System Performance Analysis of Distributed Raman Amplification With Dual-Order Forward Pumping

Arin Dutta, *Student Member, IEEE*, Youichi Akasaka, *Senior Member, IEEE*, and Rongqing Hui, *Senior Member, IEEE*

***Abstract*—Performance of distributed Raman amplification (DRA) system with dual order forward (FW) pumping is analyzed with the consideration of both pump relative intensity noise (RIN) to signal phase noise transfer and signal nonlinear interference. The efficiencies of pump RIN to signal phase noise transfer are theoretically analyzed and experimentally verified by measuring signal phase noise introduced by small index intensity modulations applied on the pump lasers. The results indicate that the efficiency of 2nd order pump RIN to signal phase noise transfer can be more than 2 orders of magnitude higher than that from the 1st order pump. Although dual order FW Raman pumping corresponds to a slight increase of amplified spontaneous emission (ASE) compared to using only a 1st order pump, its major advantage comes from the reduction of nonlinear interference noise in a dense wavelength-division multiplexing (DWDM) system. Because pump RIN to signal phase noise transfer has lowpass characteristics, systems at high baud rates, such as 100 Gbaud, are less susceptible to the impact of pump laser RIN.**

***Index Terms*—Bit-error rate (BER), dual-order pumping, forward pumping, optical signal to noise ratio (OSNR), OSNR penalty, phase noise, relative intensity noise (RIN).**

I. Introduction

Distributed Raman Amplification (DRA) provides improved optical signal to noise ratio (OSNR) compared to systems only using Erbium-doped fiber amplifiers (EDFA), and helps extending the reach of long-distance fiber-optic communication systems [1]. DRA with backward (BW) Raman pumping scheme is most often used in commercial fiber-optic systems to provide efficient Raman gain and with less stringent requirement in the RIN of the pump lasers. However, because the optical noise is mostly generated near the end of the fiber span where the pump power is the highest, the OSNR improvement of BW Raman pumping is not as efficient as FW Raman pumping. Applying a high-power 2nd-order BW Raman pump together with a low power 1st order BW Raman pump can slightly reduce the optical noise by pushing the highest Raman gain region away from the end of the fiber span, the OSNR improvement is usually about 2 dB.

Forward (FW) Raman pumping has been adopted to further improve the DRA performance as it is more efficient in OSNR improvement because the optical noise is generated near the beginning of the fiber span and attenuated along the fiber. The major concern of FW Raman pumping scheme is the RIN transfer from the pump laser to the intensity noise and phase noise of the optical signal because they propagate in the same direction. Thus, the RIN of pump lasers usually needs to be low enough when used in the FW pumping scheme. Another concern of FW pumped DRA is the increase of signal optical power near the beginning of the fiber span so that the nonlinear phase shift of the optical signal can be increased, and the system is more susceptible to fiber nonlinear effects such as self-phase modulation (SPM), cross-phase modulation (XPM), and four-wave mixing (FWM).

2nd order FW Raman pumping has also been used in DRA [2], [3], while it does not improve OSNR at the receiver, it helps reduce nonlinear effects of the optical signal. This can be applied to long haul fiber-optic systems with many fiber spans as well as metro links with long fiber spans to improve system link budget. Although experimental works [3] clearly demonstrated improved transmission performance of high capacity DWDM fiber system utilizing 2nd order FW Raman pumping, systematic analysis is still needed to better understand the mechanism and parameter tradeoffs of this performance improvement.

The mechanism of pump laser RIN transfer into signal intensity noise and phase noise has been extensively investigated for DRA with 1st order pumping [4], [5], [6]. When 2nd order Raman pumping is introduced with the required optical power much higher than that of the 1st order pump, the impact of its RIN needs to be taken into consideration. In a DRA system with dual order pumping, there are three possible paths of RIN transfer: (1) from the 1st order pump to the signal directly, (2) from the 2nd order pump to the signal directly, and (3) from 2nd order pump to the signal via the 1st order pump. When only considering pump RIN to signal intensity noise transfer, as investigated both numerically and experimentally in Ref. [7], path (2) does not exist because signal is outside the Raman gain window of the 2nd order pump. However, the RIN of the 2nd order pump can be directly translated to signal phase noise through XPM, and this important transfer path was not considered in [7]. The bandwidth of XPM is proportional to $\alpha_p/d_{s,p}$ for FW Raman

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Arin Dutta and Rongqing Hui are with the Department of Electrical Engineering and Computer Science, University of Kansas, Lawrence, KS 66045 USA (e-mail: arindutta60@ku.edu; rhui@ku.edu).

Youichi Akasaka is with the Fujitsu Network Communications, Inc., Dallas, TX 75252 USA (e-mail: youichi.akasaka@fujitsu.com).

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pumping, where α_p is fiber loss at the pump wavelength and $d_{s,p}$ is the walk-off between the pump (p) and the signal (s) [5]. Although the 2nd order pump has faster walk-off than that of the 1st order pump because of the larger wavelength separation from the signal, the ratio of XPM bandwidths between 1st and 2nd order pumps is usually less than 3 dB. But the power of the 2nd order pump can be orders of magnitude stronger than the 1st order pump. Pump RIN to signal intensity noise and phase noise transfer has been theoretically analyzed in systems with high order pumping [8], which indicates that in coherent fiber-optic systems based on complex optical field modulation and detection, system performance is more susceptive to phase noise of the optical signal than the intensity noise. Pump RIN to signal phase noise transfer in a system with 1st order Raman pumping has been investigated both theoretically and experimentally [9], but it does not include the impacts of path (2) and path (3) mentioned above in systems with dual order Raman pumping.

As the performance of DRA with backward pumping is well understood with relatively low impact of RIN transfer, this paper is focused on the FW pumping scheme, and is intended to provide a comprehensive analysis on the system performance impact of dual order FW Raman pumping, including signal phase noise induced by the RINs of both 1st and the 2nd order pump lasers, as well as the impacts of linear and nonlinear noise. Similar to that used in ref. [8], our theoretical model considers the power evolution of the 1st and the 2nd order pumps and the optical signal along the fiber span, which is essential to evaluate the efficiency of pump RIN transfer. By applying a shallow intensity modulation to the pump laser to mimic the RIN, we were able to validate the calculated pump RIN to signal phase noise transfer efficiency experimentally.

Then the performance of the dual order FW pumped DRA configurations is compared with that of single order Raman pumping to understand trade-offs of system parameters. The nonlinear interference (NLI) noise is analyzed to study the overall OSNR improvement when employing a 2nd order Raman pump. Finally, a DWDM system with 16-QAM modulation is used as an example to show the benefit of DRA with dual order Raman pumping with different pump RIN levels. We also consider a DRA system using a 1st order incoherent pump together with a 2nd order coherent pump [10], [11]. In this case, the 1st order incoherent pump has negligible impact in the RIN transfer, that means RIN transfer paths (1) and (2) defined previously are negligible, but path (3) still exists which is shown to have the strongest impact in the system performance. For simplicity, the analysis in this research only considers a single polarization. Polarization crosstalk in polarization multiplexed optical systems has been investigated in [8], and the impact can be minimized by using depolarized or polarization-multiplexed Raman pump sources.

