot}}}(s)/s \). However, analytically finding the inverse Laplace transform of \( \mathcal{L}_{\Lambda_{\text{Tot}}}(s)/s \) to have the closed-form expression of \( \text{Pr}(\Lambda_{\text{Tot}} \leq x) \) is challenging. Instead, we exploit a numerical method, the Euler algorithm [11, (36)], to numerically compute the inverse Laplace transformation of \( \mathcal{L}_{\Lambda_{\text{Tot}}}(s)/s \). This operation is a standard numerical procedure and the details are omitted here.

VI. NUMERICAL RESULTS

We examine the proposed analytical interference models by comparing them with Monte-Carlo simulations. In the simulations, obstacles model building in urban scenarios with lengths \( L \in [10, 14] \) m and widths \( W \in [5, 10] \) m. We consider concrete buildings with the reflection loss \( RL = 7 \) dB [9]. We set the \( \Delta_{\text{TX}} = 13 \) degree and \( \Delta_{\text{RX}} = 20 \) degree. The noise power in linear scale is given by \( W_0 = BW \cdot 10^{-3} \cdot \left( \frac{N_0}{F} + F \right) \), where \( BW = 1 \) GHz, thermal noise power density \( N_0 = -174 \) dBm/Hz, and noise figure \( NF = 9 \) dB, resulting in \( W_0 = -105 \) dB. Note that the same parameters were used to evaluate the \( E_{\Lambda_{\text{LoS}}} \) in (13) and \( E_{\Lambda_{\text{Ref}}} \) in (14).
Fig. 3(a) shows the average numbers of LoS and reflection interferers $E_{K_{\text{LoS}}}$ and $E_{K_{\text{Ref}}}$ in (15) and (17), respectively, across $\lambda_{\text{LoS}}$ for $\lambda_{TX} = 0.005$. Tight matching between the analysis and Monte-Carlo simulations is evidenced. Seen from Fig. 3(a), the $E_{K_{\text{LoS}}}$ is monotonically decreasing with respect to $\lambda_{\text{LoS}}$ due to the increased blockage. For reflection interferers, increasing $\lambda_{\text{LoS}}$ generates more first-order reflection paths, but at the same time, increases the blockage. When $\lambda_{\text{LoS}} < 0.0008$, the generation of reflection paths dominates the blockage of reflection paths, and thus the $E_{K_{\text{Ref}}}$ grows with $\lambda_{\text{LoS}}$. When $\lambda_{\text{LoS}} > 0.0008$, opposite trend is observed since the blockage dominates the generation of reflection paths.

For fixed $\lambda_{\text{LoS}} = 0.0005$, the $E_{K_{\text{LoS}}}$ and $E_{K_{\text{Ref}}}$ for $q \in \{\text{LoS}, \text{Ref}\}$ across $\lambda_{TX}$ are displayed at the left and right subfigures of Fig. 3(b), respectively. Seen from the left subfigure, the $E_{K_{\text{LoS}}}$ and $E_{K_{\text{Ref}}}$ are linear with $\lambda_{TX}$, which is clear from (15) and (17). In the right subfigure, it can be seen that the $E_{K_{\text{LoS}}}$ is over 10 dB larger than the $E_{K_{\text{Ref}}}$. Since the $E_{K_{\text{LoS}}}$ and $E_{K_{\text{Ref}}}$ in the left subfigure are in the same order, the large gap between $E_{K_{\text{LoS}}}$ and $E_{K_{\text{Ref}}}$ is mainly due to the fact that the reflection pathloss is much higher than the LoS pathloss. Moreover, it is observed that both $E_{K_{\text{LoS}}}$ and $E_{K_{\text{Ref}}}$ are far larger than the noise power $W_0 = -105$ dB.

Fig. 3(c) compares analytic CDF curves of the $\Lambda_{\text{Tol}}$ and $\Lambda_{\text{LoS}}$ in (19) with those of Monte-Carlo simulations when $\lambda_{TX} = 0.005$ and $\lambda_{\text{LoS}} = 0.0005$. Since $\Lambda_{\text{Tol}} \geq \Lambda_{\text{LoS}}$, when $\Lambda_{\text{Tol}} \leq x$, the $\Lambda_{\text{LoS}}$ must satisfy $\Lambda_{\text{LoS}} \leq x$, yielding $P_r(\Lambda_{\text{Tol}} \leq x) \leq P_r(\Lambda_{\text{LoS}} \leq x)$. The minimum $\Lambda_{\text{LoS}}$ (i.e., $\Lambda_{\text{LoS}} = I_q(R_{\text{LoS}})$ in (3)) for $q \in \{\text{LoS, Ref}\}$ are $-80.4$ dB and $-87.4$ dB, respectively. This reveals, in Fig. 3(c), the constant CDF $P_r(\Lambda_{\text{Tol}} \leq x)$ for $x \in [-105, -87.4]$ (resp., $P_r(\Lambda_{\text{LoS}} \leq x)$ for $x \in [-105, -80.4]$) dB. About $0.2$ CDF difference between $P_r(\Lambda_{\text{Tol}} \leq x)$ and $P_r(\Lambda_{\text{LoS}} \leq x)$ at $x \in [-105, -87.4]$ dB and the drastic increase of the CDF of $\Lambda_{\text{LoS}}$ at $x \in [-87.4, -80.4]$ dB demonstrate the impact of the reflection interference in the network. When $x \in [-80.4, -55]$ dB, we find $P_r(\Lambda_{\text{LoS}} \leq x) \approx P_r(\Lambda_{\text{Tol}} \leq x)$. This is because the $\Lambda_{\text{LoS}}$ is much smaller than the $\Lambda_{\text{LoS}}$, as seen from Fig. 3(b). When the LoS and reflection interferences coexist, the LoS interference overwhelms the reflection counterpart.

VII. CONCLUSION

We proposed analytical interference models characterizing the accumulated interference power in a dense urban mmWave network in the presence of geometry-dependent first-order reflection paths. The statistics of the numbers of LoS and reflection interferers and the mean and CDF of the accumulated network interference power were derived. Simulation results demonstrated the accuracy of the analysis and showed that the dense mmWave network is often interference-limited. The geometry-dependent reflection interference is non-negligible. It is indeed an indispensable component in characterizing the mmWave network.

REFERENCES