II. Theoretical Analysis of RIN to Phase Noise Transfer

Fig. 1 shows the basic configuration of a DRA system with FW Raman pumping. The output of the 1st order and the 2nd order pump lasers with the optical power P_1 and P_2, respectively,

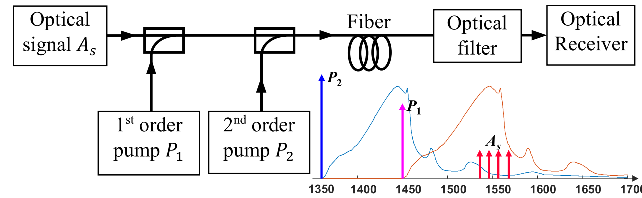

Fig. 1. Dual order FW pumped DRA system block diagram.

are combined with the optical signal A_s through WDM optical couplers, and they propagate in the same direction along the transmission fiber. As shown in the inset of Fig. 1, for a silica-based optical fiber with approximately 13 THz peak Raman shift, a high-power 2nd order pump at 1350 nm wavelength can amplify the 1st order pump at 1450 nm, which in turn provides Raman gain for the optical signals at the 1550 nm wavelength window.

The efficiency of RIN transfer from the pump lasers to the signal depends on their power profiles along the fiber. For a system with only a 1st order pump, the pump power decreases exponentially along the fiber, so that an analytical formula can be derived to predict the RIN transfer efficiency [5]. When the 2nd order pump is introduced, the power change of the 1st order pump along the fiber is no longer monotonic, which complicates the analysis. Raman interaction among multi-wavelength channels along an optical fiber can be expressed by a propagation equation [12]:

$$\frac{dP(z,f_i)}{dz} = [-\alpha(f_i) + A(z,f_i) - B(z,f_i) - D(z,f_i)] \quad P(z,f_i) + 2hf_i\Delta f_i C(z,f_i) \tag{1}$$

where, $\alpha(f_i)$ is the fiber loss at optical frequency f_i, h is the plank's constant, Δf_i is the bandwidth of optical noise, and the four power dependent terms in (1) are,

$$A(z,f_i) = \sum_{m=i+1}^{N} \frac{g_R(f_m - f_i)}{\Gamma A_{eff}} P(z,f_m) \tag{2}$$

$$B(z,f_i) = \sum_{m=1}^{i-1} \frac{g_R(f_i - f_m)}{\Gamma A_{eff}} P(z,f_m) \tag{3}$$

$$C(z,f_i) = \sum_{m=i+1}^{N} \frac{g_R(f_m - f_i)}{2A_{eff}} P(z,f_m) \left[1 + \left(e^{\frac{h(f_m-f_i)}{kT}} - 1\right)^{-1}\right] \tag{4}$$

$$D(z,f_i) = 2h \sum_{m=1}^{i-1} f_m \Delta f_m \frac{g_R(f_i - f_m)}{2A_{eff}} P(z,f_m) \left[1 + \left(e^{\frac{h(f_m-f_i)}{kT}} - 1\right)^{-1}\right] \tag{5}$$

where $g_R(f_m - f_i)$ is the Raman gain coefficient between optical frequencies f_m and f_i, A_{eff} is the fiber effective cross

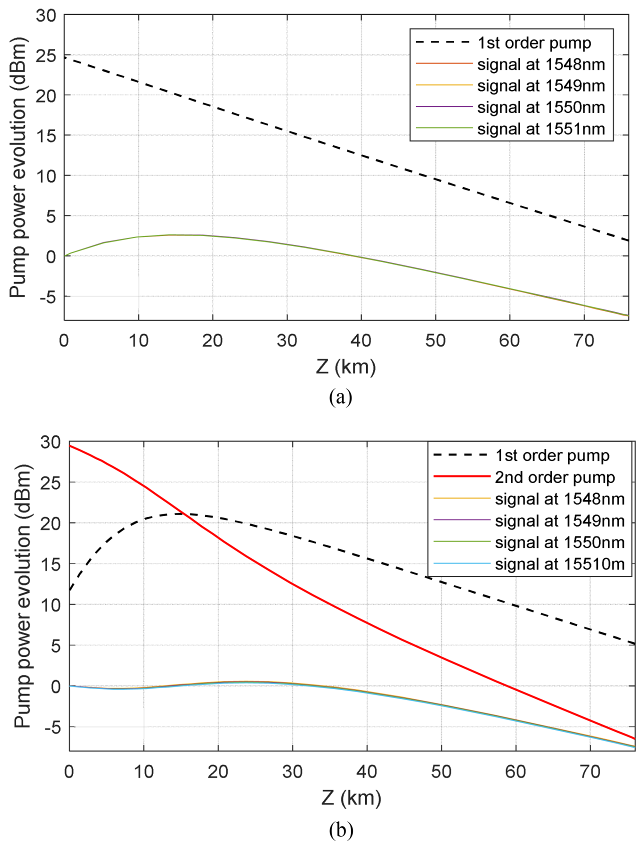

Fig. 2. Pump and signal power profiles along the fiber for DRA with only a 1st order FW pump (a) and with both 1st order and 2nd order forward pumps (b).

TABLE I
FIBER PARAMETERS USED

Fiber length	76 km
1st order pump (1450nm)	Attenuation $\alpha_1 = 0.295$ dB/km
2nd order pump (1365nm)	Attenuation $\alpha_2 = 0.36$ dB/km
Signal (1550nm)	Attenuation $\alpha_s = 0.21$ dB/km
λ of zero dispersion	$\lambda_0 = 1280$ nm
Dispersion slope	$S_0 = 0.0088$ ps/(km-nm^2)
Effective area	$A_{eff} = 80$ μm^2
Fiber nonlinear parameter	$\gamma_s = 1.1$ W^{-1}km^{-1}
Peak Raman gain	$g_{R,max} = 6.9 \times 10^{-14}$m/W
Pol. Randomization factor	$\Gamma = 1.38$ (in Eq.s (2-3))

Values are for dual-order pumped system with 9 dB raman gain.

section area, Γ is the polarization randomization effect with the value between 1 and 2, k is the Boltzmann's constant, and N is the number of wavelength channels including signal channels and Raman pumps. $A(z, f_i)$ represents Raman gain at signal frequency f_i caused by pump at frequency f_m along the fiber, $B(z, f_i)$ is the Raman loss, $C(z, f_i)$ represents the generation of spontaneous emission, and $D(z, f_i)$ is the Raman loss caused by broadband spontaneous emission. Steady state power evolution of optical signal and pumps can be calculated by numerically solving the steady state coupled wave (1)–(5) along the fiber. This allows to find power profiles of the optical signal $\bar{P}_s(z)$, and the Raman pumps $\bar{P}_1(z)$ and $\bar{P}_2(z)$ along the fiber, as well as the accumulated optical noise power spectral density (PSD).

Fig. 2 shows examples of calculated signal and pump powers along a 76 km standard single-mode fiber (SMF-28). Fiber parameters used in this paper for both experiment and modeling are listed in Table I. Raman gain profile of SMF is shown in the inset of Fig. 1.

In Fig. 2(a), only a 1st order FW pump is used at 1450 nm wavelength with 290 mW optical power. The pump power decreases exponentially along the fiber which amplifies 4 channels of optical signal in the 1550 nm wavelength window. Linear fiber loss at the signal wavelength is 16 dB, and the on/off Raman gain is 9 dB, so that the net loss of the optical signal is 7 dB over the system. Fig. 2(b) shows the power profiles of a dual pump system, where the 2nd order pump has 890 mW power and 1365 nm wavelength, and the 1st order pump has 14.7 mW power and 1450 nm wavelength. This also provides 9 dB on/off Raman gain for the signal channels. The power of the 2nd order pump decreases monotonically along the fiber which amplifies the 1st order pump. The 1st order pump reaches to its maximum power of about 21 dBm in the vicinity of $z \approx 15$ km in the fiber, where the slope of the 2nd order pump attenuation is the highest because the strong energy transfer to the 1st order pump. A comparison between Fig. 2(a) and (b) indicates that the maximum signal optical power is decreased by approximately 2 dB with dual-order pumping, which helps reduce system performance penalty associated with the nonlinear noise.

RIN transfer from the pump lasers to intensity noise and phase noise of the optical signal is an important issue, especially in DRA systems with high order pumps which require much higher power levels than only using a 1st order pump. Assuming that pump power fluctuation is much smaller than its average value, a small-signal model can be used to analyze the efficiency of RIN transfer. In this linear model, frequency-dependent modulation indices $m_1(\Omega, z)$ and $m_2(\Omega, z)$ are associated with the 1st order and the 2nd order pumps, respectively, so that,

$$P_1(z,t) = \overline{P_1}(z)\left[1 + m_1(\Omega, z)\exp(j\Omega t)\right] \quad (6)$$

$$P_2(z,t) = \overline{P_2}(z)\left[1 + m_2(\Omega, z)\exp(j\Omega t)\right] \quad (7)$$

where $\bar{P}_1(z)$ and $\bar{P}_2(z)$ are steady-state values of pump powers along the fiber as shown in Fig. 2. $m_1(\Omega, 0)$ and $m_2(\Omega, 0)$ are the indices of modulation applied on the two pump lasers at the fiber input, and the evolution of them along the fiber can be evaluated by the coupled-wave equations of $P_1(z,t)$ and $P_2(z,t)$ when they propagate in the same direction along the fiber,

$$\frac{\partial m_1(\Omega, z)}{\partial z} = j\Omega d_{s1} m_1(\Omega, z) + g_{12} m_2(\Omega, z)\overline{P_1}(z) \quad (8)$$

$$\frac{\partial m_2(\Omega, z)}{\partial z} = j\Omega d_{s2} m_2(\Omega, z) - g_{12} m_1(\Omega, z)\overline{P_2}(z) \quad (9)$$

where, g_{12} is the Raman gain coefficients between the 1st and the 2nd order pump, which is determined by the wavelength separation between the two pumps and the fiber material. In (8)–(9), the velocity of the optical signal v_s is used as the reference, so that $d_{s1} = 1/v_s - 1/v_1$ and $d_{s2} = 1/v_s - 1/v_2$ represent differential group delays of the 1st order and the 2nd order pumps with respect to the signal. Since the power profiles $\bar{P}_1(z)$ and $\bar{P}_2(z)$ need to be calculated numerically through (1)–(5), (8)–(9) also have to be numerically solved. This can

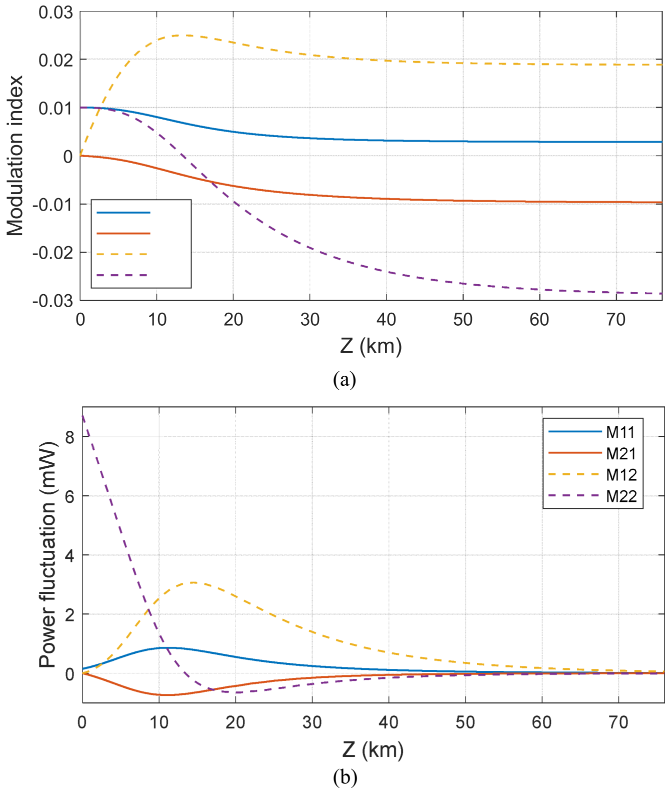

Fig. 3. Evolution of pump modulation indices (a) and pump power fluctuation (b) along 76 km SMF.

be accomplished by dividing the fiber into many short sections, and within each section of length Δz_n, power levels $\bar{P}_1(z_n)$ and $\bar{P}_2(z_n)$ can be regarded as constants. The evolution of $m_1(\Omega, z)$ and $m_2(\Omega, z)$ along the fiber can be calculated progressively from one section to the next until the end of the fiber.

$$\begin{bmatrix} m_1(\Omega, z_{n+1}) \\ m_2(\Omega, z_{n+1}) \end{bmatrix} = \begin{bmatrix} e^{j\Omega d_{s1}\Delta z_n} & g_{12}\overline{P_1}(z_n)\Delta z_n \\ -g_{12}\overline{P_2}(z_n)\Delta z_n & e^{j\Omega d_{s2}\Delta z_n} \end{bmatrix} \times \begin{bmatrix} m_1(\Omega, z_n) \\ m_2(\Omega, z_n) \end{bmatrix} \tag{10}$$

The bandwidth of coupling between m_1 and m_2 is determined by their differential walk off $d_{12} = d_{s1} - d_{s2} = 1/v_1 - 1/v_2$. As the RIN transfer efficiency is proportional to $1/[1 + (\Omega d_{12}/\alpha_2)^2]$ with α_2 the fiber loss at the 2nd order pump wavelength [5], a 3-dB bandwidth can be found as $\Omega_{3dB} = \alpha_2/d_{12}$. For SMF with a differential walk-off of approximately 700 ps/km between the 1st order pump at 1450 nm and 2nd order pump near 1350 nm, and $\alpha_2 = 0.36$ dB/km, the bandwidth of RIN transfer is about a few MHz [5], which is similar to that of RIN transfer from the 1450 nm 1st order pump to the 1550 nm optical signal.

The intensity modulation coupling between the 1st and the 2nd order pumps is through Raman gain and Raman loss. To further illustrate this mutual coupling mechanism, we use $m_{a,b}$ to represent the modulation index of pump a caused by the modulation of pump b, where $a, b = 1, 2$ represent 1st and 2nd order pumps. Fig. 3(a) shows the evolution of pump modulation indices $m_{a,b}$ along the fiber at low frequencies $\Omega \ll \Omega_{3dB}$. In this example, to show the impact of coupling between the two pumps, we applied a 1% intensity modulation on either the 1st order pump ($m_1(0) = 0.01$), or the 2nd order pump ($m_2(0) = 0.01$) to mimic the RIN of the pump lasers, and other conditions are the same as those used to create Fig. 2. The 1% modulation index was chosen because it is small enough not to cause changes in the average pump power profiles, and high enough to cause measurable crosstalk in the optical signal. Fig. 3(a) shows that when only the 1st order pump is modulated at the fiber input with $m_1(0) = 1\%$ and $m_2(0) = 0$, $m_{11}(z)$ decreases along the fiber and creates a modulation index $m_{21}(z)$ on the 2nd order pump through the stimulated Raman loss, so that $m_{21}(z)$ has an opposite sign compared to $m_{11}(z)$. Similarly, when only the 2nd order pump is modulated at the fiber input with $m_2(0) = 1\%$ and $m_1(0) = 0$, it introduces a modulation index $m_{12}(z)$ on the 1st order pump through stimulated Raman gain. Because of the high power of the 2nd order pump near the input of the fiber, $m_{12}(z)$ reaches its maximum near $z \approx 13$ km. Then the 1st order pump transfers its modulation back to the 2nd order pump $m_{22}(z)$ through stimulated Raman loss and flips its phase at $z \approx 13$ km.

While the modulation indices $m_{ab}(z)$ are normalized parameters, the actual power fluctuations defined by $M_{11}(z) = m_{11}(z)\bar{P}_1(z)$, $M_{12}(z) = m_{12}(z)\bar{P}_2(z)$, $M_{21}(z) = m_{21}(z)\bar{P}_1(z)$ and $M_{22}(z) = m_{22}(z)\bar{P}_2(z)$ are proportional to their steady state power levels as shown in Fig. 3(b). Because the 2nd order pump power is much higher than that of the 1st order pump near the input end of the fiber, M_{22} is also higher there than other terms. While the power fluctuation of the 2nd order pump does not directly generate the intensity noise on the optical signal, it can directly create phase noise of the optical signal through XPM.

Assuming that the optical signal is a continuous wave (CW) at the fiber input, the variation of its complex field along the fiber can be expressed as

$$A_s(\Omega, z) = \sqrt{P_s(\Omega, z)}e^{j\theta(\Omega, z)} \tag{11}$$

where, $P_s(\Omega, z) = |A_s(\Omega, z)|^2$ is the signal optical power and $\theta(\Omega, z)$ is the optical phase. The evolution of $A_s(\Omega, z)$ along the fiber can be evaluated by a linearized coupled-wave equation [7], [8],

$$\frac{\partial A_s(\Omega, z)}{\partial z} = \frac{A_s(\Omega, z)}{2} g_{1s}\overline{P_1}(z) m_1(\Omega, z) + j2\gamma_s\left[\overline{P_1}(z) m_1(\Omega, z) + \overline{P_2}(z) m_2(\Omega, z)\right] A_s(\Omega, z) \tag{12}$$

where, $m_1(\Omega, z)$ and $m_2(\Omega, z)$ are obtained from (10). $g_{2s} = 0$ is assumed because the 2nd order pump alone does not directly create Raman gain on the optical signal. The 2nd term on the right-hand-side of (12) represents the impact of pump RIN induced phase noises of the signal optical field, and the phase variations at the end of the fiber of length L is,

$$\theta(\Omega, L) = 2\gamma_s \int_0^L \left[\overline{P_1}(z) m_1(\Omega, z) + \overline{P_2}(z) m_2(\Omega, z)\right] dz \tag{13}$$

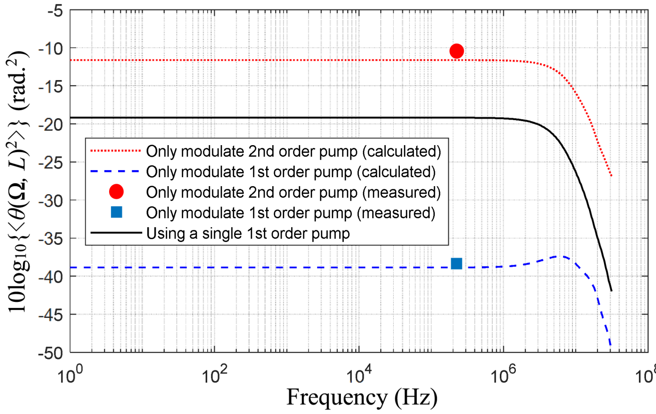

Fig. 4. Signal phase noise power spectra due to 1% intensity modulation of 1st order and 2nd order pump, respectively, also showing the data points obtained from experiment.

Fig. 4 shows the calculated phase noise variance $\langle\theta(\Omega, L)^2\rangle$ caused by 1% modulation of the 1st and the 2nd order pumps, respectively. The impact of 1st order pump RIN alone ($m_2\ (0) = 0$) on the signal phase noise is proportional to $\int_0^L [\ \bar{P}_1(z)m_{11}(\Omega, z) + \bar{P}_2(z)m_{21}(\Omega, z)]dz$, while the impact of 2nd order pump RIN along ($m_1\ (0) = 0$) is proportional to $\int_0^L [\ \bar{P}_1(z)m_{12}(\Omega, z) + \bar{P}_2(z)m_{22}(\Omega, z)]dz$. For comparison, the black solid curve in Fig. 4 shows phase noise variance in a system using only a 1st order pump to achieve the same 9 dB Raman gain and with 1% pump intensity modulation at frequency Ω.

All the spectra of phase noise variance shown in Fig. 4 have lowpass characteristics with the bandwidth of about a few *MHz* [5], [8]. The bandwidths of pump RIN to signal phase noises transfer through XPM is proportional to $\alpha_p/d_{s,p}$ with $p = 1,\ 2$ for 1st and 2nd order pumping, α_p the fiber loss and $d_{s,p}$ the walk-off between pump (p) and signal (s) [5]. Based on the parameters of SMF with dispersion represented by the Sellmeier Equation [7] and fiber losses at the 1st and 2nd order pump wavelengths, the ratio $(\alpha_1/d_{s,1})/(\alpha_2/d_{s,2})$ is only about 1.15. That is the reason why the solid and the dotted curves in Fig. 4 have very similar corner frequencies.

Fig. 4 indicates that in a system with dual-order pumps, the intensity noise of the 2nd order pump has about 27 dB higher impact than that of the 1st order pump. One reason is that the 2nd order pump has much higher power than the 1st order pump, especially near the input end of the fiber. Another reason is that the intensity noise of the 1st order pump creates an intensity noise on the 2nd order pump along the fiber but with a π phase shift with respect to the intensity noise on the 1st order pump, shown as M_{11} and M_{21} in Fig. 3(b), the partial cancelation between them further reduces the overall efficiency of 1st order pump RIN to signal phase noise transfer. In comparison, for a DRA system using only a single 1st order FW pump to achieve the same Raman gain, the efficiency of pump RIN to signal phase noise transfer is almost 9 dB lower than that of the 2nd order pump in the system based on dual-order pumping. Thus, a major limitation in dual-order FW pumped DRA system is the RIN level of the 2nd order pump laser.

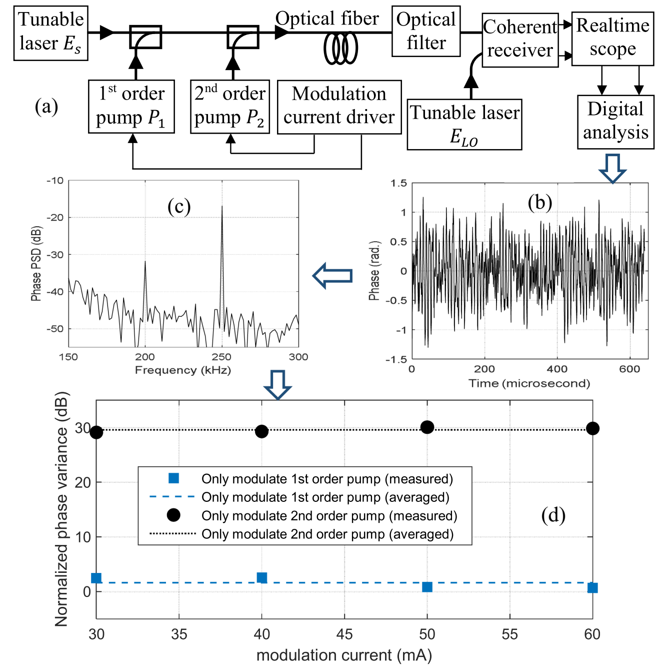

Fig. 5. (a) Experimental setup to measure the signal phase noise mean-square power caused by 1st and 2nd order pump intensity modulation. (b) Example of measured signal optical phase modulation, (c) Illustration of signal phase noise PSD, and (d) Normalized phase noise variance as the function of pump modulation current. Circles: when only 2nd order pump laser is modulated (circles) and only 1st order pump laser is modulated (squares).

III. Measurement of Pump RIN to Signal Phase Noise Transfer

Pump RIN to signal amplitude noise transfer has been analyzed and measured for the DRA systems with dual-order pumping [7], and the efficiency of RIN transfer from the 2nd order pump is shown to be more than 10 dB higher than that from the 1st order pump. Because the 2nd order pump alone does not introduce Raman gain on the optical signal ($g_{s2} = 0$), RIN transfer from the 2nd order pump to the signal intensity noise has to go through its impact in the 1st order pump.

However, the 2nd order pump RIN can be directly translated into signal phase noise through XPM without going through the 1st order pump, and this impact needs to be considered carefully. In order to verify the theoretical analysis presented in the previous section, we constructed an experimental setup as shown in Fig. 5. A tunable laser with < 10 kHz spectral linewidth is used to generate a CW optical carrier. Another identical tunable laser is used as the local oscillator (LO) which mixes with the signal optical carrier in a coherent optical receiver for complex optical field detection. The in-phase and quadrature components of the detected complex optical field are digitized and recorded by a real-time oscilloscope for offline digital processing to recover signal optical phase $\theta(\mathrm{t})$, as shown in Fig. 5(b). A diode laser and a high-power fiber laser are used as the 1st and the 2nd order pumps, respectively, to create DRA. Wavelengths and power levels of the pump lasers, and fiber parameters are the same as those used in the calculation to generate Figs. 2, 3, and 4. To mimic the RIN, a 1% intensity modulation, the same as that used in the calculation, is applied on each pump laser through the injection current (for the fiber laser, the modulation is on its pump laser). The maximum modulation frequency of < 300 kHz is primarily limited by the available current driver. The variance of optical signal phase at the pump modulation frequency Ω can be measured by the PSD of the phase noise after Fourier transform of $\theta(\mathrm{t})$, as illustrated by Fig. 5(c), where the 1st order and the 2nd order pump lasers were modulated by 250 kHz and 200 kHz sinewaves, respectively.

In the experiment, we first calibrated intensity modulation efficiencies of the 1st and the 2nd order pump lasers at their respective modulation frequencies. This was accomplished by sending the modulated pump into a DC coupled photodiode followed by an oscilloscope to measure the intensity modulated waveform $P_p(\mathrm{t}) = P_{p,ave}[1 + \xi \cdot \cos(\Omega \mathrm{t})]$ with $P_{p,ave}$ the average power, ξ the pump intensity modulation index, and Ω the modulation frequency. Then, the PSD of signal optical phase noise at the pump modulation frequencies are measured using setup shown in Fig. 5(a) and normalized by the modulation indices of the corresponding pumps. Fig. 5(d) shows the normalized variance of phase noise, defined as $\langle[\theta(\Omega, L)/m_{12}(\Omega, 0)]^2\rangle$, for 4 different modulation currents of each pump laser, where $m_{12}(\Omega, 0)$ represents the 1st order (1) and 2nd order (2) pump laser intensity modulation index at the fiber input with modulation frequency Ω. When the modulation is applied on the 2nd order pump laser, the normalized signal phase noise variance is approximately 27.5 dB higher than that with modulation applied on the 1st order pump laser, shown as the solid circles and squares in Fig. 4. This measurement validates the theoretical prediction that in a dual-order FW pumped DRA system the RIN impact of the 2nd order pump laser has much higher impact than that from the 1st order pump laser.

IV. Nonlinear Interference Induced Noise Analysis

In a DRA system, the major advantage of dual order pumping in the FW direction is to reduce signal optical power along the fiber and to reduce the impact of Kerr effect nonlinear interference (NLI). NLI in a DWDM optical system can be evaluated as an additive optical noise based on a Gaussian noise (GN) model, and the nonlinear noise PSD can be calculated through a 2-dimensional integration [13],

$$\rho_{NL}(f) = \frac{16}{27}\gamma_s^2 L_{s,eff}^2 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \rho_s(f_1)\rho_s(f_2)\rho_s(f_1+f_2-f) \times \eta(f_1,f_2,f)\, df_1 df_2 \tag{14}$$

with the mixing efficiency determined by,

$$\eta(f_1,f_2,f) = \left|\int_0^L p(z)\exp\left[j4\pi^2\beta_2(f_1-f)(f_2-f)z\right]dz\right|^2 \tag{15}$$

where, $L_{s,eff} = (1-e^{-\alpha_s L})/\alpha_s$ is the effective fiber length with α_s the fiber attenuation parameter at signal wavelength, and β_2 is the dispersion coefficient of fiber. $\rho_s(f)$ is the signal PSD, $p(z)$ is the normalized signal power profile along the fiber, and L is the fiber length, where $L > L_{s,eff}$ is assumed so that $1 - e^{-\alpha_s L} \approx 1$. In an optical system without DRA, signal optical power is attenuated along the fiber exponentially as $p(z) = e^{-\alpha_s z}$. Assuming each WDM channel has a flat-top optical spectrum with the baud rate equal to the channel spacing, a closed form expression can be derived for the nonlinear noise PSD,

$$\rho_{NL} = \frac{8P_s^3\gamma_s^2 L_{s,eff}^2\alpha_s}{27\pi|\beta_2| B_{ch}^3}\mathrm{asinh}\left(\frac{\pi^2}{2\alpha_s}|\beta_2| B_{ch}^2 N_{ch}^2\right) \tag{16}$$

where, P_s is the signal optical power at the fiber input, B_{ch} is the baud rate of each WDM channel which is equal to frequency spacing between adjacent channels, and N_{ch} is the total number of signal channels. In the calculation we assume the optical bandwidth of the WDM system is $B_{ch}\, N_{ch} = 12$ nm.

For an optical system with DRA, the signal optical power does not follow the simple exponential decay along the fiber, so that a closed-form analytical formula cannot be found. For the NLI analysis in a Raman system, the nonlinear noise power depends on the normalized signal power profile $p(z)$ along the fiber, which needs to be inserted into the integration in (15). A simple curve fitting for DRA system with a 1st order FW Raman pumping has been used in [1], where the normalized signal power profile was fitted by a 2-term exponential expansion $p(z) = b_1\exp(-\alpha_{s1}z) - b_2\exp(-\alpha_{s2}z)$. In this approach, each exponential term corresponds to an equivalent loss coefficient α_{si} and an effective length $L_{si,eff} = [1-\exp(-\alpha_{si}z)]/\alpha_{si}$ with $i = 1,\ 2$. This allows the use of simple analytic formula (16) by simply replacing $L_{s,eff}^2$ with $\left(\sum_{i=1}^{2} b_i L_{si,eff}\right)^2$, and the nonlinear noise PSD can be obtained by a superposition of all the three terms as,

$$\begin{aligned}\rho_{NL,\mathrm{Raman},1} &= \rho_{NL}\left[(b_1 L_{s1,eff})^2, \alpha_{s1}\right] \\ &+ \rho_{NL}\left[(b_2 L_{s2,eff})^2, \alpha_{s2}\right] \\ &+ \rho_{NL}\left[(b_1 b_2 L_{s2,eff} L_{s2,eff}), \sqrt{\alpha_{s2}\alpha_{s2}}\right]\end{aligned} \tag{17}$$

For a DRA system with both 1st and 2nd order FW pumps, the normalized signal power profile $p(z)$ shown in Fig. 2(b) cannot be accurately fitted by a 2-term expansion, so that we have adopted a 4-term exponential expansion to fit $p(z)$ in our analysis,

$$p(z) = b_1 e^{-\alpha_{s1}z} - b_2 e^{-\alpha_{s2}z} + b_3 e^{-\alpha_{s3}z} + b_4 e^{-\alpha_{s4}z} \tag{18}$$

where, $b_2 = b_1$, $b_3 = 1 - b_4$ are normalized for 0 dBm input signal power. The values of b_1, b_4, α_{s1}, α_{s2}, α_{s3} and α_{s4} are chosen to best fit the numerically calculated $p(z)$ profile from the solution of the propagation (1)–(5). In this case, $L_{s,eff}^2 = \sum_{i=1}^{10} b_i L_{EQ,i}^2$ has a total of 10 terms, and each term has a corresponding equivalent loss parameter α_{si} as listed in Table II. Then, the nonlinear noise PSD in the DRA system with dual-order FW pumps can be expressed as,

$$\rho_{NL,\mathrm{Raman},2} = \sum_{i=1}^{10}\rho_{NL}\left(L_{EQ,i}^2, \alpha_{si}\right) \tag{19}$$

Fig. 6(a) shows signal power profiles obtained from numerical simulation for DRA systems with 1st order and dual-order FW

TABLE II
EXPRESSIONS AND VALUES OF TERMS USED IN (19)

Terms	$L_{EQ,i}^2$ ($\times 10^4$ km^2)	α_{si} (dB/km)
Term-1	$(b_1 L_{s1,eff})^2 = 2.96$	$\alpha_{s1} = 0.21$
Term-2	$(b_2 L_{s2,eff})^2 = 0.14$	$\alpha_{s2} = 0.99$
Term-3	$(b_3 L_{s3,eff})^2 = 1.61$	$\alpha_{s3} = 0.73$
Term-4	$(b_4 L_{s4,eff})^2 = 4.12$	$\alpha_{s4} = 0.43$
Term-5	$-2\,(b_1 L_{s1,eff})(b_2 L_{s2,eff})$ = -1.32	$\sqrt{\alpha_{s2}\alpha_{s1}} = 0.46$
Term-6	$-2\,(b_2 L_{s2,eff})(b_3 L_{s3,eff})$ = -0.97	$\sqrt{\alpha_{s2}\alpha_{s3}} = 0.85$
Term-7	$2\,(b_3 L_{s3,eff})(b_4 L_{s4,eff})$ = - 5.16	$\sqrt{\alpha_{s3}\alpha_{s4}} = 0.56$
Term-8	$2(b_4 L_{s4,eff})(b_1 L_{s1,eff})$ =-6.98	$\sqrt{\alpha_{s4}\alpha_{s1}} = 0.30$
Term-9	$-2\,(b_2 L_{s2,eff})(b_4 L_{s4,eff})$ = 1.56	$\sqrt{\alpha_{s4}\alpha_{s2}} = 0.65$
Term-10	$2\,(b_1 L_{s1,eff})(b_3 L_{s3,eff})$ = 4.37	$\sqrt{\alpha_{s3}\alpha_{s1}} = 0.39$

Values are for dual order pumped system with 9db Raman gain.

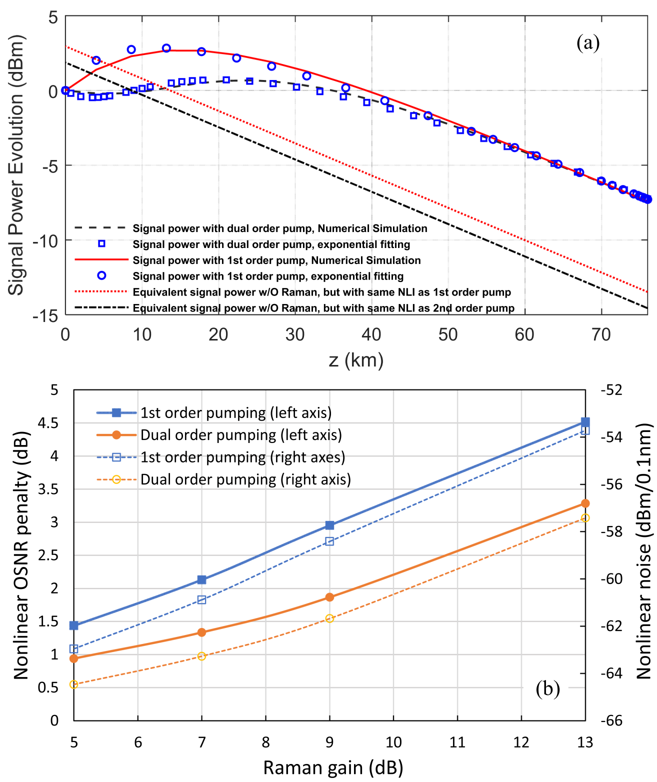

Fig. 6. (a) Solid and dashed lines: signal power profiles along fiber with 0 dBm at the input and 9 dB Raman gain for 1st order and dual order FW Raman pumping configurations. Dotted and dash-dotted straight lines: equivalent signal power profiles without Raman amplification but with the same nonlinear power as those with Raman amplifications. (b) Dashed lines: nonlinear noise PSD (right horizontal axis), solid lines: Raman induced nonlinear OSNR penalty as the function of Raman gain for the two FW pumping schemes.

pumping, respectively. Both systems have 9 dB on/off Raman gain and 0 dBm input signal power. Curve fittings with (18) are also shown in Fig. 6(a). Based on (16) and (19), and values of terms provided in Table II, the nonlinear noise can be calculated for each case. The straight lines in Fig. 6(a) show power profiles in the same fiber system without Raman amplification but with the same nonlinear noise as that with DRA. Fig. 6(a) indicates that using the 1st order FW Raman pumping with 0 dBm input signal optical power, the nonlinear noise is equivalent to the system without DRA but with 3 dBm input signal power. This is equivalent to a 3 dB increase of nonlinear OSNR penalty caused by the DRA with 1st order FW pumping at 9 dB Raman gain. For the DRA system employing dual-order pumping, the increase of nonlinear OSNR penalty is reduced to 1.9 dBm, which is 1.1 dBm better (lower) than that with only a 1st order pump.

Note that fitting parameters shown in Table II are specifically for the DRA system with 76 km SMF and 9 dB Raman gain. In general, for DRA systems with different Raman gains, the signal power profiles $p(z)$ will differ, and fitting parameters will also be different. Dashed lines in Fig. 6(b) (right vertical axis) show the nonlinear noise PSD, $\rho_{NL,\mathrm{Raman}}$, as the function of Raman gain. For the case of dual-order pumping with the Raman gain of up to 9 dB, 1st order pump power is fixed at 14.7 mW, and the Raman gain increases with the increase of 2nd order pump power. For the Raman gain of higher than 9 dB, the 1st order FW pump power is increased to limit the power of the 2nd order pump to 1 W. Nonlinear OSNR penalties caused by Raman amplification are shown as solid lines (left vertical axis) in Fig. 6(b), which were represented by $(\rho_{NL,\mathrm{Raman}}/\rho_{NL})^{1/3}$, where $\rho_{NL,\mathrm{Raman}}$ is the nonlinear noise PSD in a Raman system calculated from (17)–(19) and ρ_{NL} is that without Raman amplification calculated from (16). Although both $\rho_{NL,\mathrm{Raman}}$ and ρ_{NL} are proportional to the third power of input signal PSD, P_s^3/B_{ch}^3, Raman induced nonlinear OSNR penalty, $(\rho_{NL,\mathrm{Raman}}/\rho_{NL})^{1/3}$, is independent of signal optical power. For reference, dashed lines in Fig. 6(b)(right vertical axis) show nonlinear noise PSDs, ρ_{NL} and $\rho_{NL,\mathrm{Raman}}$, when the input signal PSD is -5 dBm/nm. The physical meaning of Raman induced nonlinear OSNR penalty is that for a system without Raman amplification, it would need an input signal power increase of $(\rho_{NL,\mathrm{Raman}}/\rho_{NL})^{1/3}$to generate the same nonlinear noise as the system with Raman amplification. Fig. 6(b) indicates that the system with dual-order Raman pumping produces less nonlinear noise compared to that using only 1st order pumping.

In a FW pumped DRA system with Raman gain partially compensating the fiber loss, overall OSNR improvement is determined by the tradeoff between the reduction of linear ASE noise and the increase of nonlinear noise compared to that only using EDFA as the amplifier. The linear ASE noise PSD, $\rho_{L,Raman}$, at the signal wavelength generated by Raman amplification can be calculated by numerically solving propagation (1)–(5) which depends on the Raman gain G_{Raman}. The ASE noise PSD generated by an EDFA is $\rho_{EDFA} = F \cdot h\upsilon(G_{EDFA} - 1)$, where F is the noise figure, $h\upsilon$ is the photon energy, and G_{EDFA} is the EDFA gain. Assuming the DRA is followed by the EDFA, the total linear ASE noise PSD is $\rho_{L,ASE} = \rho_{L,Raman} \cdot G_{EDFA} + \rho_{EDFA}$. Because FW pumped DRA has lower noise figure than that of an EDFA, $\rho_{EDFA} \gg \rho_{L,Raman}$ is valid in most practical systems, and OSNR improvement is only slightly less than the Raman gain G_{Raman}.

Fig. 7(a) shows the calculated linear OSNR improvement (only considering linear ASE noises produced by Raman and EDFA) as the function of Raman gain in the 76 km SMF system with attenuation parameter $\alpha_s = 0.21$ dB/km, and the total optical gain of $G_{Raman} + G_{EDFA} = 16$ dB which exactly compensates the fiber loss. Assume a 5 dB noise figure for the EDFA, at 9 dB Raman gain produced by a single 1st order

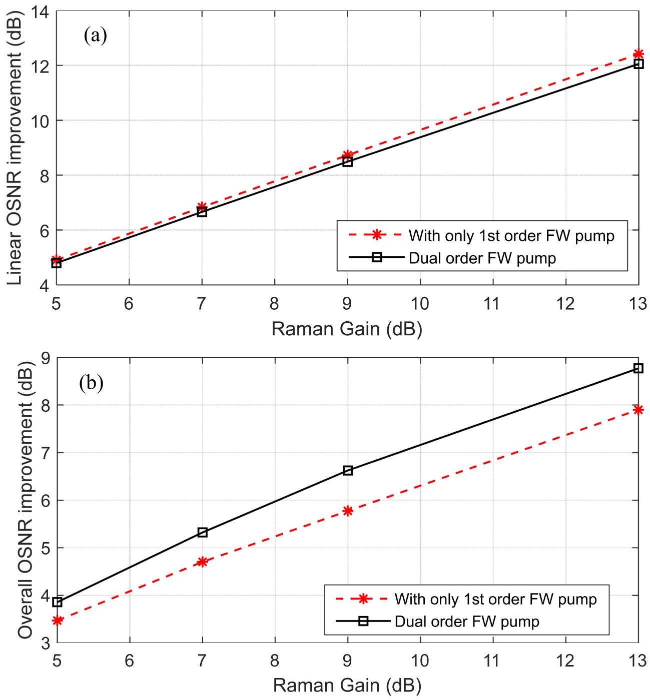

Fig. 7. (a) Linear OSNR improvement as the function of FW Raman gain, and (b) Net OSNR improvement as the function of FW Raman gain.

FW Raman pump, the linear OSNR improvement is 8.7 dB. Whereas, with dual order pumping also with 9 dB Raman gain, the linear OSNR improvement is reduced to 8.5 dB, which is 0.2 dB lower than that with 1st order pumping. This is because with the 2nd order FW pump the highest gain region, where most of ASE noise is generated, is pushed forward into the fiber link so that the ASE noise generated by Raman gain is less attenuated.

Combining the linear ASE noise $\rho_{L,ASE}$ and the nonlinear noise $\rho_{NL,\mathrm{Raman}}$ created by FW Raman amplification, Fig. 7(b) shows the net OSNR improvement as the function of Raman gain in the 76 km SMF system using 1st order and dual order FW Raman pumping schemes. At 9 dB Raman gain, for example, the OSNR is about 0.9 dB higher with dual order pumping than that using only a 1st order pump. Note that this OSNR improvement does not include the impact of pump RIN to signal phase noise conversion. In fact, the impact of pump RIN on the system performance depends on both the modulation format and the baud rate of the signal, which is discussed in the next section.

V. Pump RIN Induced OSNR Penalty for 16-QAM Signal

OSNR improvement due to the reduction of linear ASE noise and the increase of nonlinear noise has been discussed in the last section for a DRA system with FW Raman pumping. The introduction of dual order Raman pumping consisting of a weak 1st order pump and a strong 2nd pump is shown to be able to improve system OSNR compared to using a single 1st order pump at the same Raman gain. However, pump RIN to signal phase noise transfer can introduce additional system performance degradation, especially with the dual order pumping scheme where the power of the 2nd order pump is much higher than that of the 1st order pump. The impact of pump RIN on the system performance not only depends on the RIN levels of the pump lasers, but also depends on the modulation format and the baud rate of the signal. In this section, we investigate the performance of a coherent system with 16-QAM modulation to demonstrate the impact caused by the RIN of the 1st order and the 2nd order pumps. The equivalent OSNR degradation caused by pump RIN will then need to be subtracted from the OSNR improvement shown in Fig. 7(b).

Previous research indicated that for a coherent system, transmission performance is most susceptible to signal phase noise induced by pump RIN [8], we neglect the pump RIN induced signal intensity noise for simplicity. A decision aided maximum likelihood (DA-ML) phase estimation algorithm [14] can be used to derive the total phase error of QAM signals at a coherent optical receiver. Specifically, the phase error variance in the received signal constellation due to its optical carrier phase noise variance σ_{PN}^2 for a 16-QAM signal can be obtained from [5],

$$E\left[\Delta\theta^2\right] = \frac{2H^2 + 3.96H + 1.96}{3\left(H + 0.32\right)} \frac{\alpha_s}{d_{s,p}} T\sigma_{PN}^2 \qquad (20)$$

where, T and α_s are symbol time and fiber attenuation parameter, $d_{s,p} = 1/v_s - 1/v_p$ is the walk-off between the pump and the signal with group velocities of v_p and v_s, respectively. H is the averaging window length for carrier phase recovery. The phase noise variance, σ_{PN}^2, can be obtained by integrating the normalized phase noise PSD calculated from 13) over signal bandwidth for different pumping schemes. Also, for different pump RIN levels the integrated phase noise variance can be evaluated from the equation,

$$\sigma_{PN}^2 = \int_0^{2\pi/T} S_i\left(\Omega\right) \left|\theta\left(\Omega, L\right)\right|^2 d\Omega \qquad (21)$$

Where, $S_i(\Omega)$ is the PSD of pump laser RIN, and $\theta(\Omega, L)$ is the normalized pump RIN to signal phase noise transfer function defined by (13). With the presence of a phase error ($\sigma_{\Delta\theta}$) evaluated from (20), the corresponding bit error rate (BER) at the coherent receiver for a 16-QAM modulated signal can be calculated following (14)–(17) in [14]. Then, this BER penalty can be converted to an equivalent penalty of OSNR through [15],

$$\frac{E_S}{N_O} = 5\left\{Q^{-1}\left[\frac{16}{3}\left(1 - \sqrt{1 - BER}\right)\right]\right\}^2 \qquad (22)$$

where E_s is the average energy per symbol, N_o is the noise PSD, and Q represents the complementary error function.

Because OSNR versus BER relationship is not linear, we evaluate the equivalent OSNR penalty caused by pump RIN-induced signal phase noise near the HD-FEC threshold of $BER = 3.8 \times 10^{-3}$ at different baud rates. In the calculation, we have used an averaging window length of $H = 10\ T$ for carrier phase recovery, $d_{s,1} = 1.5$ ps/m and $d_{s,2} = 2.32$ ps/m for differential walk-offs between the signal and the 1st and the 2nd order pumps, respectively, and 9 dB Raman gain. Flat pump RIN spectra are assumed for simplicity with the levels of -120, -130, and -140 dB/Hz.

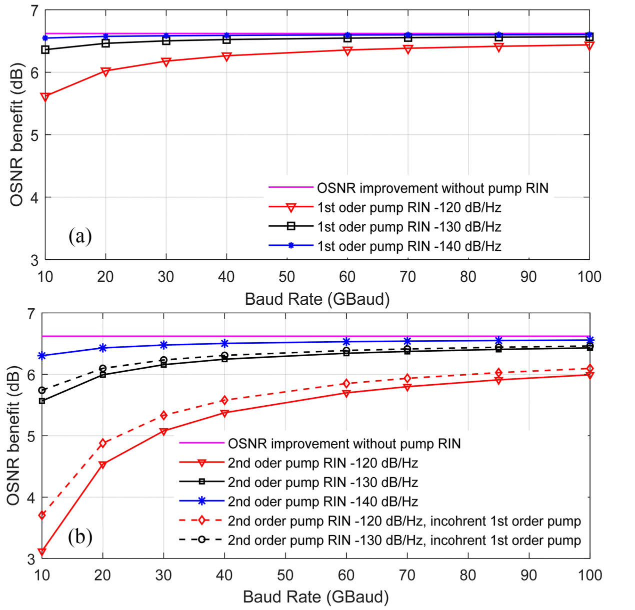

Fig. 8. Overall OSNR benefit of a dual-order Raman pumped system carrying 16-QAM signal with varying baud rates for (a) different 1st order pump RIN levels of −120, −130, and −140 dB/Hz, and ideal 2nd order pump, (b) different 2nd order pump RIN levels of −120, −130, and −140 dB/Hz, and ideal 1st order pump. Dashed lines show corresponding results when incoherent light source is used as the 1st order pump.

Fig. 8 summarizes the overall OSNR benefit as the function of signal baud rate in a system with dual-order pumping at 9 dB Raman gain, which includes OSNR improvement of 5.2 dB, shown in Fig. 7(b), and OSNR penalties due to pump RIN transfer. Because the pump RIN to signal phase noise transfer has lowpass characteristics with the bandwidth less than 10 MHz for FW pumped DRA, the impact of pump RIN transfer reduces with the increase of signal baud rate as it affects only a smaller portion of the signal spectrum. In Fig. 8(a), only 1st order pump RIN was considered and assuming the 2nd order pump laser was ideal without RIN. Whereas in Fig. 8(b), only 2nd order pump RIN was considered and assuming the 1st order pump has no RIN. Comparing Fig. 8(a) and (b), it is evident that 2nd order pump RIN has higher impact than that of the 1st order pump to degrade the overall OSNR benefit. Although 2nd order pump RIN cannot directly cause intensity noise in the optical signal, it is able to directly cause phase noise in the optical signal through XPM.

It has been shown that FW Raman pumping using incoherent pump sources with broad spectra can minimize the impact of pump RIN transfer [10], [11]. While power levels of incoherent light sources are usually not high enough to be used as the 2nd order pump, it can be used as the low power 1st order pump in a dual-order FW pumping scheme together with a high-power coherent 2nd order pump. In this configuration, the RIN of the 1st order pump, originated both by the RIN of the 1st order pump source itself and that translated from the 2nd order pump RIN (path 1 and path 3 described in the introduction), creates negligible phase noise on the signal optical carrier. However, the 2nd order pump RIN can still be directly translated into optical signal phase noise through XPM. Dashed lines in Fig. 8(b) show the overall OSNR benefits when the 1st order pump is incoherent, and only the 2nd order pump RIN to signal phase noise transfer through XPM is considered. Because the 2nd order pump has much higher power than that of the 1st order pump, replacing the 1st order pump from coherent laser to incoherent light source only slightly reduces the pump RIN induced OSNR penalty.

VI. Conclusion

We have systematically investigated the potential benefit of using dual-order FW pumping over that with only a 1st order FW pump in DRA systems by considering the impacts of linear ASE noise, nonlinear interference noise, and pump RIN to signal phase noise transfer. We showed both theoretically and experimentally that in the system with dual-order pumping, the 2nd order pump RIN to signal phase noise transfer efficiency can be more than 2 orders of magnitude higher than that from the 1st order pump. The major reason can be explained as follows: the 2nd order pump has much higher power than that of the 1st order pump. In addition to the direct transfer from 2nd order pump RIN to signal phase noise through XPM, the 2nd order pump RIN also creates intensity noise in the 1st order pump which can be, in turn, translated into signal phase noise. In a DWDM system employing DRA with FW pumping, the benefit of OSNR increase is partially offset by the increase of nonlinear noise. Example of our analysis showed that using dual order pumping, the OSNR improvement can be 0.9 dB higher than that of using only a 1st order pump at 9 dB on/off Raman gain. The impact of pump RIN transfer on transmission performance is investigated on a system with 16-QAM modulation. As pump RIN to signal phase noise transfer has lowpass characteristics, the pump RIN induced system OSNR penalty is inversely proportional to the baud rate. Because the RIN of the 2nd order pump has much higher impact than that of the 1st order pump, there is a need to set a more stringent requirement on the RIN of the 2nd order pump laser when dual order FW pumping scheme is used for DRA. Our results showed that even if an incoherent light source is used as the 1st order pump, the OSNR penalty induced by the RIN of the 2nd order pump is only reduced slightly. This paper is focused on systems with FW Raman pumping. If a system has both BW and FW Raman pumps, signal power profiles along the fiber will be different depending on the pump power levels. While general rules of FW pumping discussed in this paper still hold, system performance optimization will need to be performed based on specific system configurations.

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